Economic Equivalence

Contemporary Engineering Economics

Economic Equivalence

- What do we mean by "economic equivalence?"
- Why do we need to establish an economic equivalence?
- How do we measure and compare various cash payments received at different points in time?

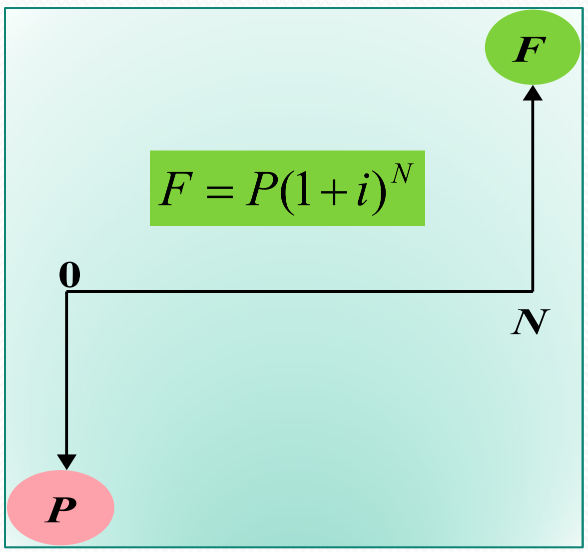
What is “Economic Equivalence?”

- Economic equivalence exists between cash flows that have the <u>same economic effect</u> and could therefore be traded for one another.
- Even though the amounts and timing of the cash flows may differ, the appropriate interest rate makes them equal in economic sense.

Equivalence

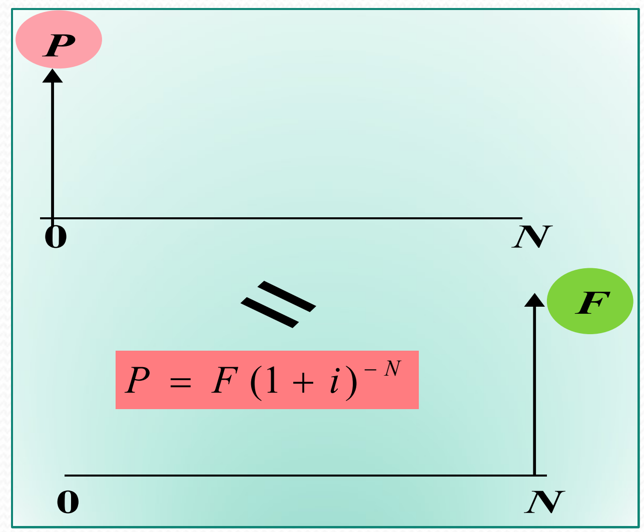
Equivalence from Personal Financing Point of View

- If you deposit *P* dollars today for *N* periods at *i*, you will have *F* dollars at the end of period *N*.



Alternate Way of Defining Equivalence

- *F* dollars at the end of period *N* is equal to a single sum *P* dollars now, if your earning power is measured in terms of interest rate *i*.



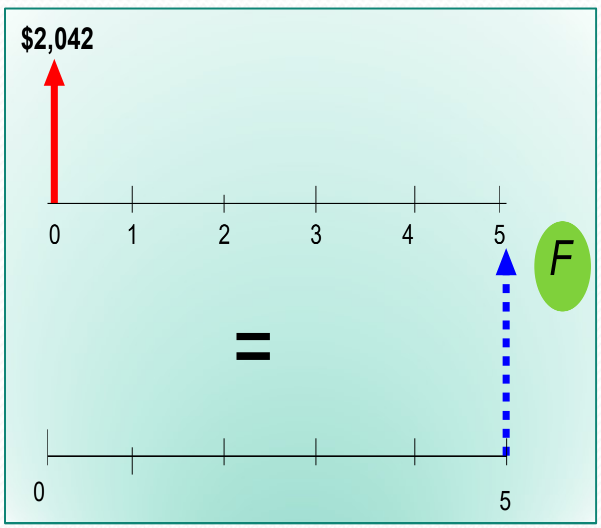
Example 3.3 Equivalence

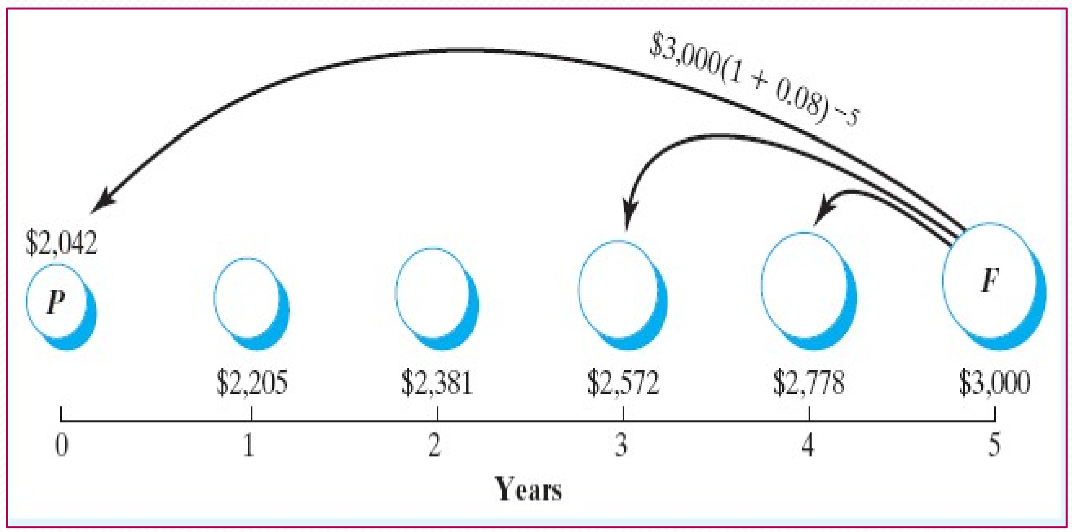
- If you deposit \$2,042 today in a savings account that pays an 8% interest annually, how much would you have at the end of 5 years?
- At an 8% interest, what is the equivalent worth of \$2,042 now in 5 years?

$$F = \$2{,}042(1 + 0.08)^5 = \$3{,}000$$

- Various dollar amounts that will be economically equivalent to \$3,000 in five years, at an interest rate of 8%

