IE 400
Principles of Engineering Management

Syllabus

Study Sets: A set of exercises will be assigned in order to prepare you for the quizzes and exams.

They will be solved during recitations by the TAs. Work on them prior to recitations!

Quizzes (24%): There will be 4 quizzes. The exact dates and places will be announced during the semester. The lowest grade will be dropped.

There is no make-up of the quizzes!

Project (8%): The project assignment (due sometime towards the end of the semester) will be a group project involving at most 3 students (mix of all Sections). We will mainly rely on Xpress software (available on campus wide computers) to solve linear and integer programming problems formulated in the MOSEL modeling language. Students are welcome to use GAMS/CPLEX, OPL/CPLEX options as well. You will be on your own in learning how to use the software. Each member is responsible from each phase. There will be **oral exams** in the end to assess your understanding.

Syllabus

Exams: One in-class midterm (28%) and one in-class comprehensive final exam (35%). Exact dates and places will be announced later.

Make-up Policy:

- A make-up examination will only be given under highly unusual circumstances (such as serious health or family problems).

- The student should contact the instructor as early as possible and provide the instructor with proper documentation (such as a medical note certified by Bilkent University's Health Center).

- The **(comprehensive)** make-up exam will be given before the final exam period

Syllabus

Grading:

5%	Class participation
8%	Project
24%	Quizzes
28%	Midterm
35%	Final exam

FZ Policy:

There is **no** FZ grade, everyone is entitled for the final exam.

Syllabus

Tentative Course Outline:

- Introduction to Operations Research and Mathematical Modeling
- Linear Programming Models and Solution Techniques
- Integer Programming and Network Models and Solution Techniques
- Project Management via CPM/PERT
- Engineering Economy

Syllabus

Rules:

- Make sure your STARS mail is valid and check your mail regularly.
- A make-up examination will only be given under extenuating circumstances (such as serious health or family problems). The student should contact the instructor as early as possible and provide the instructor with proper documentation (such as a medical note certified by Bilkent University's Health Center). The (comprehensive) make-up exam will be given during or right after the final exam period.
- No make-up will be given for quizzes under any circumstances.
- Cheating will not be tolerated and will be severely penalized. Disciplinary action will be taken.
- As a courtesy, please turn off your cell phones during class.

Engineering and Management

Engineering and Management

Definition of Engineering By ABET (Accrediting Board for Engineering and Technology)

The profession in which a knowledge of the mathematical and natural sciences gained by study, experience, and practice is applied with judgement to develop ways to utilize, economically, the materials and forces of the nature for the benefit of mankind.

Engineer: A person applying his mathematical and science knowledge properly for mankind

Engineering and Management

What is Management?

- Directing the actions of a group to achieve a goal in the most efficient manner
- Getting things done through people
- Process of achieving organizational goals by working with and through people and organizational resources

Engineering and Management

Functions of Managers

Planning: Selecting missions and objectives. Requires decision making.

Organizing: Establishing the structure for the objective.

Staffing: Keeping filled the organization structure

Leading: Influencing people to achieve the objective

Controlling: Measuring and correcting the activities

Engineering and Management

- Engineering management is a specialized form of management that is concerned with the application of engineering principles to business practice.

- Engineering management is a career that brings together the technological problem-solving savvy of engineering and the organizational, administrative, and planning abilities of management in order to oversee complex enterprises from conception to completion

Operations Research
(Management Science)

What is Operations Research (OR)?

OR is a **decision making tool** based on mathematical modeling and analysis to aid in determining **how best to design and operate a system** usually under the conditions requiring **allocation of scarce resources**.

http://www.scienceofbetter.org

Brief History of OR

- The main origin of Operations Research was during the **Second World War** in Britain.

- At the time of World War II, the military management in England invited a team of scientists to study the **strategic** and **tactical problems** related to air and land defense of the country.

Brief History of OR

- At that time the resources available with England was very limited and the objective was to win the war with available <u>scarce resources</u>.

- The resources such as food, medicines, ammunition, manpower etc., were required to manage war and for the use of the population of the country.

Brief History of OR

- It was necessary to decide upon the most effective utilization of the available resources to achieve the objective.
- It was also necessary to utilize the military resources cautiously.

Brief History of OR

- British military leaders asked an interdisciplinary group of scientists, engineers, psychologists, and sociologists to analyze some problems arising in military operations and find the optimum allocation of limited resources.
- These specialists had a brain storming session and came out with a method of solving the problem, which they coined the name **“Linear Programming”**.

Brief History of OR

- As the name indicates,

  Operations is used to refer to the problems of military,

  Research is use for inventing new method.

- As this method of solving the problem was invented during the war period, the subject is given the name **'OPERATIONS RESEARCH'** and abbreviated as **'O.R.'**

Brief History of OR

- After the World War II, there was a scarcity of industrial material and industrial productivity reached the lowest level.
- Soon after, it was discovered that OR techniques could also be applicable in various different settings.
- Today, OR techniques are heavily used to solve industrial, civil and military problems.

