

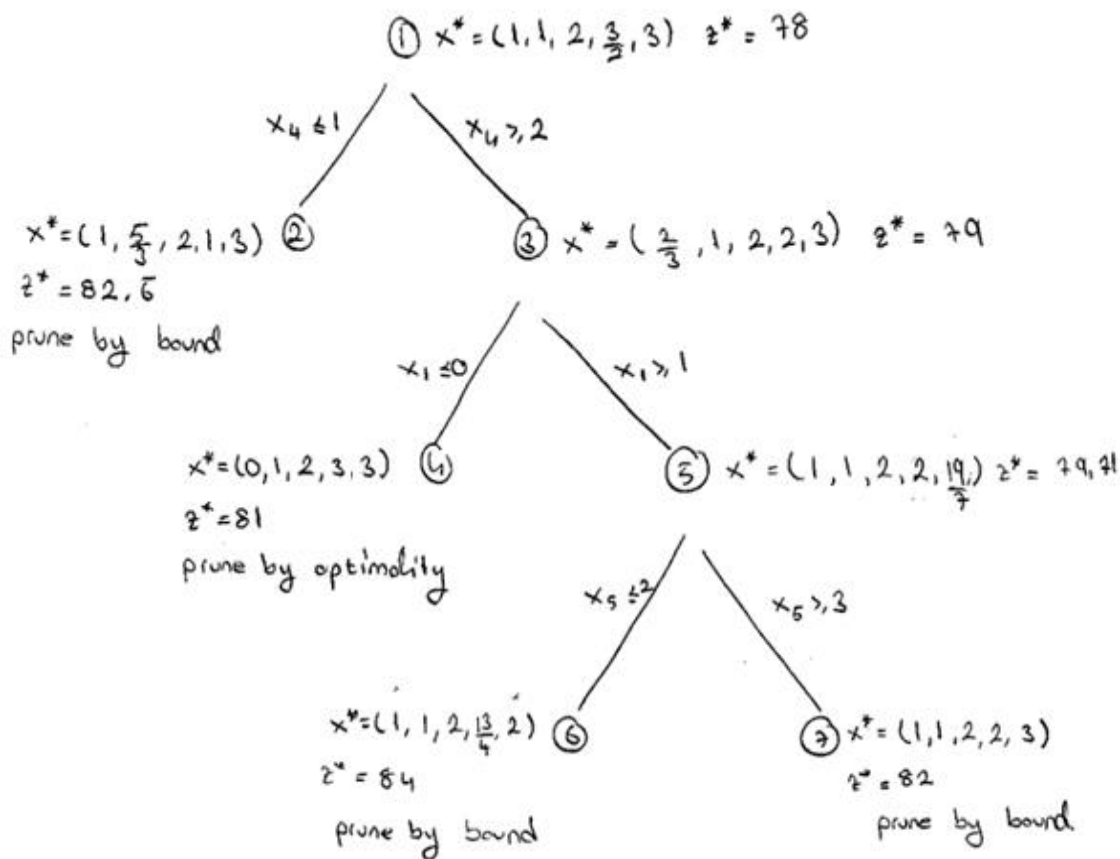
IE 400 2017-2018 Spring Study Set 3 Solutions

① First calculate the ratios:

x_1	x_2	x_3	x_4	x_5
9/6	13/3	10/2	8/4	8/4

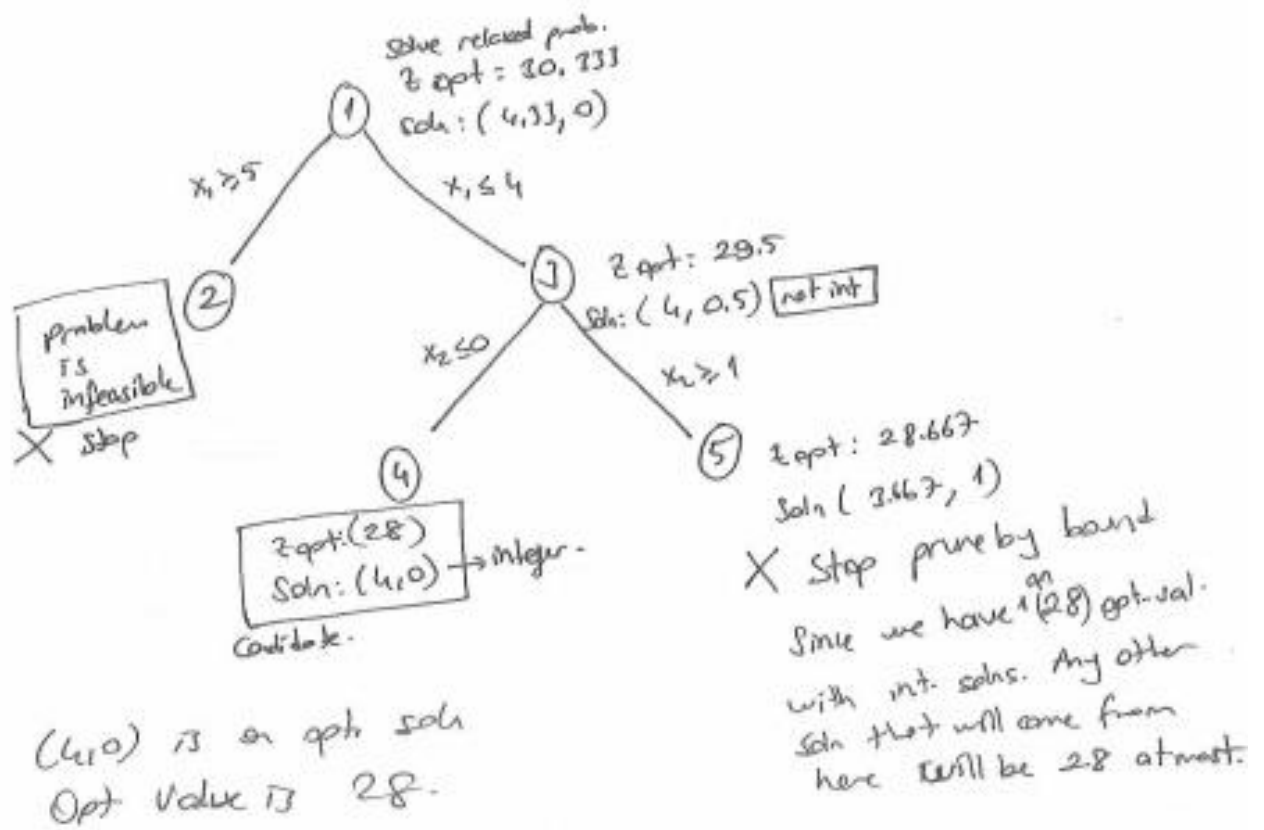
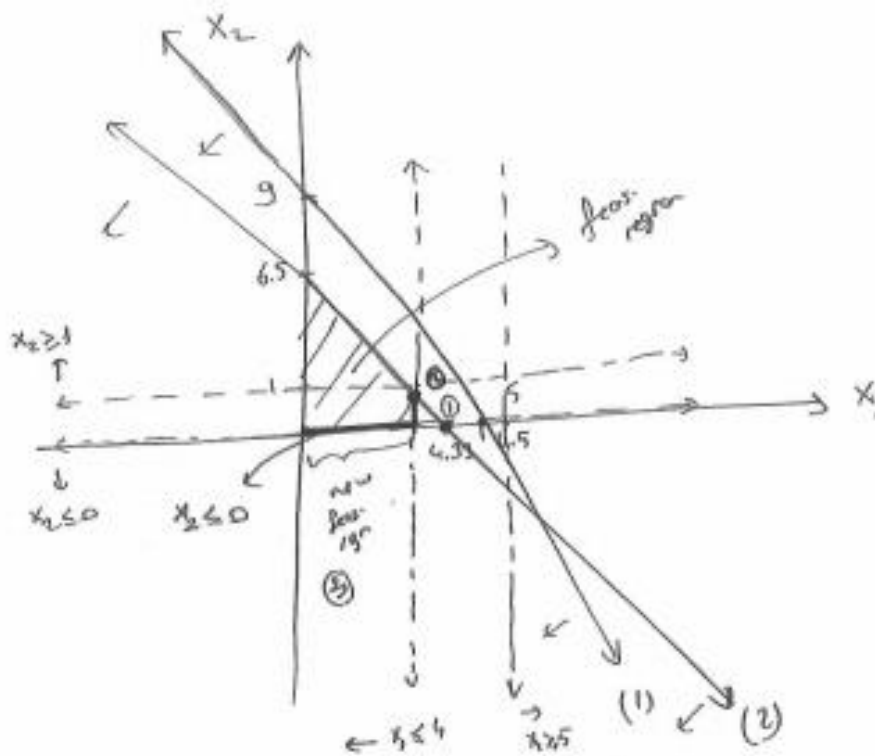
minimization so: $x_5 < x_1 < x_4 < x_2 < x_3$

