IE 400: Principles of Engineering Management

Set 1

What is Operations Research (OR)?

OR is a **decision making tool** based on mathematical modeling and analysis to aid in determining **how best to design and operate a system** usually under the conditions requiring **allocation of scarce resources**.

Brief History of OR

- The main origin of Operations Research was during the **Second World War** in Britain.
- At the time of World War II, the military management in England invited a team of scientists to study the **strategic** and **tactical problems** related to air and land defense of the country.

Brief History of OR

- At that time the resources available with England was very limited and the objective was to win the war with available <u>scarce resources</u>.

- The resources such as food, medicines, ammunition, manpower etc., were required to manage war and for the use of the population of the country.

Brief History of OR

- It was necessary to decide upon the most effective utilization of the available resources to achieve the objective.
- It was also necessary to utilize the military resources cautiously.

Brief History of OR

- British military leaders asked an interdisciplinary group of scientists, engineers, psychologists, and sociologists to analyze some problems arising in military operations and find the optimum allocation of limited resources.

Brief History of OR

- As the name indicates,

  Operations is used to refer to the problems of military,

  Research is use for inventing new method.

- As this method of solving the problem was invented during the war period, the subject is given the name **‘OPERATIONS RESEARCH’** and abbreviated as **‘O.R.’**

Brief History of OR

- After the World War II, there was a scarcity of industrial material and industrial productivity reached the lowest level.
- Soon after, it was discovered that OR techniques could also be applicable in various different settings.
- Today, OR techniques are heavily used to solve industrial, civil and military problems.

Three Main Properties of OR

- Systems Approach
- Interdisciplinary Action Based
- Scientific Methodology

Some Applications of OR

- **Manufacturing** (facility layout, machine scheduling, workforce scheduling, cost minimization, profit maximization, inventory allocation, etc.)
- **Finance** (portfolio optimization, risk minimization, etc.)
- **Statistics** (data fitting, error minimization, etc.)

Some Applications of OR

- **Networks** (design problems, shortest path, routing, etc.)
  - Transportation
  - Communication/computer
  - Physical (oil, gas, etc.)
  - Supply chain

OR Delivers Significant Value

Today, organizations and the world in which they operate continue to become more complex. Organizations worldwide in business, the military, health care, and the public sector are realizing powerful benefits from O.R., including:

Business insight: Providing quantitative and business insight into complex problems.

OR Delivers Significant Value

Business performance: Improving business performance by embedding model-driven intelligence into an organization's information systems to improve decision making.

Cost reduction: Finding new opportunities to decrease cost or investment.

Forecasting: Providing a better basis for more accurate forecasting and planning

OR Delivers Significant Value

Improved scheduling: Efficiently scheduling staff, equipment, events, and more.

Pricing: Dynamically pricing products and services.

Productivity: Helping organizations find ways to make processes and people more productive.

Profits: Increasing revenue or return on investment; increasing market share.

OR Delivers Significant Value

Quality: Improving quality as well as quantifying and balancing qualitative considerations.

Resources: Gaining greater utilization from limited equipment, facilities, money, and personnel.

Risk: Measuring risk quantitatively and uncovering factors critical to managing and reducing risk.

Throughput: Increasing speed or throughput and decreasing delays.

OR Modeling

- In spite of the fundamental differences among several settings, many problems share similar characteristics.

  Ex:

  - A physical network vs. a computer network.
  - Parallel computing vs. queuing systems in a bank (scheduling, queuing).
  - Air traffic control vs. local area network (routing).

OR Modeling

- It is the OR modeler's responsibility to observe and understand the inner workings of a system (usually in conjunction with the experts in the setting in which the problem arises).

- **Model:** A structure which has been built purposely to exhibit features and characteristics of some other object.

  Ex: Road maps, model airplanes, simulation model,
  mathematical model, etc.