Three Main Properties of OR

- Systems Approach
- Interdisciplinary Action Based
- Scientific Methodology

Some Applications of OR

- **Manufacturing** (facility layout, machine scheduling, workforce scheduling, cost minimization, profit maximization, inventory allocation, etc.)
- **Finance** (portfolio optimization, risk minimization, etc.)
- **Statistics** (data fitting, error minimization, etc.)

Some Applications of OR

- **Networks** (design problems, shortest path, routing, etc.)
  - Transportation
  - Communication/computer
  - Physical (oil, gas, etc.)
  - Supply chain

OR Delivers Significant Value

Today, organizations and the world in which they operate continue to become more complex. Organizations worldwide in business, the military, health care, and the public sector are realizing powerful benefits from O.R., including:

Business insight: Providing quantitative and business insight into complex problems.

OR Delivers Significant Value

Business performance: Improving business performance by embedding model-driven intelligence into an organization's information systems to improve decision making.

Cost reduction: Finding new opportunities to decrease cost or investment.

Forecasting: Providing a better basis for more accurate forecasting and planning

OR Delivers Significant Value

Improved scheduling: Efficiently scheduling staff, equipment, events, and more.

Pricing: Dynamically pricing products and services.

Productivity: Helping organizations find ways to make processes and people more productive.

Profits: Increasing revenue or return on investment; increasing market share.

OR Delivers Significant Value

Quality: Improving quality as well as quantifying and balancing qualitative considerations.

Resources: Gaining greater utilization from limited equipment, facilities, money, and personnel.

Risk: Measuring risk quantitatively and uncovering factors critical to managing and reducing risk.

Throughput: Increasing speed or throughput and decreasing delays.

Operations Research Over the Years

- 1947
  - Project Scoop (Scientific Computation of Optimum Programs) with George Dantzig and others. Developed the simplex method for linear programs.
- 1950's
  - Lots of excitement, mathematical developments, queuing theory, mathematical programming.
- 1960's
  - More excitement, more development and grand plans.

Operations Research Over the Years

- 1970's
  - Disappointment, and a settling down. NP-completeness. More realistic expectations.
- 1980's
  - Widespread availability of personal computers. Increasingly easy access to data. Widespread willingness of managers to use models.
- 1990's
  - Improved use of O.R. systems. Further inroads of O.R. technology, e.g., optimization and simulation add-ins to spreadsheets, modeling languages, large scale optimization. More intermixing of A.I. and O.R.

Operations Research in the 2000's

- LOTS of opportunities for OR as a field
- Data, data, data
  - E-business data (click stream, purchases, other transactional data, E-mail and more)
  - Sensor data
  - The human genome project and its outgrowth
- Need for more automated decision making
- Need for increased coordination for efficient use of resources (Supply chain management)

OR Modeling

- In spite of the fundamental differences among several settings, many problems share similar characteristics.

  Ex:

  - A physical network vs. a computer network.
  - Parallel computing vs. queuing systems in a bank (scheduling, queuing).
  - Air traffic control vs. local area network (routing).

OR Modeling

- It is the OR modeler’s responsibility to observe and understand the inner workings of a system (usually in conjunction with the experts in the setting in which the problem arises).

- **Model:** A structure which has been built purposely to exhibit features and characteristics of some other object.