The optimal solution of the LP relaxation: $x_{LP}^* = (1, 1, 2, \frac{3}{2}, 3)$

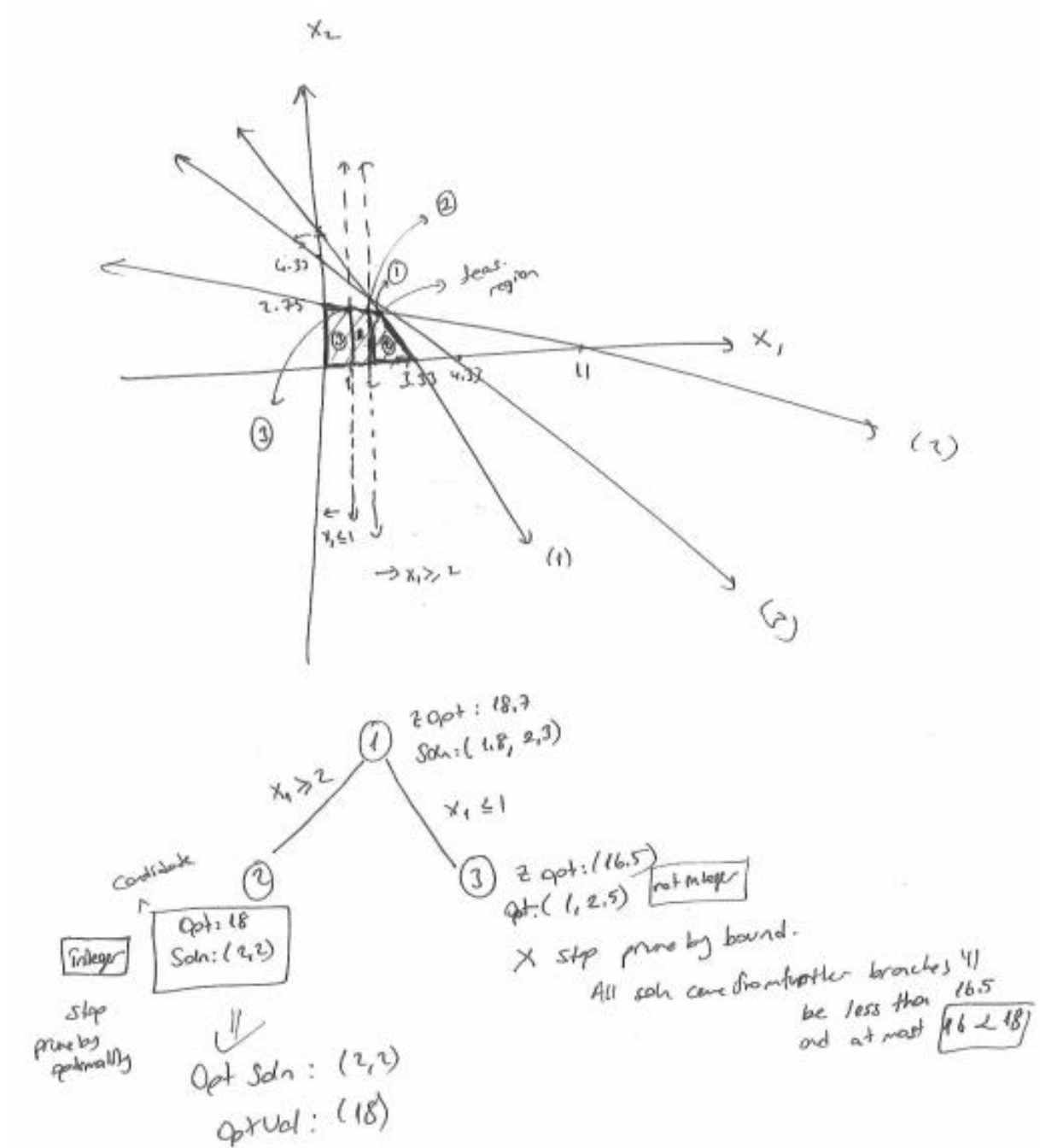


Optimal solution: $x_{IP}^* = (0, 1, 2, 2, 3)$ $z_{IP}^* = 81$

2)



3)



4) 1-2-5 with length 14

5)

This problem should be formulated as a shortest path problem.

The network will have six nodes.

- Node i is the beginning of year i and for $i < j$, an arc (i, j) corresponds to purchasing a new car at the beginning of year i and keeping it until the beginning of year j .

The length of arc (i, j) (call it c_{ij}) is the total net cost incurred from year i to j .

$$c_{ij} = \text{maintenance cost incurred during years } i, i+1, \dots, j-1 \\ + \text{cost of purchasing a car at the beginning of year } i \\ - \text{trade-in value received at the beginning of year } j$$