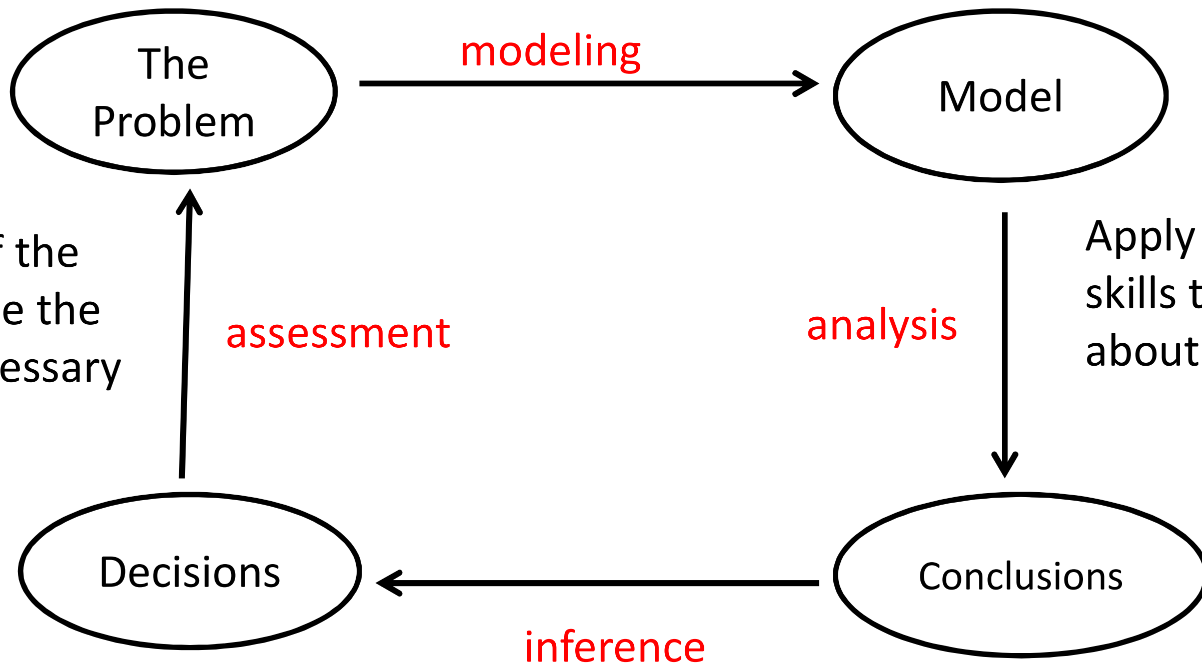
The **essential feature of a mathematical model** in OR is that it involves a set of mathematical relationships (such as equations, inequalities, logical dependencies, etc.) which correspond to the characterization of the relationships in the real world.

Why do we need mathematical modeling?

- To have a greater understanding of the object being modeled.
- To analyze it mathematically to help suggest courses that are not apparent otherwise.
- To simplify the inner workings of a complex system and to use the universally understandable language of algebra and math.

The Modeling Process

Define variables and mathematical relationships to represent the system behavior



Apply mathematical skills to make conclusions about the model

Translate the mathematical conclusions into the specific setting of the problem

Validation of the results, revise the model if necessary

MODELING EXAMPLES

The Diet Problem

	Food	Energy (kcal)	Protein (g)	Calcium (mg)	Price/serving (cents)
1	**Chicken**	205	32	12	24
2	**Egg**	160	13	54	13
3	**Whole milk**	160	8	285	9
4	**Cherry Pie**	420	4	22	20
5	**Stuffed Turkey**	260	14	80	19

The Diet Problem

Daily Nutrition Requirements:

- 2000 kcal energy
- 55 g protein
- 800 mg calcium

Balanced Diet Requirements:

# of servings of:

- Chicken: ≥ 1 and ≤3
- Eggs: ≥ 1 and ≤2
- Milk: ≥ 2 and ≤4
- Cherry pie: ≥ 0 and ≤2
- Stuffed Turkey: ≥ 0 and ≤2

Design a diet with minimum total cost to ensure that daily nutrition and balance requirements are met.

The Diet Problem

First, we need to determine the # of servings of each type of food.

Let $\mathbf{X_i}$ denote the (unknown) # of servings of food type i, i=1,2,3,4,5. These are the "decision variables"

The Diet Problem

	Food	Energy (kcal)	Protein (g)	Calcium (mg)	Price/serving (cents)
1	**Chicken**	205	32	12	24
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Objective: minimize total cost

Total Cost= $24\ X_1 + 13\ X_2 + 9\ X_3 + 20\ X_4 + 19\ X_5$

The Diet Problem

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1	**Chicken**	205	32	12	24
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Daily Nutrition Requirements:

Energy: $205\,X_1 + 160\,X_2 + 160\,X_3 + 420\,X_4 + 260\,X_5$

needs to be at least 2000 kcal

The Diet Problem

	Food	Energy (kcal)	Protein (g)	Calcium (mg)	Price/serving (cents)
1	**Chicken**	205	32	12	24
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Daily Nutrition Requirements:

Protein: $32 X_1 + 13X_2 + 8 X_3 + 4 X_4 + 14 X_5$

needs to be at least 55 g

The Diet Problem

	Food	Energy (kcal)	Protein (g)	Calcium (mg)	Price/serving (cents)
1	**Chicken**	205	32	12	24
2	**Egg**	160	13	54	13
3	**Whole milk**	160	8	285	9
4	**Cherry Pie**	420	4	22	20
5	**Stuffed Turkey**	260	14	80	19

Daily Nutrition Requirements:

Calcium: $12 X_1 + 54X_2 + 285 X_3 + 22 X_4 + 80 X_5$

needs to be at least 800 mg

The Diet Problem

Balanced Diet Requirements:

# of servings of:			
• Chicken:	≥ 1 and ≤ 3	$X_1 \geq 1$,	$X_1 \leq 3$,
• Eggs:	≥ 1 and ≤ 2	$X_2 \geq 1$,	$X_2 \leq 2$,
• Milk:	≥ 2 and ≤ 4	$X_3 \geq 2$,	$X_3 \leq 4$,
• Cherry pie:	≥ 0 and ≤ 2	$X_4 \geq 0$,	$X_4 \leq 2$,
• Stuffed Turkey:	≥ 0 and ≤ 2	$X_5 \geq 0$,	$X_5 \leq 2$.

The Diet Problem

minimize (min) $24\,X_1 + 13\,X_2 + 9\,X_3 + 20\,X_4 + 19\,X_5$

subject to (s.t.)

$$205\,X_1 + 160\,X_2 + 160\,X_3 + 420\,X_4 + 260\,X_5 \geq 2000$$

$$32\,X_1 + 13\,X_2 + 8\,X_3 + 4\,X_4 + 14\,X_5 \geq 55$$

$$12\,X_1 + 54\,X_2 + 285\,X_3 + 22\,X_4 + 80\,X_5 \geq 800$$

$X_1 \geq 1$, $X_1 \leq 3$,
$X_2 \geq 1$, $X_2 \leq 2$,
$X_3 \geq 2$, $X_3 \leq 4$,
$X_4 \geq 0$, $X_4 \leq 2$,
$X_5 \geq 0$, $X_5 \leq 2$.

The Diet Problem

minimize (min) $24 X_1 + 13 X_2 + 9 X_3 + 20 X_4 + 19 X_5$

subject to (s.t.)

constraints

$$
\left\{
\begin{aligned}
&205 X_1 + 160 X_2 + 160 X_3 + 420 X_4 + 260 X_5 \geq 2000 \\
&32 X_1 + 13 X_2 + 8 X_3 + 4 X_4 + 14 X_5 \geq 55 \\
&12 X_1 + 54 X_2 + 285 X_3 + 22 X_4 + 80 X_5 \geq 800 \\
&X_1 \geq 1, \quad X_1 \leq 3, \\
&X_2 \geq 1, \quad X_2 \leq 2, \\
&X_3 \geq 2, \quad X_3 \leq 4, \\
&X_4 \geq 0, \quad X_4 \leq 2, \\
&X_5 \geq 0, \quad X_5 \leq 2.
\end{aligned}
\right.
$$

The Diet Problem

This problem is an example of a **Linear Programming (LP)** model.

1) The function to be minimized (or maximized) is called the objective function.

2) X_1, X_2 ,…,X_5 are the decision variables whose best values need to be determined.

3) Constraints are the relationships that should be satisfied by every diet that fulfills the nutrition and balance requirements.

The Diet Problem

4) Any values of X_1, X_2 ,…,X_5 that satisfy all of the constraints simultaneously is called a feasible solution.

5) The set of all feasible solutions is called the feasible set or the feasible region.

The Diet Problem

4) Any values of X_1, X_2 ,…,X_5 that satisfy all of the constraints simultaneously is called a feasible solution.

5) The set of all feasible solutions is called the feasible set or the feasible region.

6) The objective is to find the feasible solution that yields the best objective function value.

   If there exists such a feasible solution, it is called an optimal solution.

   The objective function value evaluated at an optimal solution is called the optimal value.