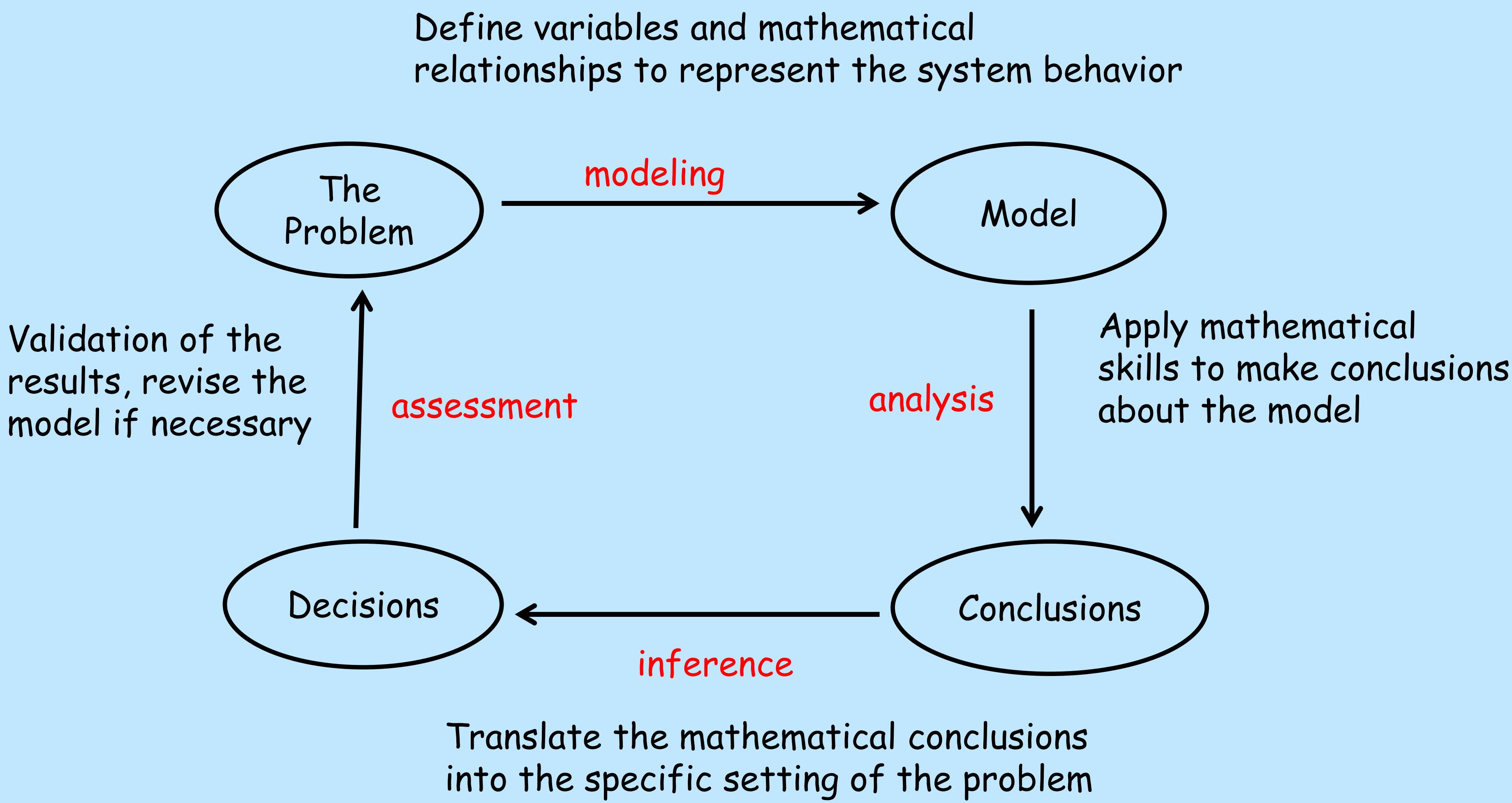
  Ex: Road maps, model airplanes, simulation model, mathematical model, etc.

The **essential feature of a mathematical model** in OR is that it involves a set of mathematical relationships (such as equations, inequalities, logical dependencies, etc.) which correspond to the characterization of the relationships in the real world.

Why do we need mathematical modeling?

- To have a greater understanding of the object being modeled.
- To analyze it mathematically to help suggest courses that are not apparent otherwise.
- To simplify the inner workings of a complex system and to use the universally understandable language of algebra and math.

The Modeling Process



Some Success Stories

- Optimal crew scheduling saves American Airlines $20 million/yr.
- Improved shipment routing saves Yellow Freight over $17.3 million/yr.
- Improved truck dispatching at Reynolds Metals improves on-time delivery and reduces freight cost by $7 million/yr.
- GTE local capacity expansion saves $30 million/yr.

Other Success Stories (cont.)

- Optimizing global supply chains saves Digital Equipment over $300 million.
- Restructuring North America Operations, Proctor and Gamble reduces plants by 20%, saving $200 million/yr.
- Optimal traffic control of Hanshin Expressway in Osaka saves 17 million driver hours/yr.
- Better scheduling of hydro and thermal generating units saves southern company $140 million.

Success Stories (cont.)

- Improved production planning at Sadia (Brazil) saves $50 million over three years.
- Production Optimization at Harris Corporation improves on-time deliveries from 75% to 90%.
- Tata Steel (India) optimizes response to power shortage contributing $73 million.
- Optimizing police patrol officer scheduling saves police department $11 million/yr.
- Gasoline blending at Texaco results in saving of over $30 million/yr.

Decision Making Problem

- What are the decision alternatives?
- Under what restrictions is the decision made?
- What is an appropriate objective criterion for evaluating the alternatives?

MODELING EXAMPLES

Milk Production

There are two machines needed to process milk to produce low-fat and regular milk. The processing times are:

	(Pasteurization) Machine 1	(Homogenization) Machine 2
Low-fat	3 minutes/liter	1 min/lt
Regular	2 min/lt	2 min/lt

In a day, machine 1 can be used for 12 hours and
machine 2 can be used for 6 hours

The sales price of low-fat milk is 2.5 TL/lt and that of
regular milk is 1.5 TL/lt.

Find how much low-fat and regular milk should be produced in a day to maximize total revenue.

1) What are the unknowns that should be determined by decision maker?

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$\mathbf{X_1}$: amount in liters of low-fat milk produced in a day
$\mathbf{X_2}$: amount in liters of regular milk produced in a day

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These are the decision variables (controllable variables)

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- Available machine hours
- Required machine hours/product
- Revenue per unit of each product

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- Required machine hours/product
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These are the parameters (uncontrollable variables)

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- Total production time on each machine cannot exceed the available machine hours.
- Production amounts should be nonnegative

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These are the constraints

4) What is the measure used to compare different decisions?

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- Total revenue

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- Total revenue

This is the objective function

Mathematical Model for Milk Production

maximize (max) total revenue

subject to (s.t.)

Total production time on Machine 1 $\leq$ 12 hours

Total production time on Machine 2 $\leq$ 6 hours

Production amounts $\geq$ 0

Mathematical Model for Milk Production

maximize (max) $2.5\ X_1 + 1.5\ X_2$

subject to (s.t.)