Applying this formula to the information the problem yields

$$C12=300+10000-7000=3300$$

$$C13=300+500+10000-6000=4800$$

$$C14=300+500+800+10000-4000=7600$$

$$C15=300+500+800+1200+10000-3000=9800$$

$$C16=300+500+800+1200+1600+10000-2000=11500$$

$$C17=300+500+800+1200+1600+2200+10000-1000=14700$$

$$C23=300+10000-7000=3300$$

$$C24=300+500+10000-6000=4800$$

$$C25=300+500+800+10000-4000=7600$$

$$C26=300+500+800+1200+10000-3000=9800$$

$$C27=300+500+800+1200+1600+10000-2000=11500$$

$$C34=300+10000-7000=3300$$

$$C35=300+500+10000-6000=4800$$

$$C36=300+500+800+10000-4000=7600$$

$$C37=300+500+800+1200+10000-3000=9800$$

$$C45=300+10000-7000=3300$$

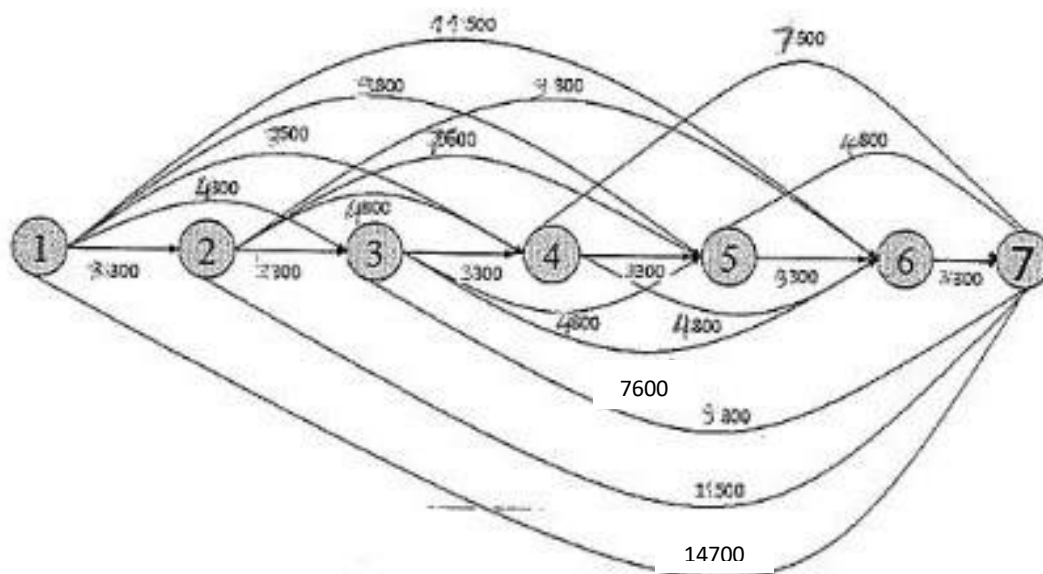
$$C46=300+500+10000-6000=4800$$

$$C47=300+500+800+10000-4000=7600$$

$$C56=300+10000-7000=3300$$

$$C57=300+500+10000-6000=4800$$

$$C67=300+10000-7000=3300$$



From the figure above, we can conclude that replacing car only at year 4 yields the least costly solution with \$12400. (path: 1-4-7)

6)

Step 0

$A_{12} = 18$ $A_{13} = 24$ $A_{16} = 20$ $A_{15} = 30$ $A_{ij} = +\infty$