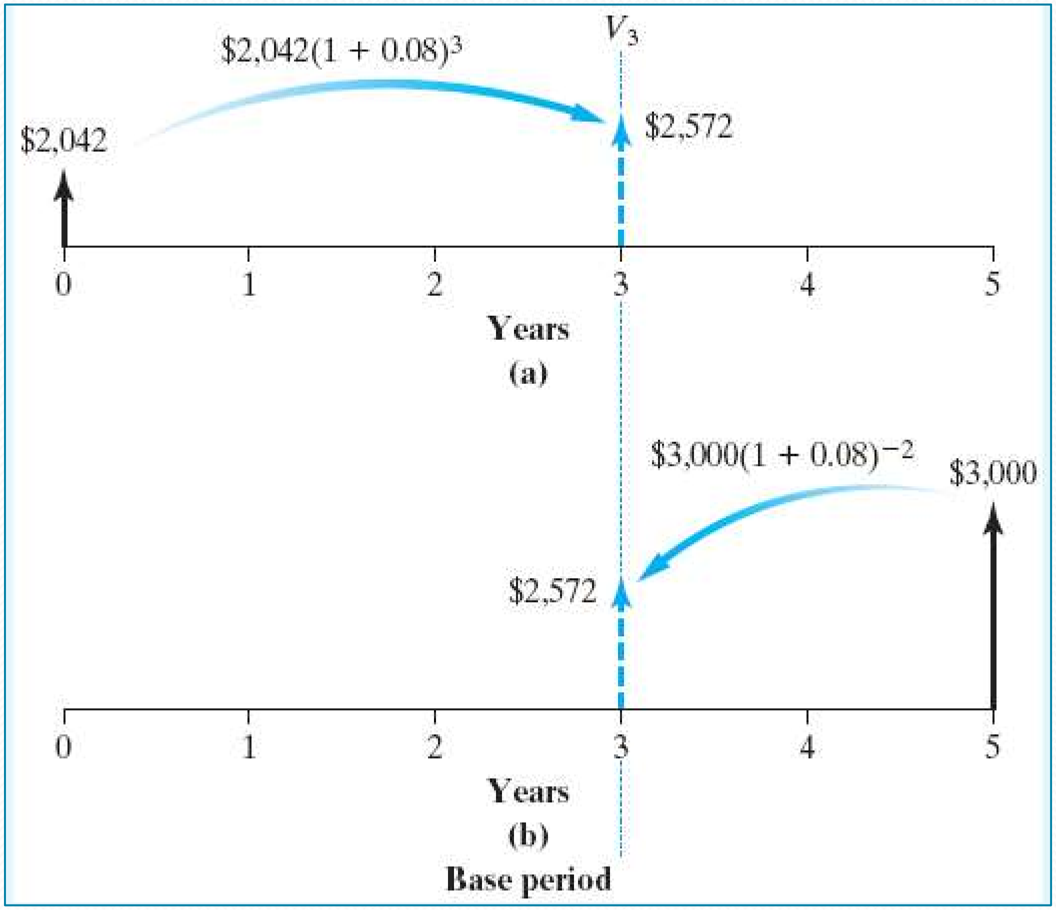
$$P = \frac{\$3{,}000}{(1 + 0.08)^5} = \$2{,}042$$





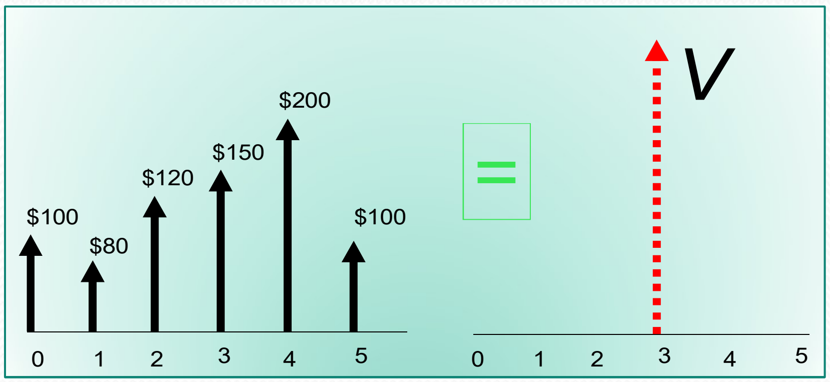
Example 3.4 Equivalent Cash Flows

- $2,042 today was equivalent to receiving $3,000 in five years, at an interest rate of 8%. Are these two cash flows are also equivalent at the end of year 3?
- Equivalent cash flows are equivalent at any common point in time, as long as we use the same interest rate (8%, in our example).



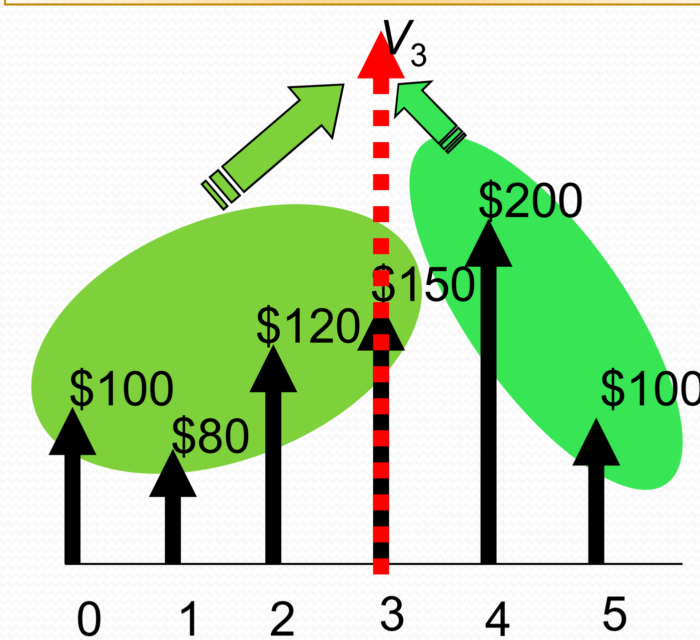
Practice Problem 1

- Compute the equivalent value of the cash flow series at n = 3, using i = 10%.



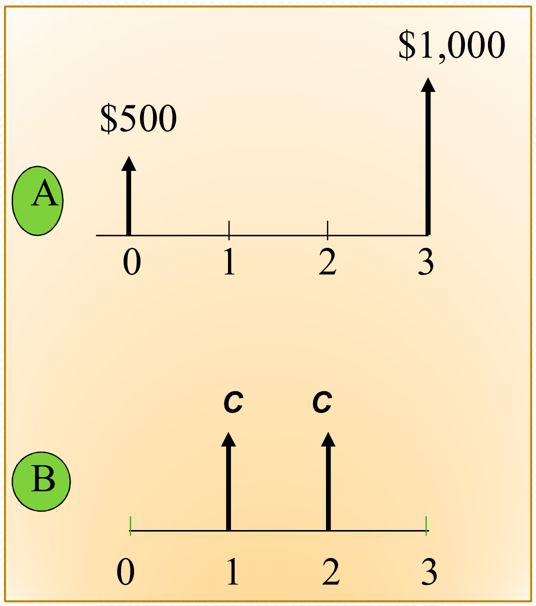
- **Solution:**

$$V_3 = \overbrace{100(1+0.10)^3 + \$80(1+0.10)^2 + \$120(1+0.10) + \$150}^{\text{Compounding Process: \$511.90}}$$
$$+ \underbrace{\$200(1+0.10)^{-1} + \$100(1+0.10)^{-2}}_{\text{Discounting Process: \$264.46}}$$
$$= \$511.90 + \$264.46$$
$$= \$776.36$$