Total production time on Machine 1 $\leq$ 12 hours

Total production time on Machine 2 $\leq$ 6 hours

Production amounts $\geq$ 0

Mathematical Model for Milk Production

maximize (max) $2.5 X_1 + 1.5 X_2$

subject to (s.t.)

$3 X_1 + 2 X_2 \leq 12x60$

Total production time on Machine 2 $\leq$ 6 hours

Production amounts $\geq$ 0

Mathematical Model for Milk Production

maximize (max) $2.5\ X_1 + 1.5\ X_2$

subject to (s.t.)

$$3\ X_1 + 2\ X_2 \leq 720$$

$$X_1 + 2\ X_2 \leq 360$$

Production amounts ≥ 0

Mathematical Model for Milk Production

maximize (max) $2.5\ X_1 + 1.5\ X_2$

subject to (s.t.)

$$3\ X_1 + 2\ X_2 \leq 720$$

$$X_1 + 2\ X_2 \leq 360$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

Mathematical Model for Milk Production

maximize (max) $2.5\ X_1 + 1.5\ X_2$ (1) objective function

subject to (s.t.)

$$3\ X_1 + 2\ X_2 \leq 720 \qquad (2)$$

$$X_1 + 2\ X_2 \leq 360 \qquad (3)$$

$$X_1 \geq 0 \qquad (4)$$

$$X_2 \geq 0 \qquad (5)$$

constraints

The Milk Production Problem

This is an example of a **Linear Programming (LP)** model.

1. Any values of decision variables X_1 and X_2 that satisfy all of the constraints ((2)-(5) simultaneously) is called a feasible solution.

2. The set of all feasible solutions is called the feasible set or the feasible region.

3. The objective is to find a feasible solution that yields the best objective function value (value of objective function (1) evaluated at the feasible solution).

   If there exists such a feasible solution, it is called an optimal solution.

   The objective function value evaluated at an optimal solution is called the optimal value (objective function value).

The Linear Programming (LP) Problem

An LP problem is an optimization problem in which

- The objective function is given by a linear combination of the decision variables.

  i.e. min (or max) $c_1x_1 + c_2x_2 + \ldots + c_nx_n$,

  where $c_1,\ldots,c_n \in R$ are given and $x_1,\ldots,x_n$ are the decision variables.

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  where $c_1,\ldots,c_n \in R$ are given and $x_1,\ldots,x_n$ are the decision variables.

- The constraints are linear inequalities or linear equations.

$$\text{i.e. } a_1x_1 + a_2x_2 + \cdots + a_nx_n \begin{Bmatrix} \leq \\ = \\ \geq \end{Bmatrix} b, \quad \text{where } a_1,\ldots,a_n \in R,\ b \in R,$$

and $x_1,\ldots,x_n$ are the decision variables.

General Form of an LP Problem

$$\max\,(\min)\quad c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$$

s.t.

$$a_{i1} x_1 + a_{i2} x_2 + \cdots + a_{in} x_n \le b_i,\quad i = 1,\ldots,l$$

$$a_{j1} x_1 + a_{j2} x_2 + \cdots + a_{jn} x_n \ge b_j,\quad j = l+1,\ldots,l+p$$

$$a_{k1} x_1 + a_{k2} x_2 + \cdots + a_{kn} x_n = b_k,\quad k = l+p+1,\ldots,l+k+q$$

$$\text{The sign restrictions, i.e.}\quad \begin{pmatrix} x_1,\ldots,x_m \ge 0, \\ x_{m+1},\ldots,x_{m+s}\ \text{urs}, \\ x_{m+s+1},\ldots,x_n \le 0 \end{pmatrix}$$

General Form of an LP Problem

$$\max (\min) \quad c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$$

$x_1, \ldots, x_n$ are the decision variables.

s.t.

$$a_{i1} x_1 + a_{i2} x_2 + \cdots + a_{in} x_n \le b_i, \quad i = 1, \ldots, l$$

l "$\le$" constraints

$$a_{j1} x_1 + a_{j2} x_2 + \cdots + a_{jn} x_n \ge b_j, \quad j = l+1, \ldots, l+p$$

p "$\ge$" constraints

$$a_{k1} x_1 + a_{k2} x_2 + \cdots + a_{kn} x_n = b_k, \quad k = l+p+1, \ldots, l+k+q$$

q "$=$" constraints

$$\text{The sign restrictions, i.e.} \begin{pmatrix} x_1, \ldots, x_m \ge 0, \\ x_{m+1}, \ldots, x_{m+s} \text{ urs}, \\ x_{m+s+1}, \ldots, x_n \le 0 \end{pmatrix}$$

c_j's, a_{ij}'s, b_j's $\in$ R and given. They are the parameters.

Assumptions of an LP Problem

1) Proportionality & Additivity:

The contribution of each decision variable to the objective function or to each constraint is proportional to the value of that variable.

The contribution from decision variables are independent from one another and the total contribution is the sum of the individual contributions.