$A_{21} = +\infty$

$P = \{1\}$ $T = \{2, \dots, 9\}$

$u_1 = 0$ $u_3 = 22$ $u_5 = 30$

$u_2 = 18$ $u_4 = 20$ $u_6, u_7 = +\infty$

u_8, u_9

$\text{Pred}(1) = 1$

$\text{Pred}(j) = \{j\}$

$\forall j = 2, \dots, 9$

Step 1: $k = 2$

Step 2:

$P = \{1, 2\}$ $T = \{3, \dots, 9\}$ $u_1 = 0$ $u_2 = 18$ $u_6 = 45$

$\text{Pred}(3) = 1$ $\text{Pred}(7) = 1$ $\text{Pred}(6) = 2$ $u_7 = 51$

$\text{Pred}(8) = 2$

Step 1: $k = 4$

Step 2: $P = \{1, 2, 4\}$ $T = \{3, 5, 6, 7, 8, 9\}$ $u_6 = 56$

$\text{Pred}(4) = 1$ $\text{Pred}(6) = 4$ $u_7 = 50$

$\text{Pred}(7) = 4$

Step 1: $k = 5$

Step 2: $P = \{1, 2, 3, 4\}$ $T = \{5, \dots, 9\}$ no updates

Step 1: $k = 5$

Step 2: $P = \{1, \dots, 5\}$ $T = \{6, \dots, 9\}$ $u_9 = 58$ $\text{Pred}(9) = 5$

$\text{Pred}(8) = 1$

Step 1: $k = 6$

Step 2: $P = \{1, \dots, 6\}$ $T = \{7, \dots, 9\}$ no updates

$u_7 = \min \{u_7, u_6 + A_{67}\}$

$50 \quad 30 + 20$

Step 1: $k = 7$

Step 2: $P = \{1, \dots, 7\}$ $T = \{8, 9\}$ no update

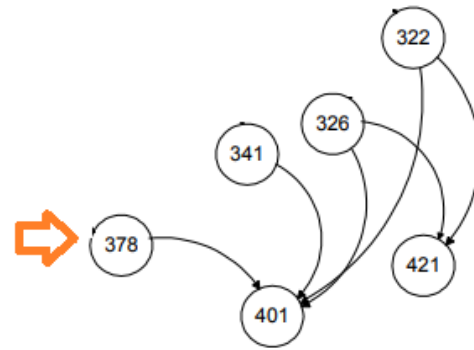
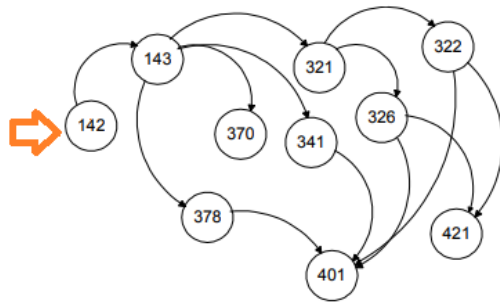
Step 1: $k = 8$

