

IE375 Fall 2020

Further Issues in Scheduling

Characteristics of the Job Shop Scheduling Problem

- Machine environment :
 - Number and Variety of Machines,
 - Flow Patterns
 - Processing Characteristics
- Sometimes decomposable
- Bottleneck machine
- Objective
- Complicated: Evaluation of Alternative Rules
 - Job Arrival Pattern
 - Number and skill level of workers

Static Stochastic Scheduling

- All jobs are available at time zero
- Processing times are uncertain
- Single machine scheduling
 - Objective is to minimize expected weighted flow time
 - Use Shortest Weighted Expected Processing Time
 - $E(t_{[1]})/u_{[1]} < E(t_{[2]})/u_{[2]} < \dots < E(t_{[n]})/u_{[n]}$
 - Objective is to minimize maximum over all jobs of the probability that a job is late
 - Use EDD if the due dates are deterministic
 - Use expected earliest due date if the due dates are random

Multiple machines stochastic case

- Processing times are exponential
 - Parallel machine scheduling
 - Processing times $t_1, t_2, \dots, t_n$ are exponential with rates $\mu_1, \mu_2, \dots, \mu_n$. Expected processing times are $1/\mu_1, 1/\mu_2, \dots, 1/\mu_n$.
 - List schedule with longest expected processing time (LEPT) minimizes makespan
- (Remember LPT list schedule is not always optimal in the deterministic case)

Multiple machines stochastic case

- Two machine flow shop
 - Deterministic case job j precedes job $j+1$ if
$$\min(A_j, B_{j+1}) < \min(A_{j+1}, B_j) \text{ (Check why)}$$
- Stochastic case: Processing times on machine 1: $A_1, A_2, \dots, A_n$ and processing times on machine 2: $B_1, B_2, \dots, B_n$ are exponential with rates $a_1, a_2, \dots, a_n$ and $b_1, b_2, \dots, b_n$
- $E[\min(A_j, B_{j+1})] = 1/(a_j + b_{j+1})$ $E[\min(A_{j+1}, B_j)] = 1/(a_{j+1} + b_j)$
- Job j precedes job $j+1$ if
$$1/(a_j + b_{j+1}) < 1/(a_{j+1} + b_j), \text{ or } a_j - b_j > a_{j+1} - b_{j+1}$$

Stochastic flow shop example

- What is the optimal sequence of jobs on both machines if the expected processing times are given as below?

Job	Expected Times	
	A	B
1	5	2
2	1	6
3	9	7
4	3	8
5	10	4

Dynamic Stochastic Scheduling

- Jobs are arriving randomly over time
- Queuing theory is used to analyze the system
- Wait time in the queuing system is equivalent to flow time in scheduling terminology
- If there is a single machine, processing times and inter-arrival times are exponential, it is an M/M/1/FCFS queue.
 - Processing rate is μ and arrival rate is λ , $\rho = \lambda/\mu$
 - $\Pr \{\text{number of jobs in system} = i\} = \rho^i(1-\rho)$
 - Expected waiting time = $E[W] = 1/(\mu - \lambda)$
 - Variance of the waiting time = $\text{Var}[W] = 1/(\mu - \lambda)^2$

Stochastic Scheduling: Dynamic Analysis

- Means of analyzing the results
 - M/M/1, ..., G/GI/1 queues: random arrivals to a single machine with random processing times.
 - **Little's Law** : $L = \lambda W$
- If the selection discipline does not depend on the flow times, the mean flow times are the same, but the variance of the flow times will differ.
- If job times are realized when the job joins the queue rather than when the job enters service, SPT generally results in lowest expected flow time.

Objectives in Job Shop Scheduling

- Meet due dates
 - Minimize work-in-process (WIP) inventory
 - Minimize average flow time
 - Maximize machine/worker utilization
 - Reduce set-up times for changeovers
 - Minimize direct production and labor costs
- (note: that these objectives can be conflicting)

Scheduling Problems in Operations

- Job Shop Scheduling
- Assembly Line Balancing
 - (6th Edition - 8.10 or 7th Edition –9.10)
- Project Scheduling
 - (6th Edition - Chapter 9 or 7th Edition – Chapter 10)
- Dynamic versus Static Scheduling
- Personnel Scheduling
- Vehicle Scheduling and Routing
- Scheduling Courses, Exams

Use of simulation

- A reasonable way of evaluating a few policies
- A reasonable way of showing your policy versus current
- Not straightforward!
 - Detail level
 - Comparison metrics
 - You need to have an approach to simulate
 - Analysis of output

How to approach a Real Machine Scheduling Problem?

- Analyze and deal only with the “significant” part
 - Bottleneck Scheduling
- Set measures that are important
- Check whether there is a reported “optimal” approach for any of the measures
- If none – heuristic – use of known results to come up with an intuitive approach (approaches)
 - Check special cases – should work
 - Approaches to improvement of the solution
 - Test your approach – artificial data – simulation - ...