The Linear Programming (LP) Problem

An LP problem is an optimization problem in which

- The objective function is given by a linear combination of the decision variables.

  i.e. min (or max) $c_1x_1 + c_2x_2 + \ldots + c_nx_n$,

  where $c_1,\ldots,c_n \in R$ are given and $x_1,\ldots,x_n$ are the decision variables.

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  where $c_1,\ldots,c_n \in R$ are given and $x_1,\ldots,x_n$ are the decision variables.

- The constraints are linear inequalities or linear equations.

$$\text{i.e. } a_1x_1 + a_2x_2 + \cdots + a_nx_n \begin{Bmatrix} \le \\ = \\ \ge \end{Bmatrix} b, \quad \text{where } a_1,\ldots,a_n \in R,\ b \in R,$$

and $x_1,\ldots,x_n$ are the decision variables.

General Form of an LP Problem

$$\max\ (\min)\quad c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$$

s.t.

$$a_{i1} x_1 + a_{i2} x_2 + \cdots + a_{in} x_n \quad \le b_i, \quad i = 1, \dots, l$$

$$a_{j1} x_1 + a_{j2} x_2 + \cdots + a_{jn} x_n \ge b_j, \quad j = l+1, \dots, l+p$$

$$a_{k1} x_1 + a_{k2} x_2 + \cdots + a_{kn} x_n = b_k, \quad k = l+p+1, \dots, l+k+q$$

The sign restrictions, i.e. $\begin{pmatrix} x_1, \dots, x_k \ge 0, \\ x_{k+1}, \dots, x_m \le 0, \\ x_{m+1}, \dots, x_n \text{ urs} \end{pmatrix}$

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l "≤" constraints

$$a_{j1}x_1 + a_{j2}x_2 + \cdots + a_{jn}x_n \ge b_j, \quad j = l+1,\ldots,l+p$$

p "≥" constraints

$$a_{k1}x_1 + a_{k2}x_2 + \cdots + a_{kn}x_n = b_k, \quad k = l+p+1,\ldots,l+k+q$$

q "=" constraints

The sign restrictions, i.e. $\begin{pmatrix} x_1,\ldots,x_k \ge 0, \\ x_{k+1},\ldots,x_m \le 0, \\ x_{m+1},\ldots,x_n \text{ urs} \end{pmatrix}$

c_j's, a_{ij}'s, b_j's ∈ R and given. They are the parameters.

Assumptions of an LP Problem

1) **Proportionality & Additivity:**

The contribution of each decision variable to the objective function or to each constraint is proportional to the value of that variable.

The contribution from decision variables are independent from one another and the total contribution is the sum of the individual contributions.

Assumption of an LP Problem

2) Divisibility:

Each decision variable is allowed to take fractional values.

Assumption of an LP Problem

2) Divisibility:

Each decision variable is allowed to take fractional values.

3) Certainty:

All input parameters (coefficients of the objective function and of the constraints) are known with certainty.

Transportation Problem

There are two power plants generating electricity, and three factories that need electricity in their production.

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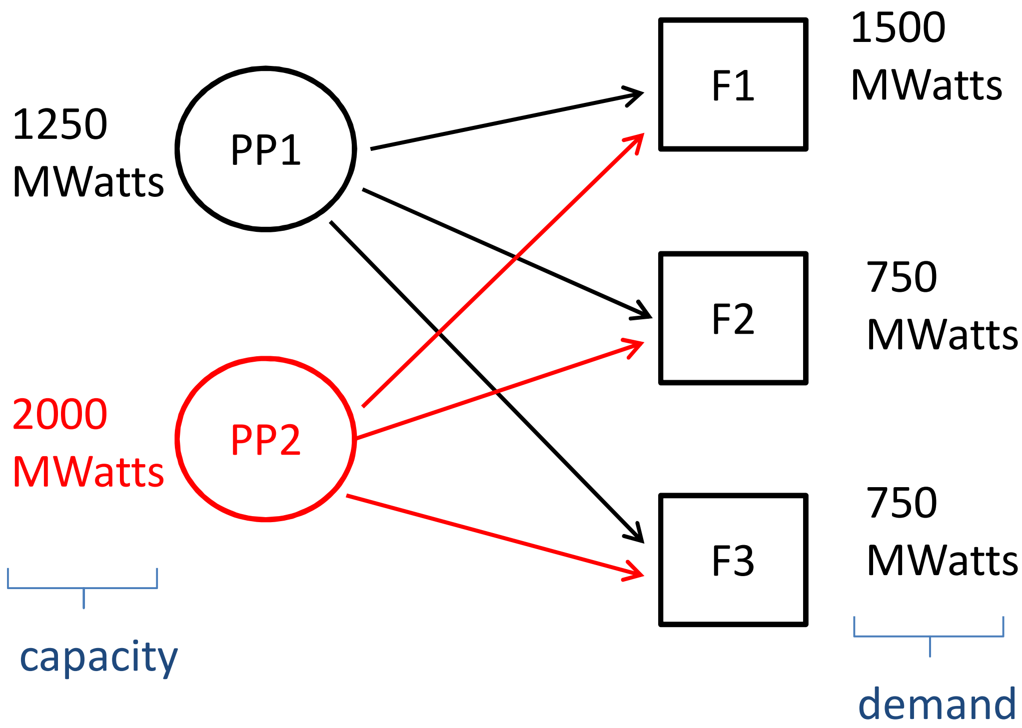