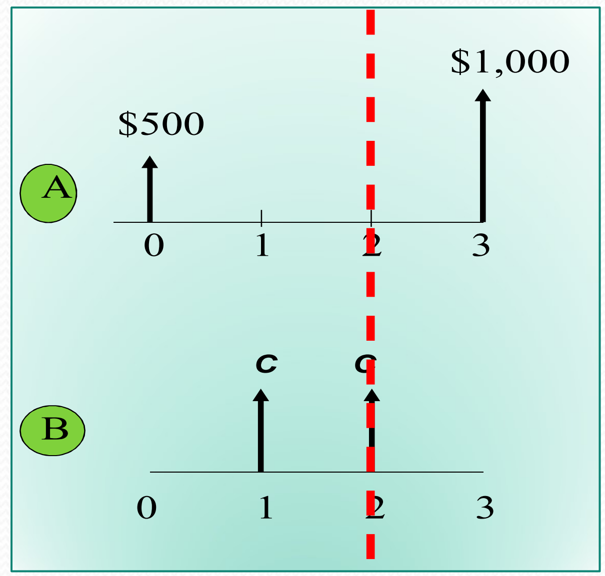
Practice Problem 2

- Find C that makes the two cash flow transactions equivalent at i = 10%



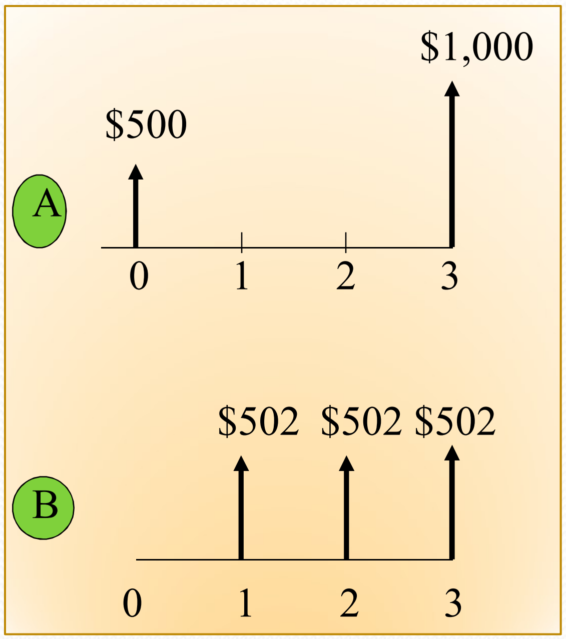
- **Approach:**
  - Step 1: Select a base period to use, say n = 2.
  - Step 2: Find the equivalent lump sum value at n = 2 for both A and B.
  - Step 3: Equate both equivalent values and solve for unknown C.

$$\begin{aligned}
\text{For A: } V_2 &= \$500(1+0.10)^2 + \$1{,}000(1+0.10)^{-1} \\
&= \$1{,}514.09 \\
\text{For B: } V_2 &= C(1+0.10) + C \\
&= 2.1C \\
2.1C &= \$1{,}514.09 \\
C &= \$721
\end{aligned}$$



Practice Problem 3

- At what interest rate would you be indifferent between the two cash flows?

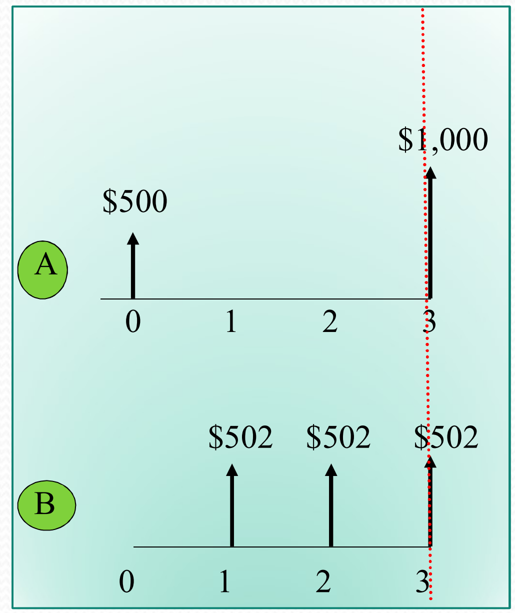


- **Approach:**
  - **Step 1: Select a base period to compute the equivalent value (say, n = 3)**
  - **Step 2: Find the equivalent worth of each cash flow series at n = 3.**

$$\text{Option A}: F_3 = \$500(1+i)^3 + \$1{,}000$$
$$\text{Option B}: F_3 = \$502(1+i)^2 + \$502(1+i) + \$502$$

i = 8%

$$\text{Option A}: F_3 = \$500(1.08)^3 + \$1{,}000 = \$1{,}630$$
$$\text{Option B}: F_3 = \$502(1.08)^2 + \$502(1.08) + \$502 = \$1{,}630$$



Interest Formulas for Single Cash Flows

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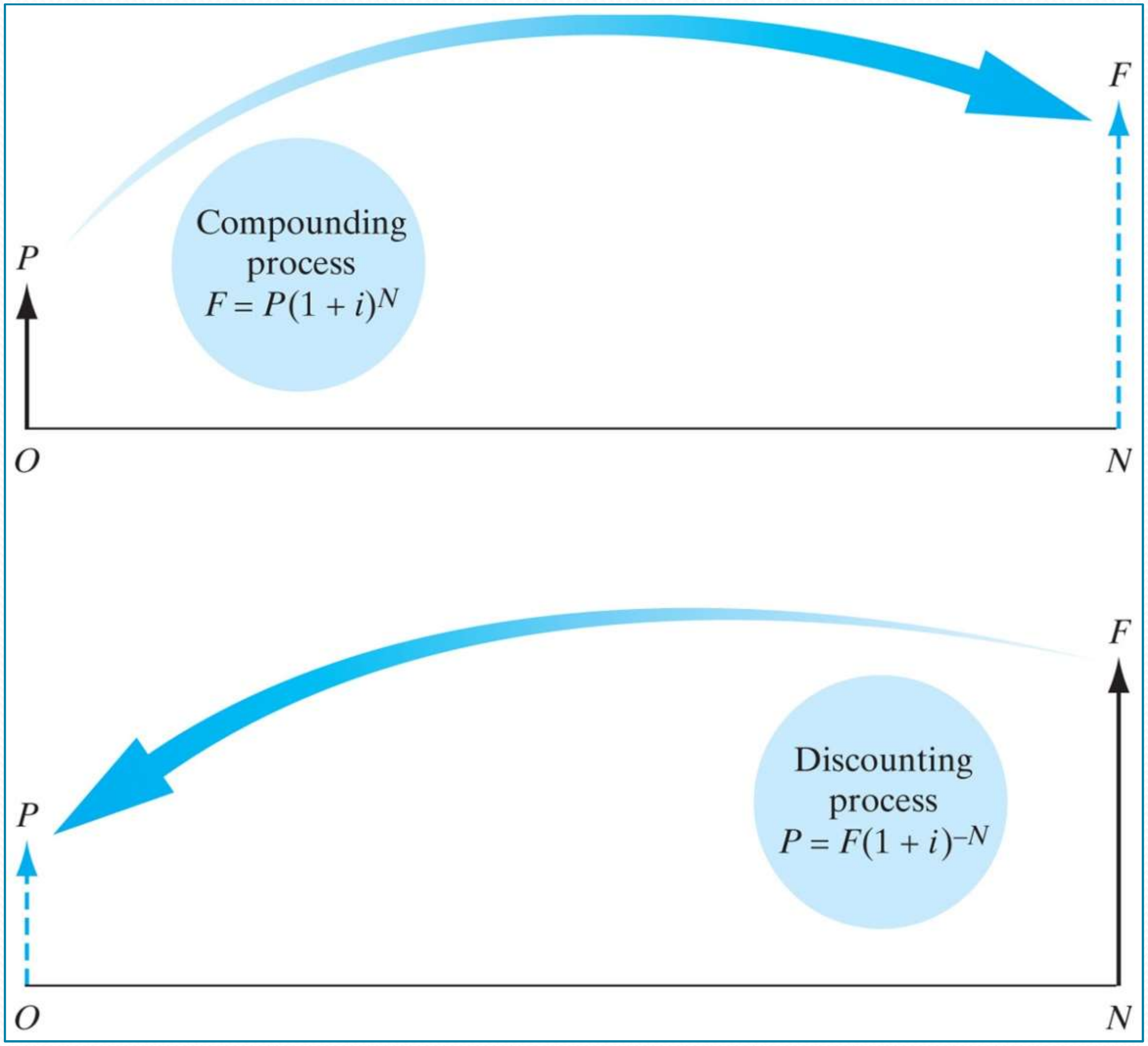
Equivalence Relationship Between *P* and *F*