Step 2: $P = \{1, \dots, 8\}$ $T = \{9\}$ no update

Step 1: $k = 9$

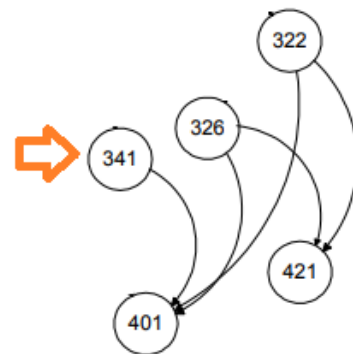
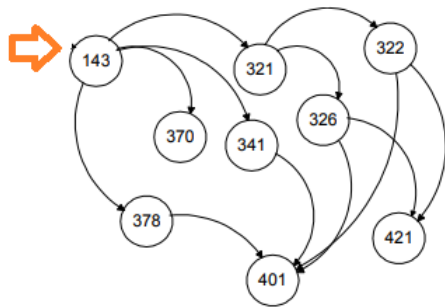
Step 2: $P = \{1, \dots, 9\}$ $T = \{ \}$

7) Pick the node without any incoming arcs:



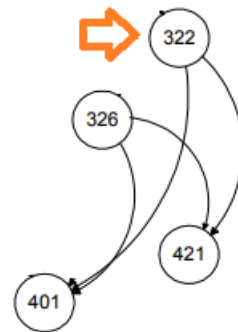
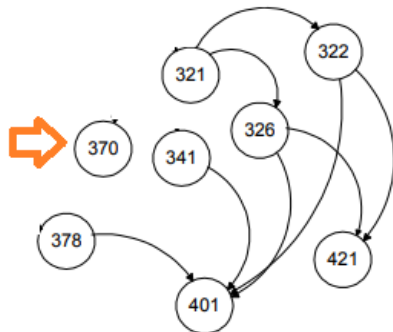
142-143-370-321-378

Randomly select another node without any incoming arcs and repeat:



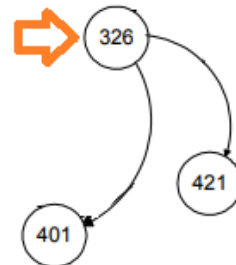
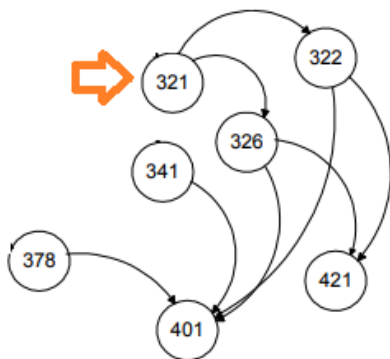
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142-143



