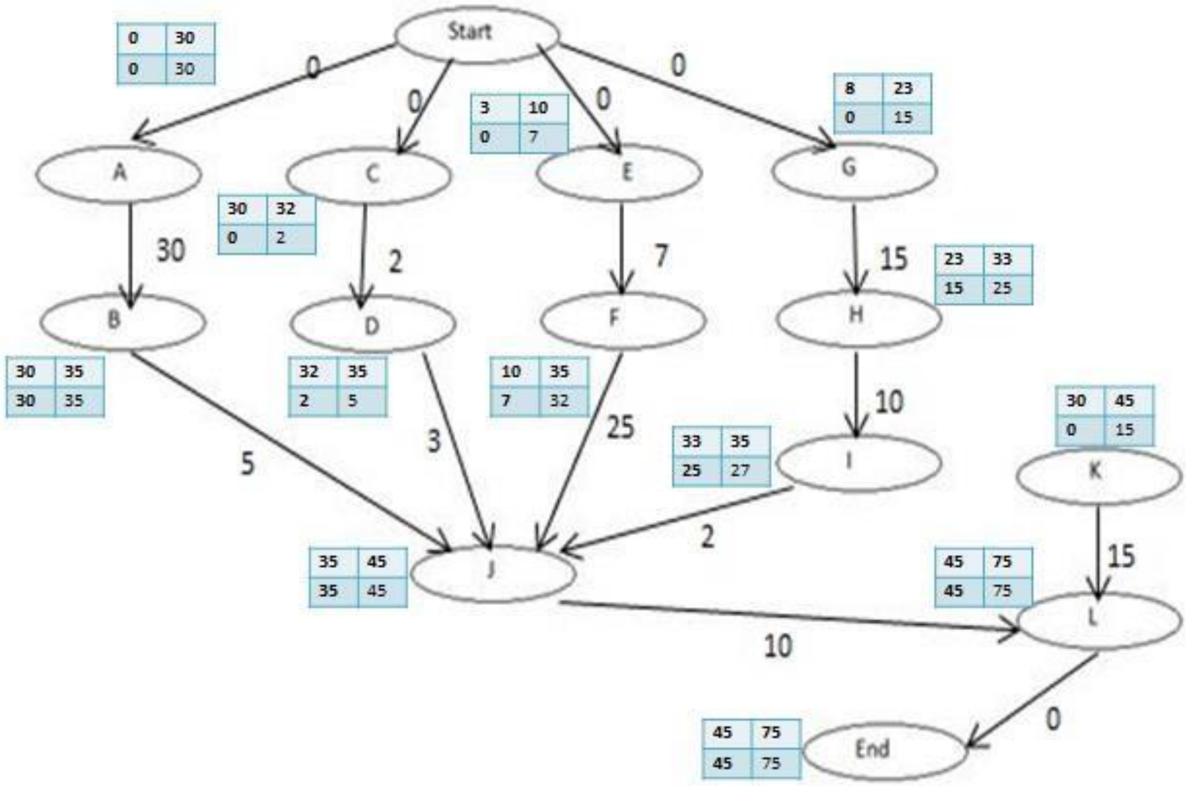
IE 400 Fall 2019 Study Set 4 Solutions

1)

a) b)



Activity	Slack
A	0
B	0
C	30
D	30
E	3
F	3
G	8
H	8
I	8
J	0
K	30
L	0

c) Since the slack for cutting the onions and mushrooms is 3, the dinner will be delayed 6-3=3 minutes. If the cutting time is 2 minutes, slack is 10 (latest finish time) – 2 (task duration) = 8 minutes. Since the slack time is more than the delay time, it does not delay the dinner.

d) Let N be the set of activities and A be the set of arc in the project network.

Parameters

a_i : task duration of activity i, for all i ϵ

N d: project duration (75)