Assumptions of an LP Problem

2) Divisibility:

Each decision variable is allowed to take fractional values.

Assumptions of an LP Problem

2) **Divisibility:**

   Each decision variable is allowed to take fractional values.

3) **Certainty:**

   All input parameters (coefficients of the objective function and of the constraints) are known with certainty.

Blending Problem (1)

An oil refinery distills crude oil from Saudi Arabia and Venezuela into gasoline, jet fuel and lubricants. The crude differ in chemical composition and yields.

1 ton of S.A. crude oil → 0.3 tons of gasoline, 0.4 tons of jet fuel, 0.2 tons of lubricants

1 ton of Ven. crude oil → 0.4 tons of gasoline, 0.2 tons of jet fuel, 0.3 tons of lubricants

Remaining 0.1 tons is lost in refining

Blending Problem (1)

At most 90 tons can be purchased daily from S.A. at a cost of $600/ton

At most 60 tons can be purchased daily from Ven. at a cost of $550/ton

The daily demand is 20 tons of gasoline,
15 tons of jet fuel, and
5 tons of lubricants

How should the demand be met at a minimum total cost?

LP Model For Blending Problem (1)

Decision Variables:

LP Model:

Transportation Problem

There are two power plants generating electricity, and three factories that need electricity in their production.

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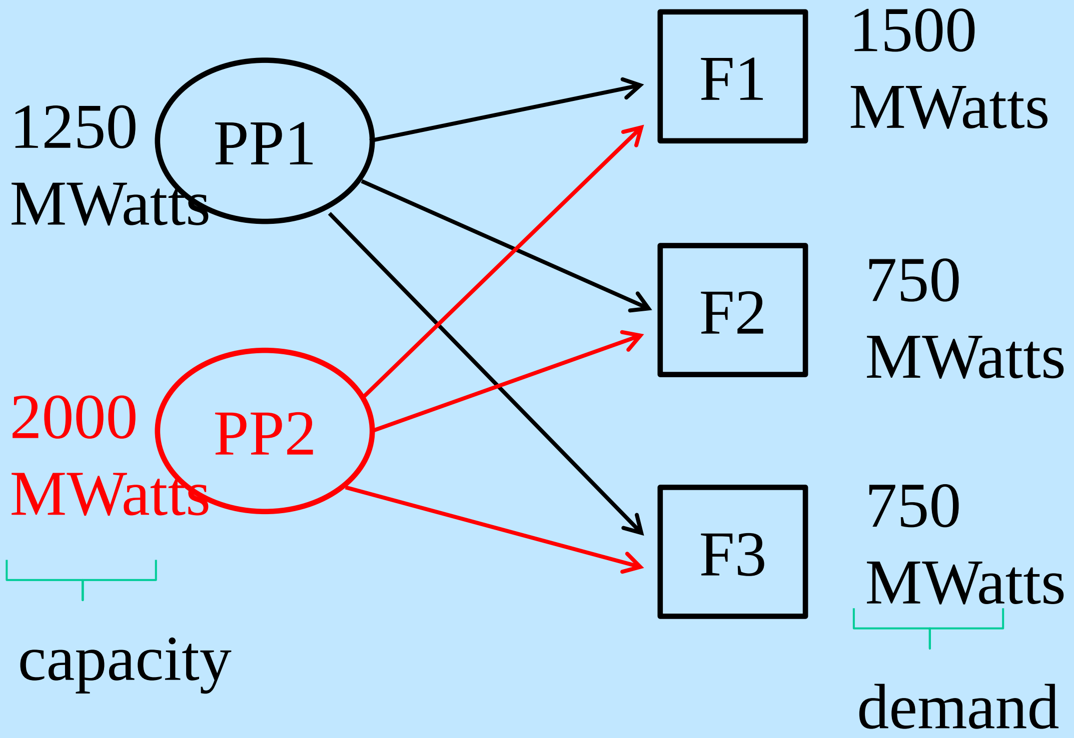