- **Compounding Process** – Finding an equivalent future value of current cash payment
- **Discounting Process** – Finding an equivalent present value of a future cash payment



Example 3.7 Single Amounts: Find *F*, Given *i*, *N*, and *P*

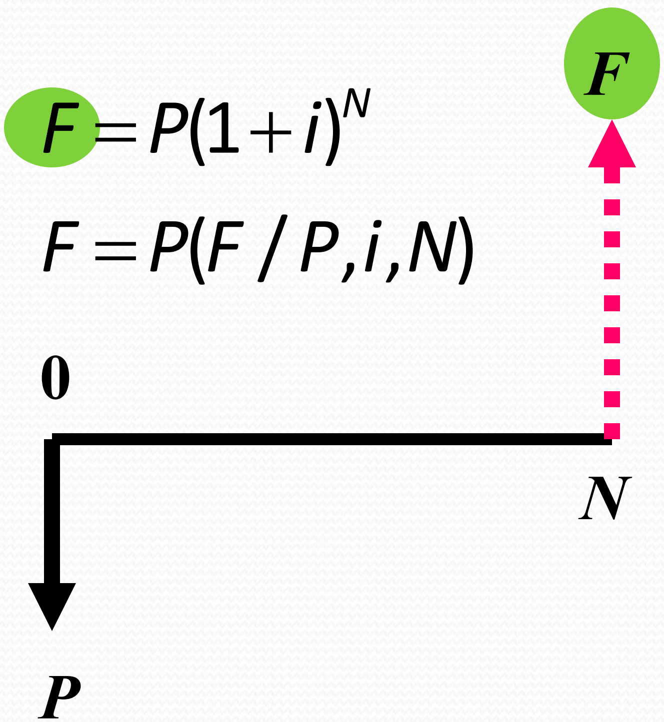
- **Given**: P = \$2,000, i = 10%, N = 8 years
- **Find**: F

$$\begin{aligned} F &= \$2,000(1+0.10)^8 \\ &= \$2,000(F/P,10\%,8) \\ &= \$4,287.18 \end{aligned}$$

- Single Cash Flow Formula – Compound Amount Factor

$$F = P(1+i)^N$$

$$F = P(F/P,i,N)$$



Example 3.8 Single Amounts: Find *P*, Given *i*, *N*, and *F*

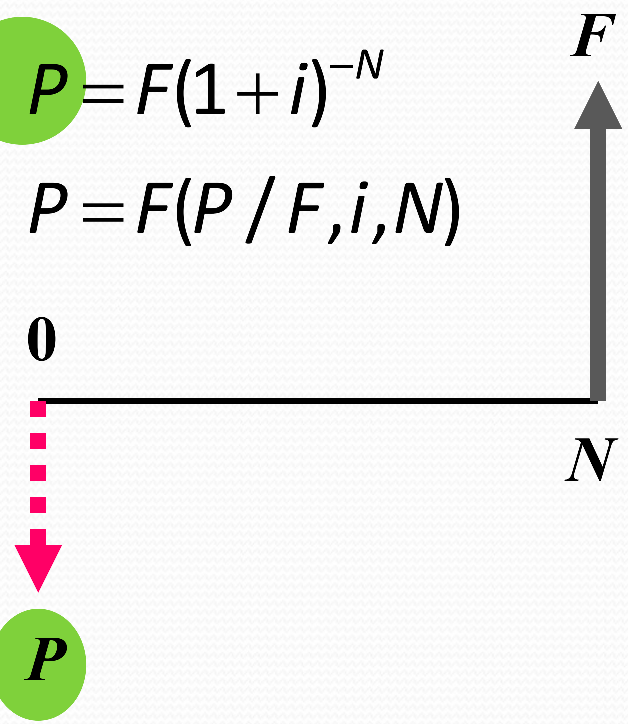
- **Given**: F = \$1,000, i = 12%, N = 5 years
- **Find**: P

$$\begin{aligned} P &= \$1{,}000(1+0.12)^{-5} \\ &= \$1{,}000(P/F, 12\%, 5) \\ &= \$567.43 \end{aligned}$$

- **Single Cash Flow Formula – Present Worth Amount Factor**

$$P = F(1+i)^{-N}$$

$$P = F(P/F, i, N)$$

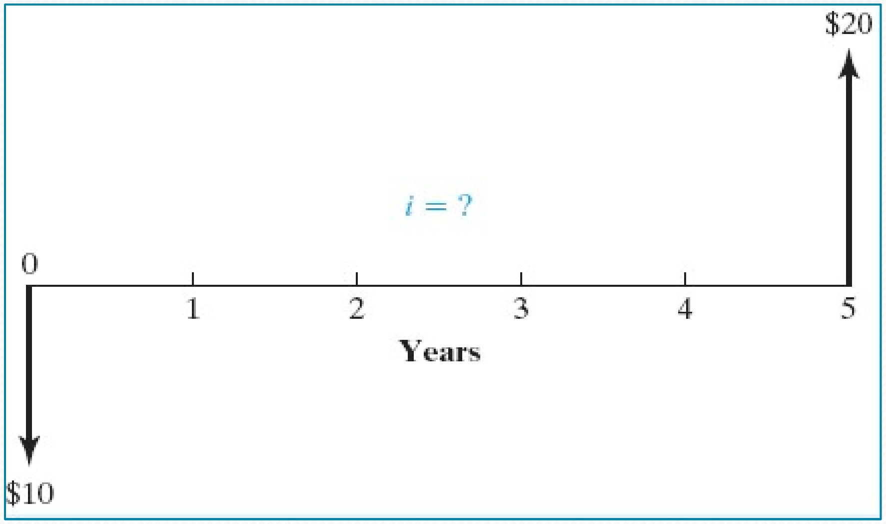


Example 3.9 Single Amounts: Find *i*, Given *P*, *F*, and *N*

- **Given**: *F* = \$20, *P* = \$10, *N* = 5 years
- **Find**: *i*

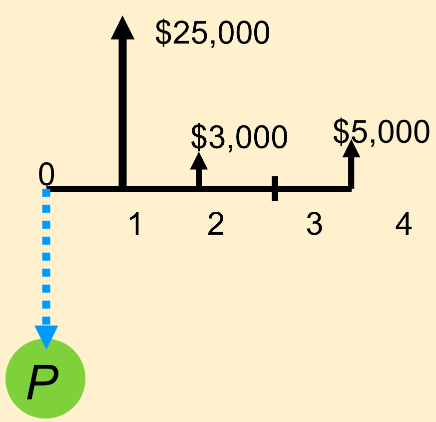
$$F = P(1+i)^N$$
$$\$40 = \$20(1+i)^5;\ \text{solve for } i$$
$$i = 2^{1/5} - 1$$
$$= 14.87\%$$

