Class Exercises

Question 1) Rylon Corporation manufactures Brute and Channelle perfumes from a raw material costing $3/pound.

- Processing 1 pound of the material takes 1 hour and yields 3 ounces of Regular Brute and 4 ounces of Regular Chanelle. This perfume can be sold directly or can be further processed to make luxury versions of the perfumes.
- One ounce of Regular Brute can be sold for $7, but 3 hours of processing and $4 of cost will convert it into one ounce of Luxury Brute, which sells for $18.
- One ounce of Regular Chanelle can be sold for $6, but 2 hours of processing and $4 of cost will convert it into one ounce of Luxury Chanelle, which sells for $14.
- Rylon can purchase up to 4000 pounds of raw material and use 6000 hours of processing time per year, and will sell everything it produces.

Determine how Rylon can maximize its profit for the year.

Decision Variables

X_1 amount of Regular Brute sold yearly (ounces)

Y_1 amount of Regular Chanelle sold yearly (ounces)

X_2 amount of Luxury Brute sold yearly (ounces)

Y_2 amount of Luxury Chanelle sold yearly(ounces)

Z amount of raw materials purchased yearly(pounds)

Linear Programming Model

$$\max \; 7X_1 + 6Y_1 + (18-4)X_2 + (14-4)Y_2 - 3Z$$

s.t.

$$Z = (X_1 + X_2)/3$$

$$Z = (Y_1 + Y_2)/4$$

$$Z \leq 4000$$

$$Z + 3X_2 + 2Y_2 \leq 6000$$

$$X_1 \geq 0, \; X_2 \geq 0, \; Y_1 \geq 0, \; Y_2 \geq 0, \; Z \geq 0$$

NOTE !!!

If you choose Decision Variables as:

X_1 amount of raw material in Regular Brute (pounds)

Y_1 amount of raw material in Regular Chanelle (pounds)

X_2 amount of raw material in Luxury Brute (pounds)

Y_2 amount of raw material in Luxury Chanelle (pounds)

It is not going to be easy to write the profit. The problem is with these variables you cannot account for the fact that 1 pound of raw material jointly produces 3 ounces of regular brute and 4 ounces of regular chanelle. You implicitly impose a sharing of the raw material which imposes an incorrect sharing on the profit.

Class Exercises

Question 2) Bel Aire Pillow Company needs to manufacture 12,000 pillows in their factory next week. They have two kinds of employees: line workers (who do the actual work) and supervisors (who oversee the line workers). Bel Aire currently has 120 line workers and 20 supervisors. Each line worker can make 300 pillows per week. Supervisors do not engage in actual pillow-making.

A recent study done on Bel Aire's operation concluded that, for proper supervision, there must be at least 1 supervisor for every 5 line workers. The study also suggested that Bel Aire might be able to cut its costs by doing some or all of the following:

- hiring new employees as line workers (the training cost is $100/worker)
- firing some current line workers (the fired worker is given $240 severance pay)
- promoting some current line workers to supervisors (the training cost is $200/worker)
- demoting some current supervisors to line workers (no cost associated with this) Assume supervisors are not fired.

Pay for a line worker is $400/week. Pay for a supervisor is $700/week. All hiring and firing, and training occur before the upcoming work week, and the costs of those activities will be charged against the upcoming week. Bel Aire wishes to perform the appropriate hiring, firing, promotions and demotions that will minimize its operations cost for next week.

Decision variables

X_1 denote # of hired line workers
X_2 denote # of fired line workers
Y_1 denote # of lines worker promoted to supervisor
Y_2 denote # of supervisors demoted to line worker

$(120 + X_1 - X_2 - Y_1 + Y_2)$: # of line workers
$(20 + Y_1 - Y_2)$: # of supervisors

Linear Programming Model :

$$\min 100 X_1 + 240 X_2 + 200 Y_1 + 400(120+X_1 - X_2 - Y_1 + Y_2) + 700 (20+ Y_1 - Y_2)$$

s.t.

$$5(20 + Y_1 - Y_2) \geq (120 + X_1 - X_2 - Y_1 + Y_2)$$

$$300(120 + X_1 - X_2 - Y_1 + Y_2) \geq 12000$$

$$X_1 \geq 0$$
$$X_2 \geq 0$$
$$Y_1 \geq 0$$
$$Y_2 \geq 0$$

Can these variables take fractional values? Does divisibility assumption apply here?

No!!! The model should be an integer programming model

Integer Programming Model :

$$\min 100 X_1 + 240 X_2 + 200 Y_1 + 400(120 + X_1 - X_2 - Y_1 + Y_2) + 700 (20 + Y_1 - Y_2)$$

s.t.

$$5(20 + Y_1 - Y_2) \geq (120 + X_1 - X_2 - Y_1 + Y_2)$$

$$300(120 + X_1 - X_2 - Y_1 + Y_2) \geq 12000$$

$X_1 \geq 0$ and integer
$X_2 \geq 0$ and integer
$Y_1 \geq 0$ and integer
$Y_2 \geq 0$ and integer