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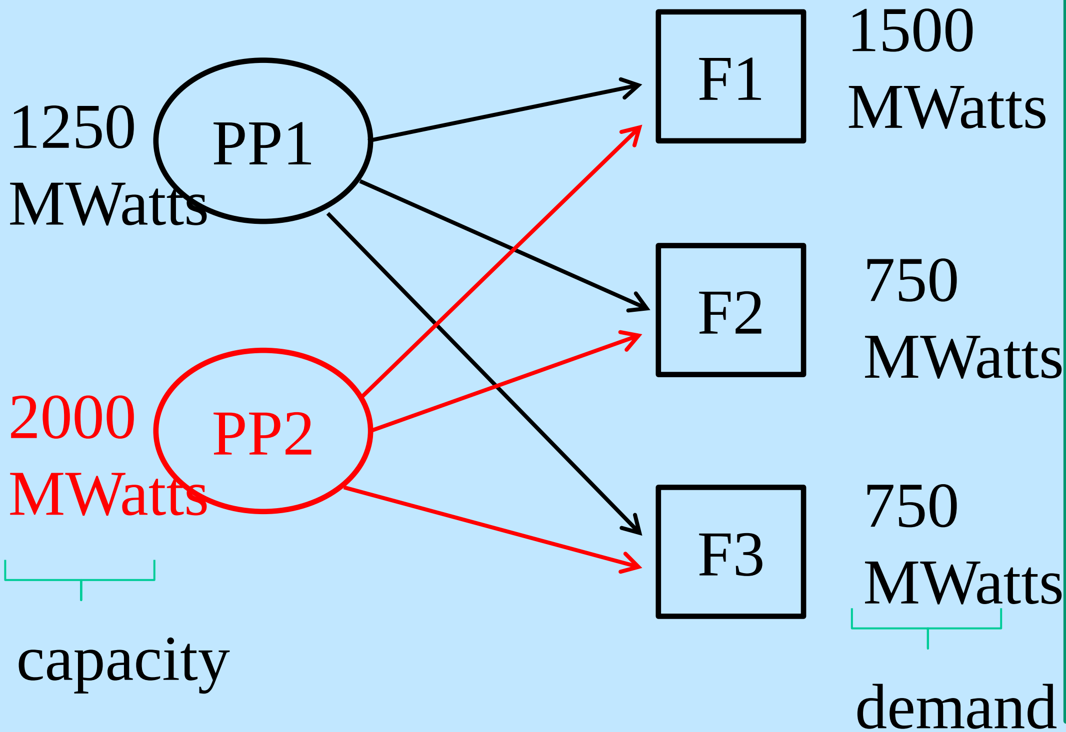

Transmission costs per MWatt (in USD):

	F1	F2	F3
PP1	50	100	60
PP2	30	20	35

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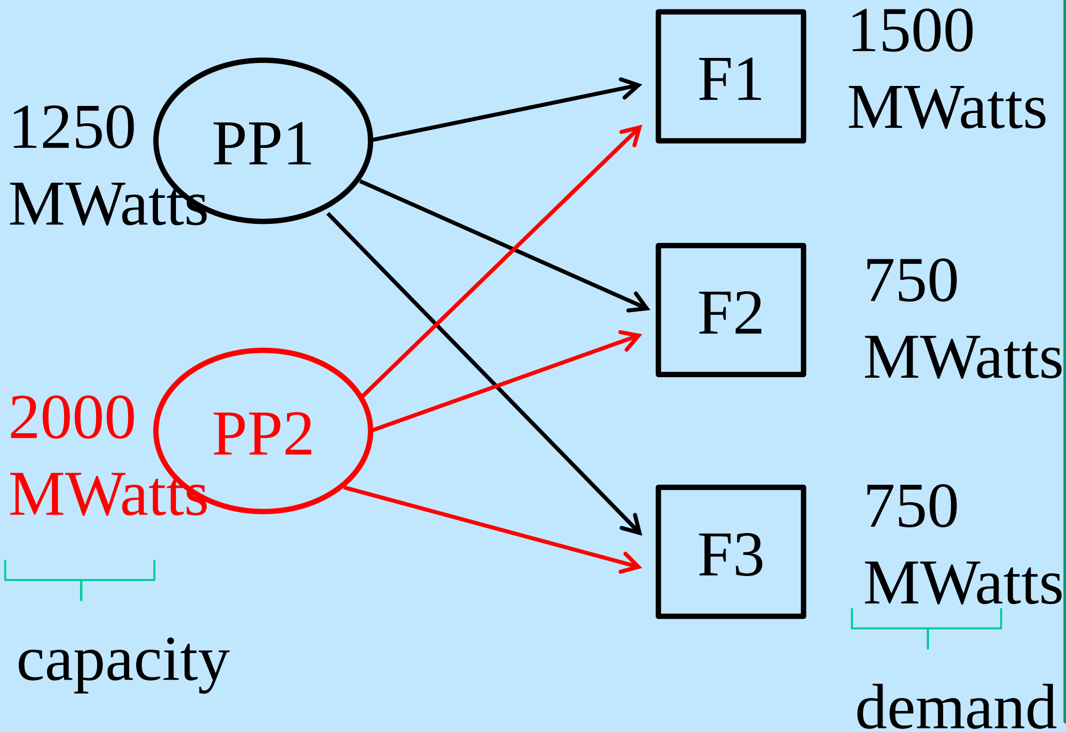

Transmission costs per MWatt (in USD):

	F1	F2	F3
PP1	50	100	60
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How to satisfy the demand of each factory from the two power plants with minimum total cost?

LP Model For Transportation Problem

Decision Variables:

LP Model:

General Transportation Problem

Say there are m warehouses and n stores.

Let a_i denote the amount of stock at warehouse i for $i=1, \ldots, m$

Let d_j denote the demand of store j for $j=1, \ldots, n$

Let c_{ij} denote the cost of transporting a unit of product from warehouse i to store j, for $i=1, \ldots, m$ and $j=1, \ldots, n$

How should the demands of stores be met at a minimum total transportation cost while respecting the stock limitations of the warehouses?

LP Model For General Transportation Problem

Decision Variables:

LP Model:

Blending Problem (2)

OJ Inc. produces orange juice from 3 different grades of Oranges. Oranges are graded 1(poor), 2(medium), and 3(good) depending on their quality.