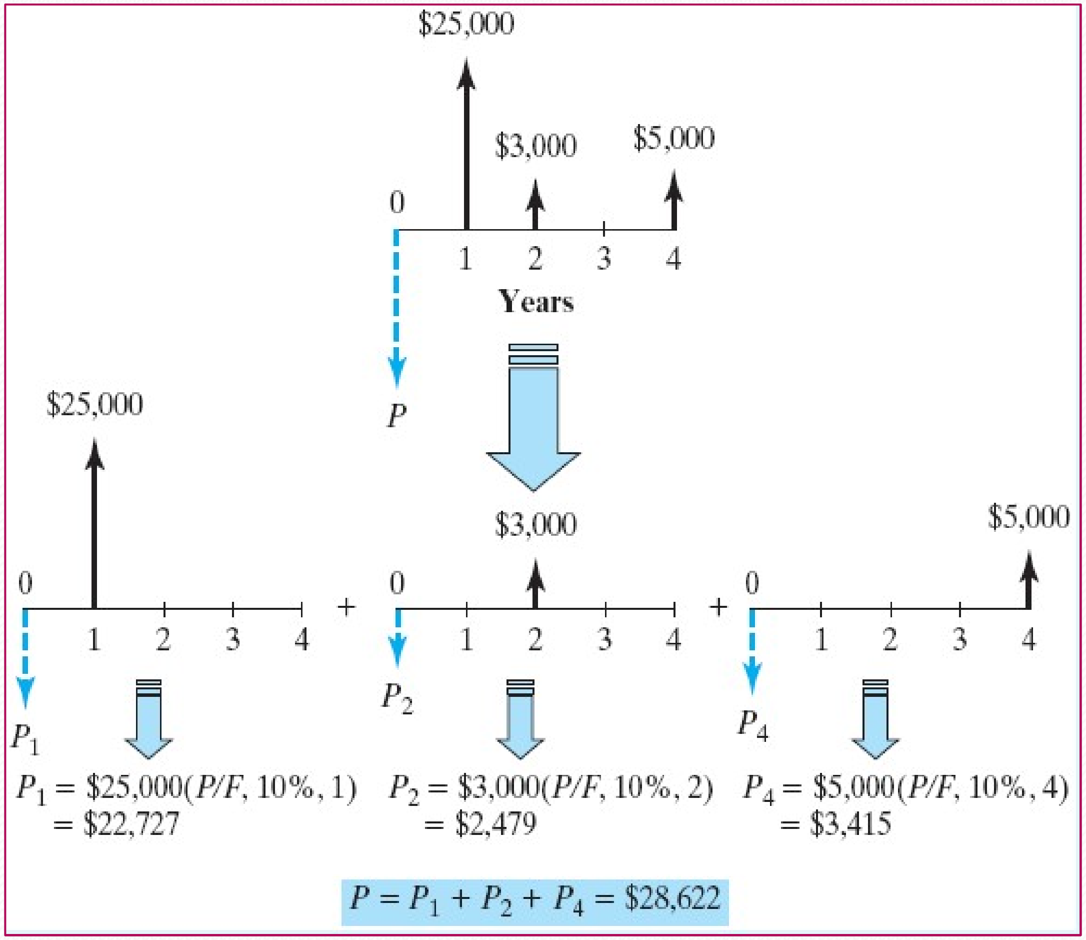
- Solving for *i*



Example 3.11 Uneven Payment Series

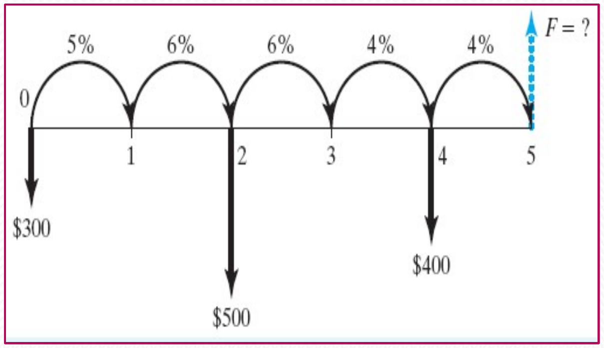
- How much do you need to deposit today (*P*) to withdraw \$25,000 at *n* =1, \$3,000 at *n* = 2, and \$5,000 at *n* =4, if your account earns 10% annual interest?





Example 3.12 Future Value of an Uneven Series with Varying Interest Rates

- **Given**: Deposit series as given over 5 years



- **Find**: Balance at the end of year 5

- Contribution of \$300 at $n = 0$ toward F_5:

$$\underbrace{\underbrace{\overbrace{\$300(F/P,5\%,1)}^{\text{Balance at } n=1}(F/P,6\%,2)}_{\text{Balance at } n=3}(F/P,4\%,2)}_{\text{Balance at } n=5} = \$382.82$$

- Contribution of \$500 at $n = 2$ toward F_5:

$$\underbrace{\overbrace{\$500(F/P,6\%,1)}^{\text{Balance at } n=3}(F/P,4\%,2)}_{\text{Balance at } n=5} = \$573.25$$

- Contribution of \$400 at $n = 4$ toward F_5:

$$\$400(F/P,4\%,1) = \$416$$

- Total balance at $n = 5$:

$$F_5 = \$382.82 + \$573.25 + \$416.00 = \$1,372.06$$

Interest Formulas – Equal Payment Series

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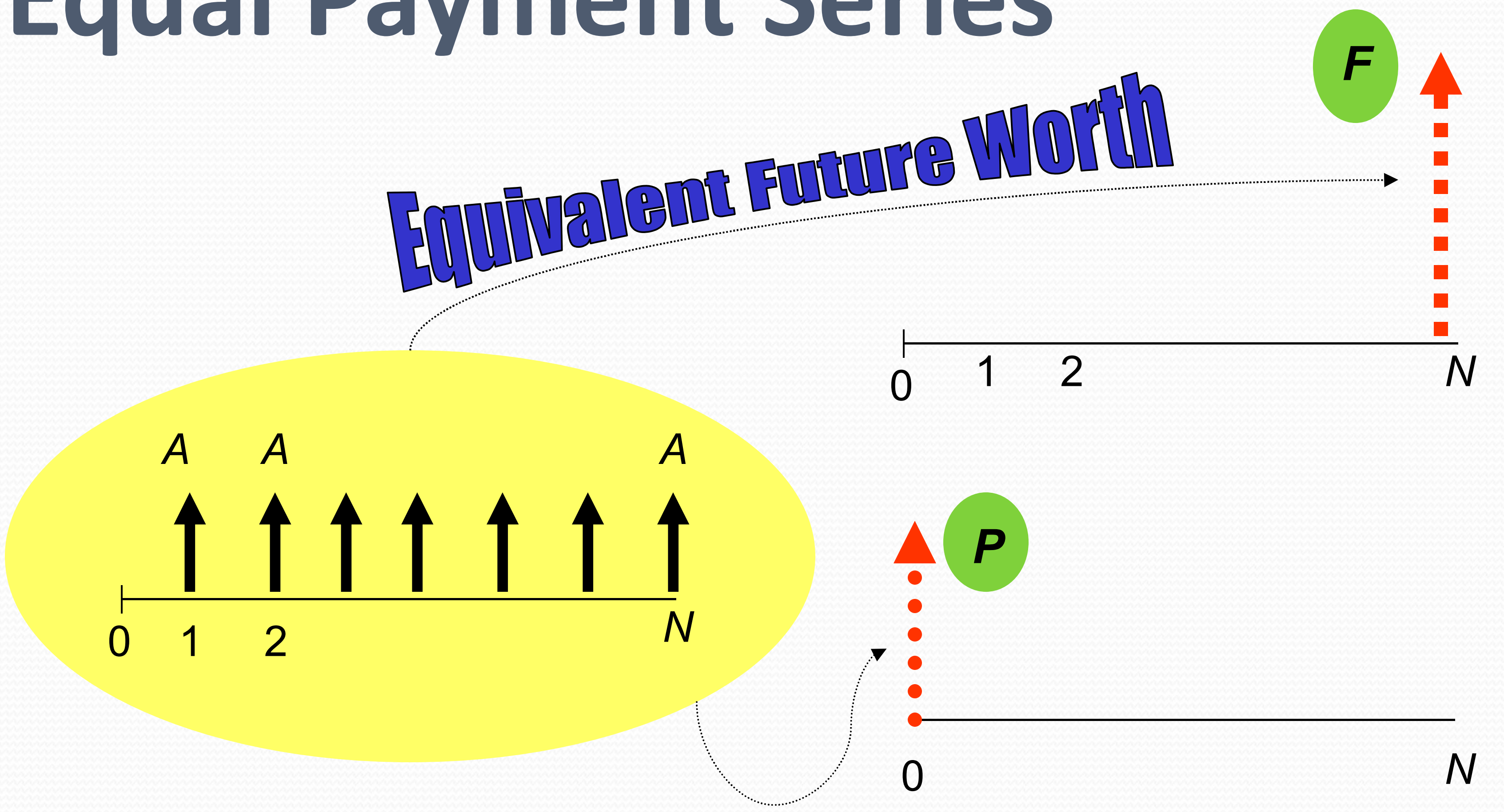
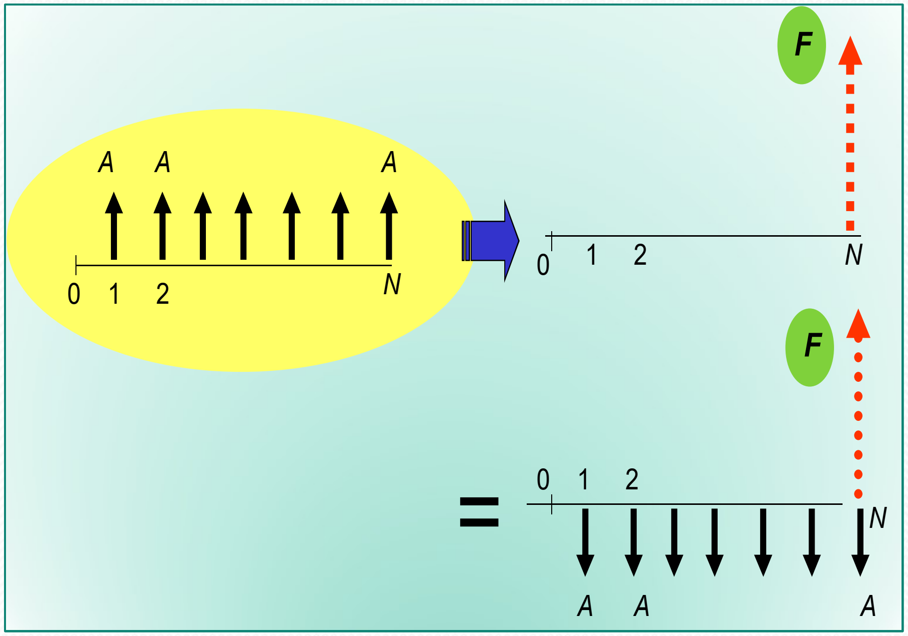
Equal Payment Series

