IE 400 2019-2020 Fall Study Set 3

1) Use the branch-and-bound method to find the optimal solution to the following IP:

$$\text{Minimize } 9x_1 + 13x_2 + 10x_3 + 8x_4 + 8x_5$$

$$\text{s.t. } 6x_1 + 3x_2 + 2x_3 + 4x_4 + 7x_5 \geq 40$$

$$x_1 \leq 1, x_2 \geq 1, x_3 \geq 2, x_4 \geq 1, x_5 \leq 3$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0 \text{ integer}$$

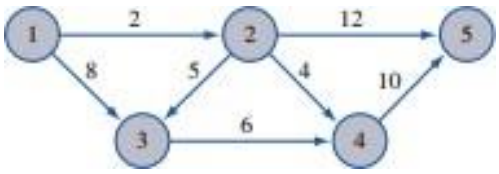
2) Use the branch-and-bound method to find the optimal solution to the following IP:

$$\begin{aligned} \max z &= 7x_1 + 3x_2 \\ \text{s.t.} \quad 2x_1 + x_2 &\leq 9 \\ 3x_1 + 2x_2 &\leq 13 \\ x_1, x_2 &\geq 0;\ x_1, x_2 \text{ integer} \end{aligned}$$

3) Use the branch-and-bound method to find the optimal solution to the following IP:

$$\begin{aligned} \max z &= 4x_1 + 5x_2 \\ \text{s.t.} \quad 3x_1 + 2x_2 &\leq 10 \\ x_1 + 4x_2 &\leq 11 \\ 3x_1 + 3x_2 &\leq 13 \\ x_1, x_2 &\geq 0;\ x_1, x_2 \text{ integer} \end{aligned}$$

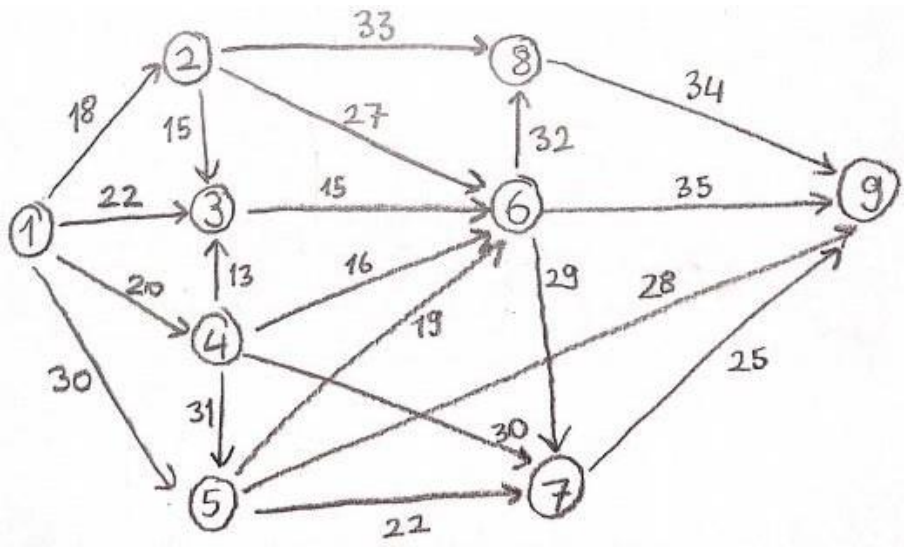
4) Find the shortest path from node 1 to node 5 in the figure below:



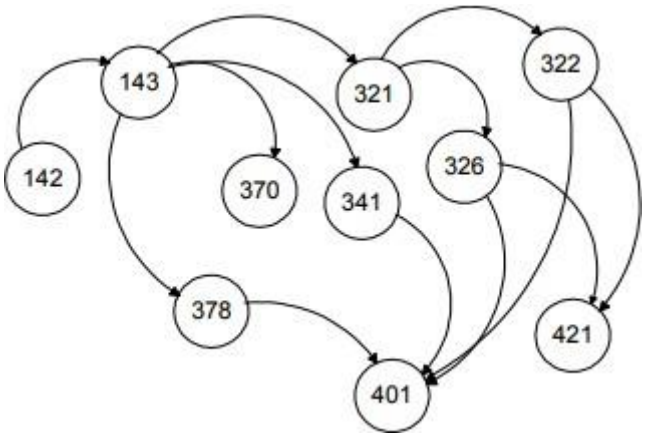
5) Suppose it costs $10,000 to purchase a new car. The annual operating cost and resale value of a used car are shown in Table 4. Assuming that one now has a new car, model and solve replacement problem that minimizes the net costs of owning and operating a car for the next six years using shortest path.

Age of Car (Years)	Resale Value ($)	Operating Cost ($)
1	7,000	300 (year 1)
2	6,000	500 (year 2)
3	4,000	800 (year 3)
4	3,000	1,200 (year 4)
5	2,000	1,600 (year 5)
6	1,000	2,200 (year 6)

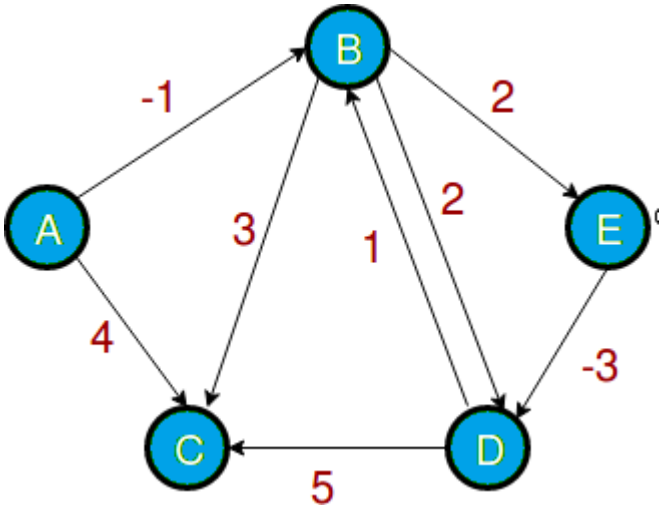
6) Find shortest paths from 1 to all other nodes using Dijkstra's algorithm.



7) Find an order in which all these courses can be taken using topological ordering.



8) Use Bellman Ford algorithm to find the shortest path of the following graph where A is the source node:



9) Given a network, a special source node s and weights on arcs corresponding to heights, find from s to every other node a path on which the highest height encountered is as few as possible.

a) Does principle of optimality apply for this problem? Why or why not?

b) Write down the functional equations corresponding to this variant of the shortest path problem.

c) Solve the above functional equation by adapting an algorithm (state which one) designed for solving the single source shortest path problem

10) For the following statements, indicate whether they are True or False. Either provide a counter example or explain your answer.

a) If all arcs in a network have different costs, the network has a unique shortest path tree.

b) Given a graph with positive edge weights, Dijkstra's algorithm and the Bellman-Ford algorithm can produce different shortest-path trees despite always producing the same shortest-path weights.

c) Every directed acyclic graph has exactly one topological ordering.

d) In a shortest path problem, if each arc length increases by k units, shortest path distances increases by a multiple of k.

e) In a directed network with positive arc lengths, if we eliminate the direction on every arc (i.e. make it undirected), the shortest path distances will not change.

f) If the arc costs all increase (decrease) by the same positive constant k, shortest paths do not change.