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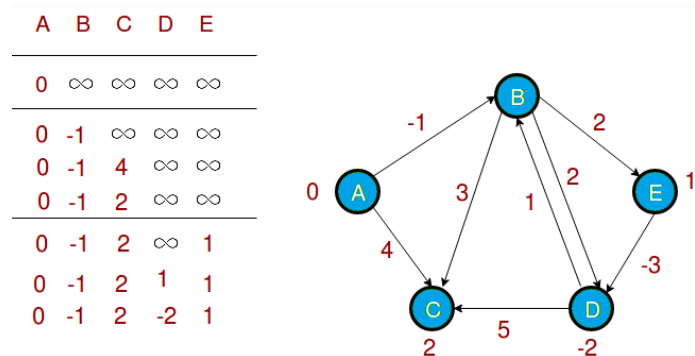
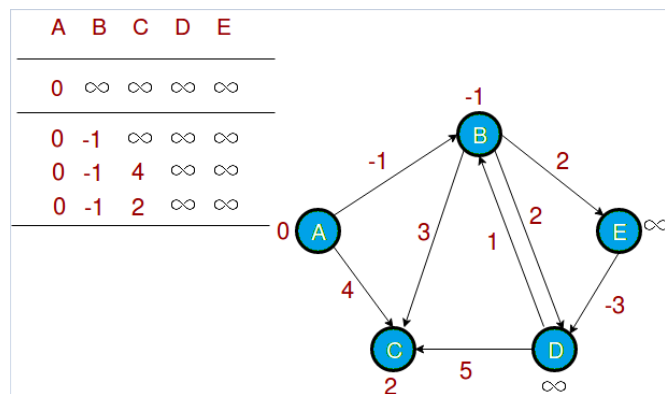
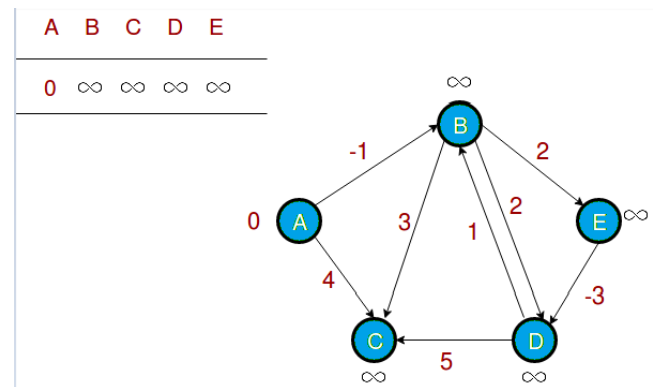
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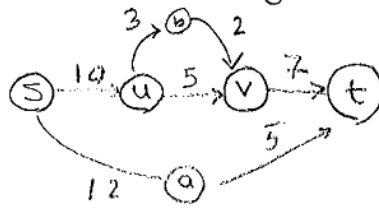
8)



The distances are minimized after the second iteration hence third and fourth iterations don't update the distances.

⑨ a) No, principle of optimality doesn't apply.

Counterexample



Shortest path
from s to t $\begin{cases} s-u-v-t = 10 \\ \text{or} \\ s-u-b-v-t = 10 \end{cases}$

but

Shortest path
from u to v $\begin{cases} u-b-v = 3 \end{cases}$

b)
$$v[i] = \min \{ v[i], \max \{ v[p], c_{pi} \} \}$$

c) you can adapt dijkstra algorithm with the functional equations stated in part (b)

10)

a) False. Consider nodes A,B,C, cost of edge (A,B) is 3, edge (A,C) is 1 and (B,C) is 2. There are 2 shortest paths from A to B both of length 3.

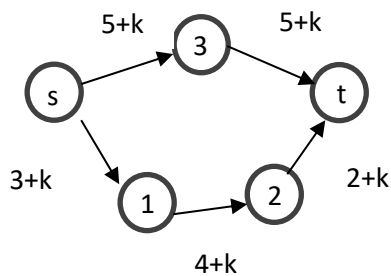
b) True. Both algorithms are guaranteed to produce the same shortest path weight, but if there are multiple shortest paths, Dijkstra's will choose the shortest path according to the greedy strategy, and Bellman-Ford will choose the shortest path depending on the order of relaxations, and the two shortest path trees may be different.

c) False. For example a graph $G = (V, E) = (\{a, b, c\}, \{(a, b), (a, c)\})$ has valid topological orderings $[a, b, c]$ or $[a, c, b]$.

d) False. Undirected graph on nodes A,B,C. Edges (A,B) and (B,C) have cost 1 and edge (A,C) has cost 3. Length of shortest A to C path is 2. If each edge cost is increased by $k = 10$ the shortest path length becomes 13. But $13 - 2$ is not a multiple of 10.

e) False. Consider graph with nodes A,B,C, arcs (B,A), (A,C) and (C,B) have cost 1. In the directed graph, shortest distance from A to B is $1 + 1 = 2$, if it becomes undirected, the shortest distance becomes 1.

f) False



without adding k , the shortest path is $s-1-2-t$ with objective function of 9 but if $k > 1$ then the shortest path changes to $s-3-t$.