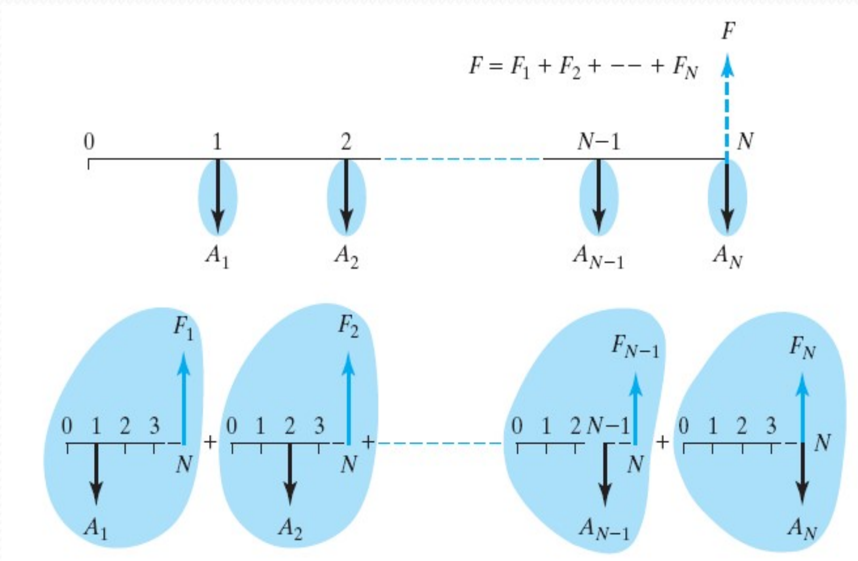


Equal-Payment Series Compound Amount Factor

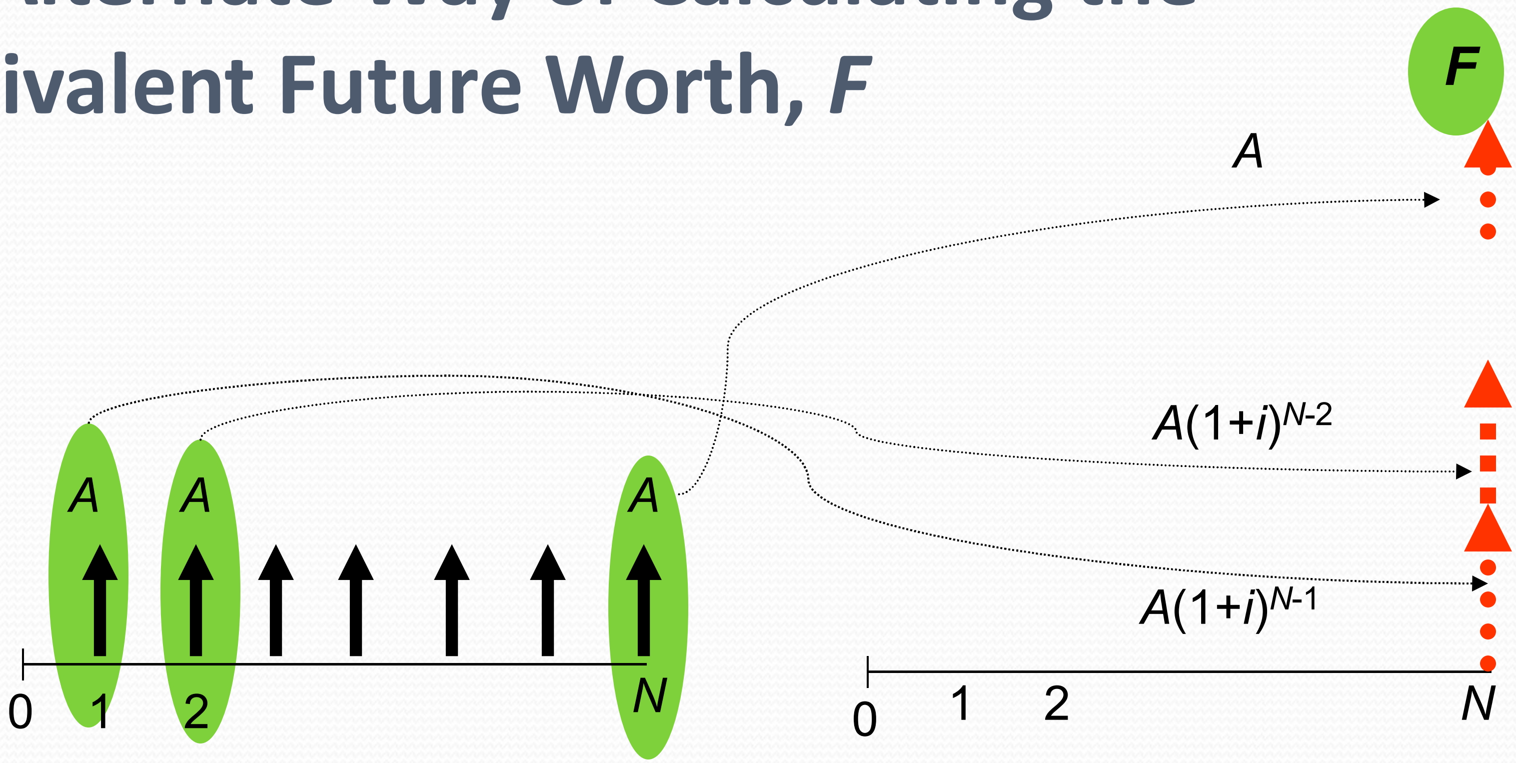


- **Formula**

$$F = A\left[\frac{(1+i)^N - 1}{i}\right] = A(F/A, i, N)$$



An Alternate Way of Calculating the Equivalent Future Worth, *F*



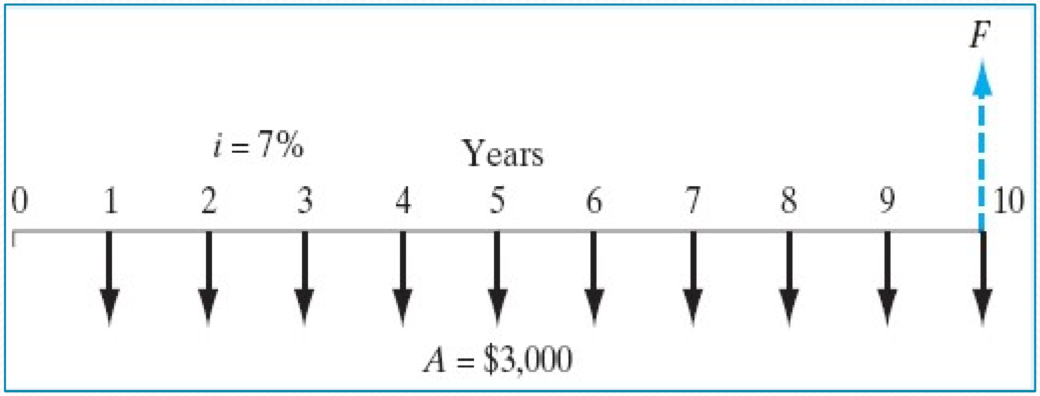
$$F = A(1+i)^{N-1} + A(1+i)^{N-2} + \cdots + A = A\left[\frac{(1+i)^N - 1}{i}\right]$$

Example 3.14 Uniform Series: Find *F*, Given *i*, *A*, and *N*

- **Given**: A = \$3,000, N = 10 years, and i = 7% per year

- **Find**: F

$$
\begin{aligned}
F &= \$3{,}000(F/A, 7\%, 10) \\
&= \$3{,}000(13.8164) \\
&= \$41{,}449.20
\end{aligned}
$$



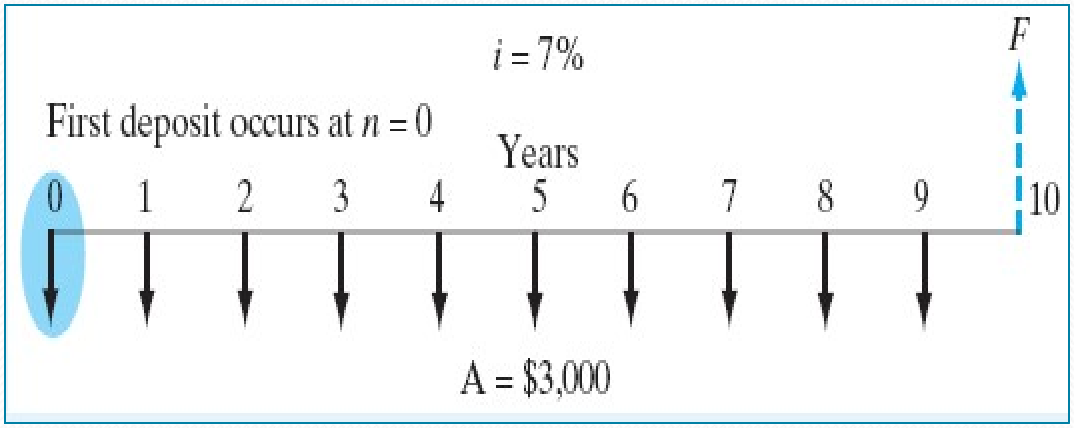
Example 3.15 Handling Time Shifts: Find *F*, Given *i*, *A*, and *N*