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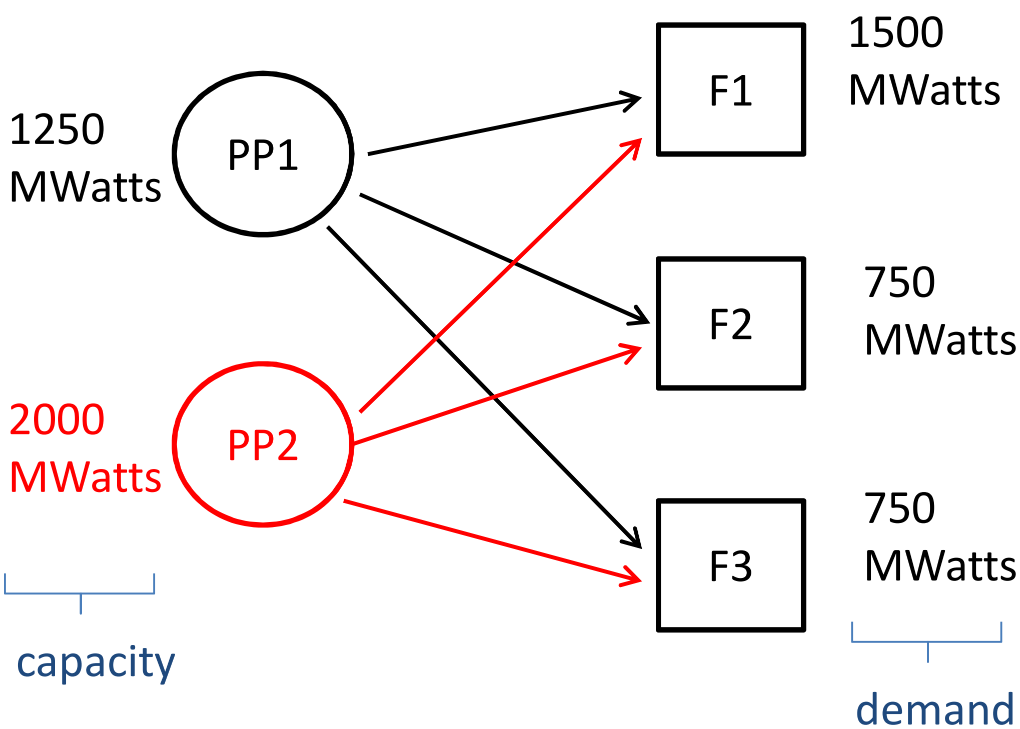

Transmission costs per MWatt (in USD):