Decision Variables

x_i : start time of activity i, for all i ϵ N

Model

Max Σx_i

s.t $x_j \geq x_i + a_i$ for all i,j ϵ N

$x_{end} \leq d$

$x_i \geq 0$ for all i ϵ N

$x_{start} = 0$

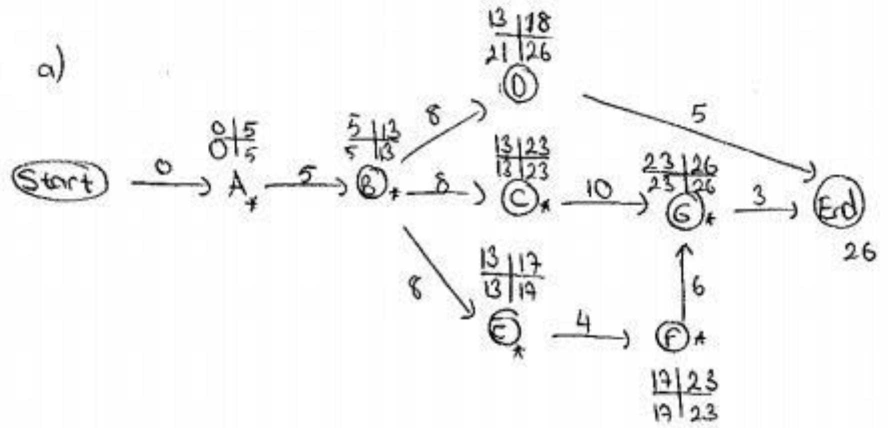
2)

End of Year	Principal Repayment	Interest payment	Remaining Balance
0			$10,000
1	$1,671	$900	$8,329
2	$1,821	$750	$6,508
3	$1,985	$586	$4,523
4	$2,164	$407	$2,359
5	$2,359	$212	$0

3) P= $7,000(P/F, 6%, 2) + $6,000(P/F, 6%, 5) + $5,000(P/F, 6%, 7) = $14,038

4)

a)



Critical paths: A-B-C-G and A-B-E-F-G

Duration: 26 days

b) D can be delayed for 8 days.

c) x_J: start time of activity J

y_J: # days duration of activity J is reduced. } Decision variables

min $30y_A + 15y_B + 20y_C + 40y_D + 20y_E + 30y_F + 40y_G$

st

$x_A \geq x_{start}$ $\quad y_A \leq 2$

$x_B \geq x_A + 5 - y_A$ $\quad y_B \leq 3$

$x_C \geq x_B + 8 - y_B$ $\quad y_C \leq 1$

$x_D \geq x_B + 8 - y_B$ $\quad y_D \leq 2$

$x_E \geq y_B + 8 - y_B$ $\quad y_E \leq 2$

$x_F \geq x_E + 4 - y_E$ $\quad y_F \leq 3$

$x_G \geq x_C + 10 - y_C$ $\quad y_G \leq 1$

$x_G \geq x_F + 6 - y_F$

$x_{end} \geq x_G + 3 - y_G$

$x_{end} \geq x_D + 5 - y_D$

$x_{start} = 0$

$x_{end} \leq 20$

$y_A, y_B, y_C, y_D, y_E, y_F, y_G \geq 0$ and integer

OR define the parameters:

c_J: cost of reduction

r_J: max reduction

d_J: duration of activity

min $\sum_J c_J y_J$

st

$y_J \leq r_J$

$x_J \geq x_i + d_i - y_i \quad \forall (i,j) \in A$

$x_{start} = 0$

$x_{end} \leq 20$

$y_J \geq 0$ and integer $\forall J$

5)

$F = 2P = P(1+0.15)^N$

$\log 2 = N \log (1.15)$

N= 4.96 years

6)

Selecting the base period at $n = 0$, we find

$$\$100(P/A,13\%,5) + \$20(P/A,13\%,3)(P/F,13\%,2) = A(P/A,13\%,5)$$

$$\$351.72 + \$36.98 = (3.5172)A$$

$$A = \$110.51$$

7)

7 You need a machine (mch) for the next 5 years

A new mch costs $1000

Cost of maintaining the mch during its i^{th} year of operation is m_i.

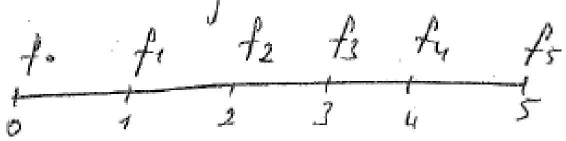
$m_1 = \$60 \quad m_2 = \$80 \quad m_3 = \$120$

A mch can be kept up to 3 years.

The trade in value after i years is s_i:

$s_1 = \$800 \quad s_2 = \$600 \quad s_3 = \$500$

min cost over 5 years.



$f_k(i)$ = min cost from time k to time 5 if the mch is i years old at time k.

Base case: $f_5(i) = -s_i \quad , i = 1,2,3$

Recursion: for $k = 4,3,2,1$

$$f_k(3) = -500 + 1000 + 60 + f_{k+1}(1)$$
$$f_k(2) = \min\{-600+1000+60+f_{k+1}(1),\ 120 + f_{k+1}(3)\}$$
$$f_k(1) = \min\{-800+1000+60+f_{k+1}(1),\ 80 + f_{k+1}(2),\ \dots\}$$

Opt. Value $\Rightarrow f_0 = 1000 + 60 + f_1(1)$!