Grade	Yield (Juice (lt)/kg)
1	0.4
2	0.5
3	0.6

The company currently has:

100 kgs of Grade 1 oranges

150 kgs of Grade 2 oranges

200 kgs of Grade 3 oranges

OJ Inc produces 3 types of orange juices from these oranges:

Superior, premium and regular

Blending Problem (2)

Production Requirements:

Type	Minimum average grade	Profits per lt	Minimum daily production
Superior	2.4	1.5 TL	45 lt
Premium	2.2	1 TL	60 lt
Regular	2.0	0.75 TL	100 lt

Assume the company may sell more than demand at the same prices.

How can the company maximize total profit while satisfying production requirements?

LP Model For Blending Problem (2)

Decision Variables:

LP Model:

Short Term Investment Planning

You have 12,500 TL at the beginning of year 2018 and would like to invest your money. Your goal is to maximize your total amount of money at the beginning of year 2021. You have the following options:

Short Term Investment Planning

You have 12,500 TL at the beginning of year 2018 and would like to invest your money. Your goal is to maximize your total amount of money at the beginning of year 2021. You have the following options:

Option	Duration (yrs)	Total interest rate	Available at the beginning of
1	2	26%	2018,2019
2	1	12%	2018,2019,2020
3	3	38%	2018
4	2	27%	2019

Short Term Investment Planning

Note that, normally:

Cash available at time t = cash invested at time t

+

uninvested cash at time t that

is carried over to time t+1

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However, since Option 2 is available at each year, we can always invest in this option rather than keeping some money for later better options.

Short Term Investment Planning

Hence, for this problem, the following holds:

Cash available at time t = cash invested at time t

LP Model For Short Term Investment Planning

Decision Variables:

LP Model:

Product Mix Problem

- Milk chocolate is produced with three ingredients.
  1 kg of milk chocolate contains:
  0.5 kg of milk,
  0.4 kg of cocoa,
  0.1 kg of sweetener.

- Each of these three ingredients must be processed before they can be used in the production of milk chocolate.

Product Mix Problem

- The factory has two departments that can process these ingredients:

	Dept 1 (hrs/kg)	Dept 2 (hrs/kg)
Milk	0.4	0.6
Cocoa	0.3	0.2
Sweetener	0.5	0.6

- Both departments have 150 hours of available time per week for processing.

Product Mix Problem

- The factory has two departments that can process these ingredients:

	Dept 1 (hrs/kg)	Dept 2 (hrs/kg)
Milk	0.4	0.6
Cocoa	0.3	0.2
Sweetener	0.5	0.6

- Both departments have 150 hours of available time per week for processing.

Formulate an LP problem that maximizes the total amount of milk chocolate produced in a week.

LP Model For Production Problem

Decision Variables:

LP Model:

Error Minimization Problem

- You estimate that there is a linear relationship between the stock price of a beverage company and the # of beverages sold (in thousands) in a day:

Day	Stock Price	# of beverages sold
1	2	10
2	3	12
3	2	8
4	3	11
5	4	14

Error Minimization Problem

- You predict the relationship as $\mathbf{s_i = a n_i + b}$, where s_i is the stock price and n_i is the # of beverages sold (in thousands) on day i, i=1,…,5.

  For a given choice of a and b, your estimation error on day i is $|\mathbf{s_i - a n_i - b}|$, i=1,…,5.

  Choose the unknown coefficients a and b such that the largest estimation error in any one of the 5 days is as small as possible.

LP Model For Error Minimization Problem

Decision Variables:

LP Model:

Production Planning Problem

XYZ Company produces three products A, B, and C and has to meet the demand for the next four weeks.

	Weekly Demand			
Product	**1**	**2**	**3**	**4**
A	25	30	15	22
B	15	16	25	18
C	10	17	12	12

Each unit of:

Product A requires 3 hours of labor,

Product B requires 2 hours of labor ,

Product C requires 4 hours of labor .

Production Planning Problem

- During a week, 120 hours of regular-time labor is available at a cost of $10/hour.

- In addition 25 hours of overtime labor is available weekly at a cost of $15/hour.

- **No shortages are allowed** and XYZ Company has 18 A, 13 B, and 15 C initial stocks for these products.

Production Planning Problem

Provide a Linear Programming formulation to determine a production schedule that minimizes the total labor costs for the next four weeks.

Production Planning Problem

Variables:

X_{ij}: the amount of product i produced during week j, $i \in \{A,B,C\}$ and $j=1,2,3,4$.

Inv_{ij}: the amount of inventory of product i at the end of week j after meeting demand. $i \in \{A,B,C\}$ and $j=0,1,2,3,4$ (in this case, Inv_{i0} is the notation for the initial stock)

OT_j: the amount of overtime hours used in week j, $j=1,2,3,4$.