	F1	F2	F3
PP1	50	100	60
PP2	30	20	35

Transportation Problem

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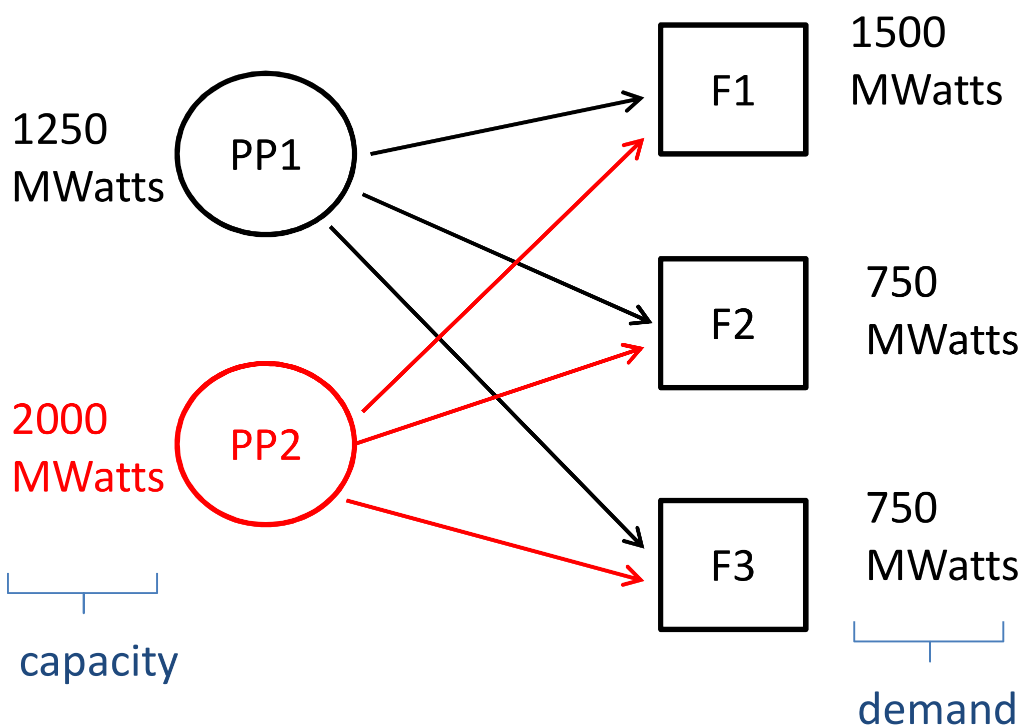

Transmission costs per MWatt (in USD):

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PP1	50	100	60
PP2	30	20	35

How to satisfy the demand of each factory from the two power plants with minimum total cost?

Short Term Investment Planning

You have 12,500 TL at the beginning of year 2008 and would like to invest your money. Your goal is to maximize your total amount of money at the beginning of year 2011. You have the following options:

Short Term Investment Planning

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Option	Duration (yrs)	Total interest rate	Available at the beginning of
1	2	26%	2008,2009
2	1	12%	2008,2009,2010
3	3	38%	2008
4	2	27%	2009

Short Term Investment Planning

Additionally,

- You can invest at most 5,000 TL in any option at a time,
- You can invest at most 4,000 TL in option 4.

Short Term Investment Planning

Note that, normally:

Cash available at time t = cash invested at time t

+

uninvested cash at time t that

is carried over to time t+1

However, since Option 2 is available all the time, we can always invest in one of the options rather than not investing some portion of the money.

Short Term Investment Planning

Hence, for this problem, the following holds:

Cash available at time t = cash invested at time t

Short Term Investment Planning

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Blending Problem

The oil company produces three types of gasoline by blending three types of crude oil.

Gas	Sales Price per Barrel (USD)	Crude Oil	Sales Price per Barrel (USD)
1	70	1	45
2	60	2	35
3	50	3	25

Blending Problem

- The company can purchase up to 5,000 barrels of each type of crude oil daily.
- It costs $4 to transform one barrel of crude oil into one barrel of gasoline.
- The company can process up to 14,000 barrels of crude oil daily.

Crude Oil Specifications:

Crude Oil	Octane Rating	Sulfur Content
1	12	0.5 %
2	6	2.0 %
3	8	3.0 %

Blending Problem

Production Requirements:

Gas	Octane Rating	Sulfur Content
1	at least 10	at most 1%
2	at least 8	at most 2%
3	at least 6	at most 1%

The company has to meet the daily demand:

Gas	Demand (barrels)
1	4,000
2	5,000
3	3,000

Blending Problem

How can the company maximize total profit while satisfying daily demand and production requirements?

(assume octane levels and sulfur contents of different crudes blend linearly)

Production Problem

- Milk chocolate is produced with three ingredients.
  1 kg of milk chocolate contains:
  - 0.5 kg of milk,
  - 0.4 kg of cocoa,
  - 0.1 kg of sweetener.

- Each of these three ingredients must be processed before they can be used in the production of milk chocolate.