Stage 5

$f_5(1) = -800$. Sell

$f_5(2) = -600$ Sell

$f_5(3) = -500$ Sell

Stage 4

$f_4(3) = -500+1000+60+\overbrace{f_5(1)}^{-800} = -240$ Sell

$f_4(2) = \min\{\underbrace{-600+1000+60+f_5(2)}_{-240},\ \underbrace{120+f_5(3)}_{-380}\}$
$= -380$ Keep

$f_4(1) = \min\{\underbrace{-800+1000+60+f_5(1)}_{-540},\ \underbrace{80+f_5(2)}_{80-600}\} = -540$ Sell

Stage 3

$f_3(3) = -500+1000+60+\overbrace{f_4(1)}^{-540} = 20$ Sell

$f_3(2) = \min\{\underbrace{-600+1000+60+\underbrace{f_4(1)}_{-540}}_{-80},\ 120+\underbrace{f_4(3)}_{-240}\} = -120$ Keep

$f_3(1) = \min\{\underbrace{-800+1000+60+f_4(1)}_{-280},\ \underbrace{80+\underbrace{f_4(2)}_{-380}}_{-300}\} = -300$ Keep

Stage 2

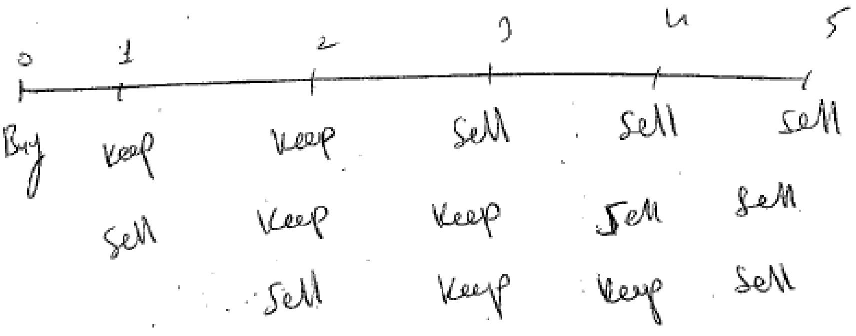
$f_2(2) = \min\{\underbrace{-600+1000+60+\underbrace{f_3(1)}_{-300}}_{160},\ 120+\underbrace{f_3(3)}_{20}\} = 140$ Keep

$f_2(1) = \min\{\underbrace{-800+1000+60+\underbrace{f_3(1)}_{-300}}_{-40},\ 80+\underbrace{f_3(2)}_{-120}\} = -40$ Keep or Sell

Stage 1

$f_1(1) = \min\{\underbrace{-800+1000+60+\underbrace{f_2(1)}_{-40}}_{220},\ 80+\underbrace{f_2(2)}_{140}\} = 220$ Keep or Sell

Opt. Value $\Rightarrow$ $f_0 = 1000 + 60 + \underbrace{f_1(1)}_{220} = 1280$



8)

$$\max \sum_{j=1}^{n} c_j x_j$$

st

$$\sum_{j=1}^{n} a_j x_j \le b_1$$

$$\sum_{j=1}^{n} d_j x_j \le b_2$$

$x_j \ge 0 \ \forall j$ and integer

Think b_1 and b_2 as resources and we have n type items

x_j : decision at stage j, $j = 1, \dots, n$ (amount of type i items taken)

ℓ_1, ℓ_2 : amount of resources available

$(\ell_1, \ell_2) \rightarrow$ state

$f_j(\ell_1, \ell_2)$: maximum value of the objective function considering stages (items) $j, j+1, \dots, n$ with available resources ℓ_1 and ℓ_2

so the recursion function is as follows:

$$f_j(\ell_1, \ell_2) = \max_{\substack{a_j x_j \le \ell_1 \\ d_j x_j \le \ell_2 \\ x_j \ge 0,\ \text{integer}}} \left\{ c_j x_j + f_{j+1}(\ell_1 - a_j x_j, \ell_2 - d_j x_j) \right\} \quad \forall j = 1, \dots, n$$

Base case: $f_{n+1}(k_1, k_2) = 0 \quad \forall k_1, k_2$

Optimal solution value: $f_1(b_1, b_2)$