Production Planning Problem

Production Constraints:

For any product i=1,2,3 and any week j=1,2,3,4,

$$Inv_{i,j-1} + X_{i,j} - D_{i,j} = Inv_{i,j}$$

(assume $D_{i,j}$ is the parameter that represents the demand of product i in month j)

For Product A:

$$8 + X_{A,1} - 25 = Inv_{A,1}$$
$$Inv_{A,1} + X_{A,2} - 30 = Inv_{A,2}$$
$$Inv_{A,2} + X_{A,3} - 25 = Inv_{A,3}$$
$$Inv_{A,3} + X_{A,4} - 22 = Inv_{A,4}$$

Production Planning Problem

Production Constraints:

For Product B:

$$13 + X_{B,1} - 15 = Inv_{B,1}$$
$$Inv_{B,1} + X_{B,2} - 16 = Inv_{B,2}$$
$$Inv_{B,2} + X_{B,3} - 25 = Inv_{B,3}$$
$$Inv_{B,3} + X_{B,4} - 18 = Inv_{B,4}$$

For Product C:

$$15 + X_{C,1} - 10 = Inv_{C,1}$$
$$Inv_{C,1} + X_{C,2} - 17 = Inv_{C,2}$$
$$Inv_{C,2} + X_{C,3} - 12 = Inv_{C,3}$$
$$Inv_{C,3} + X_{C,4} - 12 = Inv_{C,4}$$

Production Planning Problem

Demand is met on time constraint:

Production Planning Problem

Demand is met on time constraint:

$Inv_{i,j} \geq 0$ for $i \in \{A,B,C\}$ and $j=1,2,3,4$.

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Labor Constraints:

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Labor Constraints:

$$3X_{A,1} + 2X_{B,1} + 4X_{C,1} \leq 120 + OT_1$$

$$3X_{A,2} + 2X_{B,2} + 4X_{C,2} \leq 120 + OT_2$$

$$3X_{A,3} + 2X_{B,3} + 4X_{C,3} \leq 120 + OT_3$$

$$3X_{A,4} + 2X_{B,4} + 4X_{C,4} \leq 120 + OT_4$$

Production Planning Problem

Demand is met on time constraint:

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Labor Constraints:

$$3X_{A,1} + 2X_{B,1} + 4X_{C,1} \leq 120 + OT_1$$

$$3X_{A,2} + 2X_{B,2} + 4X_{C,2} \leq 120 + OT_2$$

$$3X_{A,3} + 2X_{B,3} + 4X_{C,3} \leq 120 + OT_3$$

$$3X_{A,4} + 2X_{B,4} + 4X_{C,4} \leq 120 + OT_4$$

$OT_j \leq 25$, $j=1,2,3,4$.

Production Planning Problem

Sign restrictions:

$X_{i,j} \geq 0, \quad i \in \{A,B,C\}$ and $j=1,2,3,4.$

$OT_j \geq 0, \quad j=1,2,3,4.$

Objective Function:

Production Planning Problem

Sign restrictions:

$X_{i,j} \geq 0$, $i \in \{A,B,C\}$ and $j=1,2,3,4$.

$OT_j \geq 0$, $j=1,2,3,4$.

Objective Function:

$$\text{Min} \quad 10 \cdot \left(\sum_{j=1}^{4} 3X_{A,j} + 2X_{B,j} + 4X_{C,j} \right) + 5 \sum_{j=1}^{4} OT_j$$

Class Exercises

Question 1) Rylon Corporation manufactures Brute and Channelle perfumes from a raw material costing $3/pound.

- Processing 1 pound of the material takes 1 hour and yields 3 ounces of Regular Brute and 4 ounces of Regular Chanelle. This perfume can be sold directly or can be further processed to make luxury versions of the perfumes.
- One ounce of Regular Brute can be sold for $7, but 3 hours of processing and $4 of cost will convert it into one ounce of Luxury Brute, which sells for $18.
- One ounce of Regular Chanelle can be sold for $6, but 2 hours of processing and $4 of cost will convert it into one ounce of Luxury Chanelle, which sells for $14.
- Rylon has available 4000 pounds of raw material, 6000 hours of processing time, and will sell everything it produces.

Determine how Rylon can maximize its profit.

LP Model For Rylon Corporation

Decision Variables:

LP Model:

Class Exercises

Question 2) Bel Aire Pillow Company needs to manufacture 12,000 pillows in their factory next week. They have two kinds of employees: line workers (who do the actual work) and supervisors (who oversee the line workers). Bel Aire currently has 120 line workers and 20 supervisors. Each line worker can make 300 pillows per week. Supervisors do not engage in actual pillow-making.

A recent study done on Bel Aire's operation concluded that, for proper supervision, there must be at least 1 supervisor for every 5 line workers. The study also suggested that Bel Aire might be able to cut its costs by doing some or all of the following:

- hiring new employees as line workers (the training cost is $100/worker)
- firing some current line workers (the fired worker is given $240 severance pay)
- promoting some current line workers to supervisors (the training cost is $200/worker)
- demoting some current supervisors to line workers (no cost associated with this)

Pay for a line worker is $400/week. Pay for a supervisor is $700/week. All hiring and firing, and training occur before the upcoming work week, and the costs of those activities will be charged against the upcoming week. Bel Aire wishes to perform the appropriate hiring, firing, promotions and demotions that will minimize its operations cost for next week.

Mathematical Model For Bel Aire Company

Decision Variables:

Model:

What is different about this mathematical model?

Is it a linear programming problem?