Production Problem

- The factory has two departments that can process these ingredients:

	Dept 1 (hrs/kg)	Dept 2 (hrs/kg)
Milk	0.4	0.6
Cocoa	0.3	0.2
Sweetener	0.5	0.6

- Both departments have 150 hours of available time per week for processing.

Production Problem

- The factory has two departments that can process these ingredients:

	Dept 1 (hrs/kg)	Dept 2 (hrs/kg)
Milk	0.4	0.6
Cocoa	0.3	0.2
Sweetener	0.5	0.6

- Both departments have 150 hours of available time per week for processing.

Formulate an LP problem that maximizes the total amount of milk chocolate produced in a week.

Error Minimization Problem

- You estimate that there is a linear relationship between the stock price of a beverage company and the # of beverages sold (in thousands) in a day:

Day	Stock Price	# of beverages sold
1	2	10
2	3	12
3	2	8
4	3	11
5	4	14

Error Minimization Problem

- You predict the relationship as $\mathbf{s_i=an_i+b}$, where s_i is the stock price and n_i is the # of beverages sold (in thousands) on day i, i=1,…,5.

  For a given choice of a and b, your estimation error on day i is $|\mathbf{s_i-an_i-b}|$, i=1,…,5.

  Choose the unknown coefficients a and b such that the largest estimation error in any one of the 5 days is as small as possible.

Production Planning Problem

XYZ Company produces three products A, B, and C and has to meet the demand for the next four weeks.

	Weekly Demand			
Product	**1**	**2**	**3**	**4**
A	25	30	15	22
B	15	16	25	18
C	10	17	12	12

Production Planning Problem

XYZ Company produces three products A, B, and C and has to meet the demand for the next four weeks.

	Weekly Demand			
Product	**1**	**2**	**3**	**4**
A	25	30	15	22
B	15	16	25	18
C	10	17	12	12

Each unit of:

Product A requires 3 hours of labor,

Product B “ 2 hours “ ,

Product C “ 4 hours “ .

Production Planning Problem

- During a week, 120 hours of regular-time labor is available at a cost of \$10/hour.
- In addition 25 hours of overtime labor is available weekly at a cost of \$15/hour.
- **No shortages are allowed** and XYZ Company has 18 A, 13 B, and 15 C initial stocks for these products.
- For simplicity, assume that items produced during a week are available to meet the demand for that week.

Production Planning Problem

Provide a Linear Programming formulation to determine a production schedule that minimizes the total labor costs for the next four weeks.

Production Planning Problem

Variables:

Production Planning Problem

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X_{ij}: the amount of product i produced during week j,
$i \in \{A,B,C\}$ and $j=1,2,3,4$.

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NI_{ij}: the amount of inventory of product i at the end of week j after meeting demand. $i \in \{A,B,C\}$ and $j=0,1,2,3,4$ (in this case, NI_{i0} is the notation for the initial stock)

Production Planning Problem

Variables:

X_{ij}: the amount of product i produced during week j, $i \in \{A,B,C\}$ and $j=1,2,3,4$.

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OT_j: the amount of overtime hours used in week j, $j=1,2,3,4$.

Production Planning Problem

Production Constraints:

Production Planning Problem

Production Constraints:

For any product i and any week j,

$$NI_{i,j-1} + X_{i,j} - D_{i,j} = NI_{i,j}$$

Production Planning Problem

Production Constraints:

For any product i and any week j,

$$NI_{i,j-1} + X_{i,j} - D_{i,j} = NI_{i,j}$$

For Product A:

$$18 + X_{A,1} - 25 = NI_{A,1}$$
$$NI_{A,1} + X_{A,2} - 30 = NI_{A,2}$$
$$NI_{A,2} + X_{A,3} - 25 = NI_{A,3}$$
$$NI_{A,3} + X_{A,4} - 22 = NI_{A,4}$$

Production Planning Problem

Production Constraints:

For Product B:

$$13 + X_{B,1} - 15 = NI_{B,1}$$
$$NI_{B,1} + X_{B,2} - 16 = NI_{B,2}$$
$$NI_{B,2} + X_{B,3} - 25 = NI_{B,3}$$
$$NI_{B,3} + X_{B,4} - 18 = NI_{B,4}$$

For Product C:

$$15 + X_{C,1} - 10 = NI_{C,1}$$
$$NI_{C,1} + X_{C,2} - 17 = NI_{C,2}$$
$$NI_{C,2} + X_{C,3} - 12 = NI_{C,3}$$
$$NI_{C,3} + X_{C,4} - 12 = NI_{C,4}$$

Production Planning Problem

Demand is met on time constraint:

Production Planning Problem

Demand is met on time constraint:

$NI_{i,j} \geq 0$ for $i \in \{A,B,C\}$ and $j=1,2,3,4$.