- **Given**: A = \$3,000, N = 10 years, and i = 7% per year

- **Find**: F

$$
\begin{aligned}
F &= \$3{,}000(F/A, 7\%, 10) \\
&= \$3{,}000(13.8164) \\
&= \$41{,}449.20
\end{aligned}
$$

$$F_{10} = \$41{,}449.20(1.07) = \$44{,}350.64$$



- Each payment has been shifted to one year earlier, thus each payment would be compounded for one extra year

Sinking-Fund Factor: Find *A*, Given *i*, *N*, and *F*

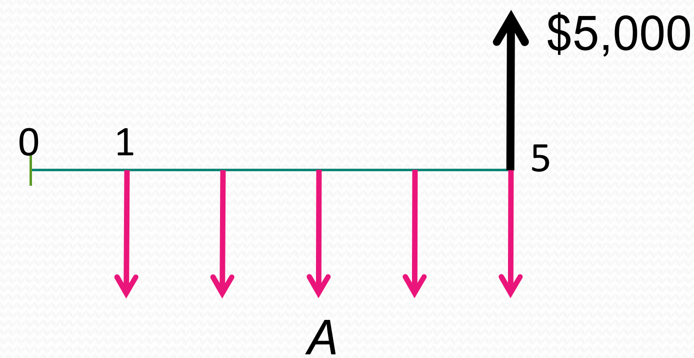
- **Given**: F = \$5,000, N = 5 years, and i = 7% per year
- **Find**: A

$$A = \$5{,}000(A/F, 7\%, 5) = \$869.50$$

- **Formula – Sinking Fund Factor**

$$A = F\left[\frac{i}{(1+i)^N - 1}\right] = F(A/F, i, N)$$

$$A = \$5{,}000(A/F, 7\%, 5) = \$869.50$$



Example 3.18 Uniform Series: Find *A*, Given *P*, *i*, and *N*

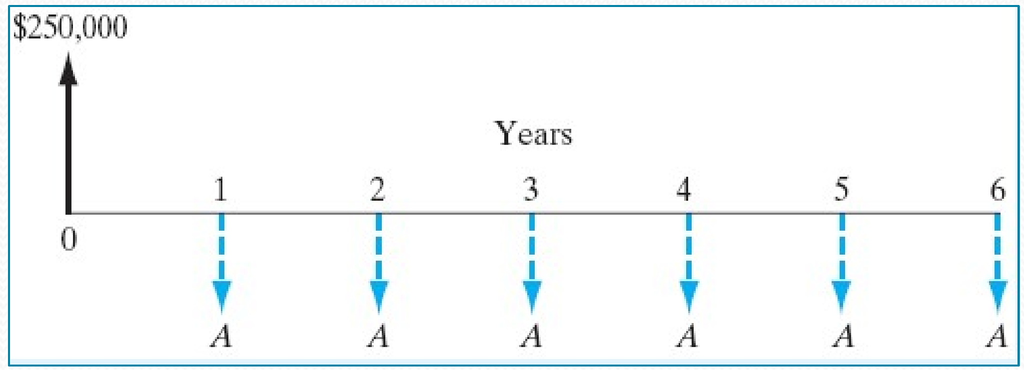
- **Given**: *P* = $250,000, *N* = 6 years, and *i* = 8% per year
- **Find**: *A*
- **Formula to use:**

$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right] = P(A/P, i, N)$$

$$\begin{aligned} A &= \$250{,}000(A/P, 8\%, 6) \\ &= \$250{,}000(0.2163) \\ &= \$54{,}075 \end{aligned}$$

- **Excel Solution:**

• Capital Recovery Factor



	A	B
1	*P*	$ 250,000
2	*i*	8%
3	*N*	6
4		
5	*A*	$54,078.85

=PMT(8%,6,−250000)

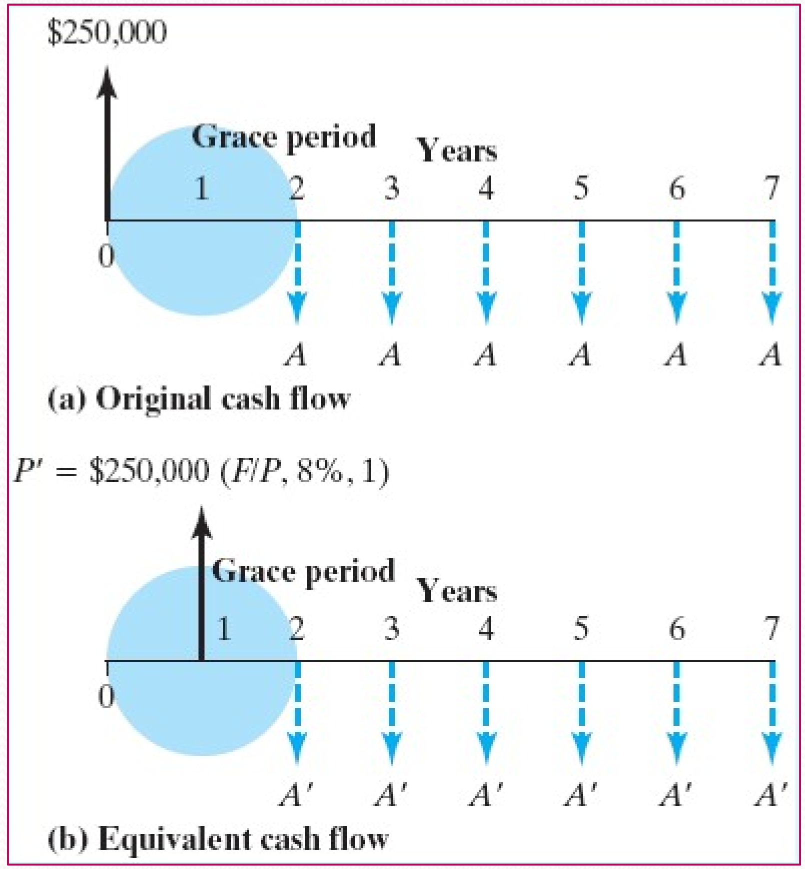
Example 3.19 – Deferred Loan Repayment

- **Given**: P = \$250,000, N = 6 years, and i = 8% per year, but the first payment occurs at the end of year 2
- **Find**: A
  - Step 1: Find the equivalent amount of borrowing at the end of year 1:

$$P' = \$250{,}000(F/P, 8\%, 1) = \$270{,}000$$

  - Step 2: Use the capital recovery factor to find the size of annual installment:

$$A' = \$270{,}000(A/P, 8\%, 6) = \$58{,}401$$



Example 3.20 Uniform Series: Find *P*, Given *A*, *i*, and *N*

- **Given**: A = \$10,576,923, N = 26 years, and i = 5% per year
- **Find**: P
- **Formula to use:**

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] = A(P/A, i, N)$$

$$\begin{aligned} P &= \$10{,}576{,}923(P/A, 5\%, 26) \\ &= \$152{,}045{,}228 \end{aligned}$$

- **Excel Solution:**

```
= PV(5%,26,-10576923)=$152,045,228
```

- **Present Worth Factor**