Production Planning Problem

Demand is met on time constraint:

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Labor Constraints:

Production Planning Problem

Demand is met on time constraint:

$NI_{i,j} \geq 0$ for $i \in \{A,B,C\}$ and $j=1,2,3,4$.

Labor Constraints:

$$3X_{A,1} + 2X_{B,1} + 4X_{C,1} \leq 120 + OT_1$$

$$3X_{A,2} + 2X_{B,2} + 4X_{C,2} \leq 120 + OT_2$$

$$3X_{A,3} + 2X_{B,3} + 4X_{C,3} \leq 120 + OT_3$$

$$3X_{A,4} + 2X_{B,4} + 4X_{C,4} \leq 120 + OT_4$$

Production Planning Problem

Demand is met on time constraint:

$NI_{i,j} \geq 0$ for $i \in \{A,B,C\}$ and $j=1,2,3,4$.

Labor Constraints:

$$3X_{A,1} + 2X_{B,1} + 4X_{C,1} \leq 120 + OT_1$$

$$3X_{A,2} + 2X_{B,2} + 4X_{C,2} \leq 120 + OT_2$$

$$3X_{A,3} + 2X_{B,3} + 4X_{C,3} \leq 120 + OT_3$$

$$3X_{A,4} + 2X_{B,4} + 4X_{C,4} \leq 120 + OT_4$$

$OT_j \leq 25$, $j=1,2,3,4$.

Production Planning Problem

Sign restrictions:

$X_{i,j} \geq 0$, $i \in \{A,B,C\}$ and $j=1,2,3,4$.

$OT_j \geq 0$, $j=1,2,3,4$.

Objective Function:

Production Planning Problem

Sign restrictions:

$X_{i,j} \geq 0$, $i \in \{A,B,C\}$ and $j=1,2,3,4$.

$OT_j \geq 0$, $j=1,2,3,4$.

Objective Function:

$$\text{Min} \quad 10 \cdot \left(\sum_{j=1}^{4} 3X_{A,j} + 2X_{B,j} + 4X_{C,j} \right) + 5\sum_{j=1}^{4} OT_j$$

Class Exercises

Question 1) Rylon Corporation manufactures Brute and Channelle perfumes from a raw material costing $3/pound. Processing 1 pound of the material takes 1 hour and yields 3 ounces of Regular Brute and 4 ounces of Regular Chanelle. This perfume can be sold directly or can be further processed to make luxury versions of the perfumes. One ounce of Regular Brute can be sold for $7, but 3 hours of processing and $4 of cost will convert it into one ounce of Luxury Brute, which sells for $18. One ounce of Regular Chanelle can be sold for $6, but 2 hours of processing and $4 of cost will convert it into one ounce of Luxury Chanelle, which sells for $14. Rylon has available 4000 pounds of raw material, 6000 hours of processing time, and will sell everything it produces. Determine how Rylon can maximize its profit.

Question 2) Bel Aire Pillow Company needs to manufacture 12,000 pillows in their factory next week. They have two kinds of employees: line workers (who do the actual work) and supervisors (who oversee the line workers). Bel Aire currently has 120 line workers and 20 supervisors. Each line worker can make 300 pillows per week. Supervisors do not engage in actual pillow-making.

A recent study done on Bel Aire's operation concluded that, for proper supervision, there must be at least 1 supervisor for every 5 line workers. The study also suggested that Bel Aire might be able to cut its costs by doing some or all of the following:

- hiring new employees as line workers (the training cost is $100/worker)
- firing some current line workers (the fired worker is given $240 severance pay)
- promoting some current line workers to supervisors (the training cost is $200/worker)
- demoting some current supervisors to line workers (no cost associated with this)

Pay for a line worker is $400/week. Pay for a supervisor is $700/week. All hiring and firing, and training occur before the upcoming work week, and the costs of those activities will be charged against the upcoming week. Bel Aire wishes to perform the appropriate hiring, firing, promotions and demotions that will minimize its operations cost for next week.