

Q.1)

Decision Variables :

X_{ij} = amount of land allocated by kibbutz j for crop i

i : 1 (Sugar beets), 2 (Cotton), 3 (Sorghum)

j : 1, 2, 3

Model

$$\text{Max } 1000(X_{11} + X_{12} + X_{13}) + 750(X_{21} + X_{22} + X_{23}) + 250(X_{31} + X_{32} + X_{33})$$

s.t

$$\left. \begin{aligned} X_{11} + X_{12} + X_{13} &\leq 600 \\ X_{21} + X_{22} + X_{23} &\leq 500 \\ X_{31} + X_{32} + X_{33} &\leq 325 \end{aligned} \right\} \text{for max quota}$$

$$\left. \begin{aligned} 3X_{11} + 2X_{21} + X_{31} &\leq 600 \\ 3X_{12} + 2X_{22} + X_{32} &\leq 800 \\ 3X_{13} + 2X_{23} + X_{33} &\leq 375 \end{aligned} \right\} \text{for water Const.}$$

$$\left. \begin{aligned} X_{11} + X_{21} + X_{31} &\leq 400 \\ X_{12} + X_{22} + X_{32} &\leq 600 \\ X_{13} + X_{23} + X_{33} &\leq 300 \end{aligned} \right\} \text{for usable land}$$

$$\left(\frac{X_{11} + X_{21} + X_{31}}{400} = \frac{X_{12} + X_{22} + X_{32}}{600} = \frac{X_{13} + X_{23} + X_{33}}{300} \right)$$

$$X_{ij} \geq 0 \quad \begin{matrix} i=1,2,3 \\ j=1,2,3 \end{matrix}$$

$$\begin{aligned} (X_{12} + X_{22} + X_{32}) - 2(X_{13} + X_{23} + X_{33}) &= 0 \\ 3(X_{11} + X_{21} + X_{31}) - 2(X_{12} + X_{22} + X_{32}) &= 0 \\ 3(X_{11} + X_{21} + X_{31}) - 4(X_{13} + X_{23} + X_{33}) &= 0 \end{aligned}$$

linearize the equation.

Dec. Var.

C_i = Cash on hand at the end of month i , $i = 1, \dots, 4$

X_i = amount of tons purchased at the beginning of month i , $i = 1, \dots, 4$

Y_i = amount of tons sold at the end of month i , $i = 1, \dots, 4$

I_i = amount of tons on hand at month i , $i = 1, \dots, 4$ (after purchase, before selling)

Parameters

P_i = price of buying at month i , $i = 1, \dots, 4$

S_i = price of selling at month i , $i = 1, \dots, 4$

Model

$$\max C_4$$

s.t

$$I_i = I_{i-1} + X_i - Y_{i-1} \quad i = 1, \dots, 4 \quad (\text{Inventory balance})$$

$$C_i = C_{i-1} - P_i X_i + S_i Y_i \quad i = 1, \dots, 4 \quad (\text{Cash balance})$$

$$I_i \leq 400 \quad i = 1, \dots, 4 \quad (\text{Capacity Constraint})$$

$$Y_i \leq I_i \quad i = 1, \dots, 4 \quad (\text{Can't sell more corn than you have})$$

$$P_i X_i \leq C_{i-1} \quad i = 1, \dots, 4 \quad (\text{Can't spend more money than you have})$$

$$I_0 = 50$$

$$C_0 = 1000$$

$$Y_0 = 0$$

$$C_i, X_i, Y_i, I_i \geq 0 \quad i = 1, \dots, 4$$

Dec. Var. : X_i : number of hours process i runs, $i=1,2,3$

$$\begin{aligned} \text{a) max } & \overset{\text{rev of WB}}{9(2X_1+2X_3)} + \overset{\text{rev of BC}}{10(X_1+3X_2-3X_3)} + \overset{\text{rev of GST}}{24X_3} - \overset{\text{running Costs of processes}}{(5X_1+9X_2+X_3)} \\ & - \underset{\text{Cost of A}}{2(2X_1+X_2)} - \underset{\text{Cost of B}}{3(3X_1+3X_2+2X_3)} \end{aligned}$$

$$\begin{aligned} \text{s.t } & 2X_1+X_2 \leq 200 \quad (\text{Sugar type A}) \quad (i) \\ & 3X_1+3X_2+2X_3 \leq 300 \quad (\text{Sugar type B}) \quad (ii) \\ & X_1+X_2+X_3 \leq 100 \quad (\text{hour limitation}) \quad (iii) \\ & X_1+3X_2 \geq 3X_3 \quad (\text{max BC that can be used in process 3}) \quad (iv) \\ & X_1, X_2, X_3 \geq 0 \quad (v) \end{aligned}$$

b) Instead of (i) and (ii), add the following one:

$$(2X_1+X_2) + (3X_1+3X_2+2X_3) \leq 500 \rightarrow 5X_1+4X_2+2X_3 \leq 500$$

c) Add dec. var y : # of Wonka Box

$$y = \min\left\{2X_1+2X_3, \frac{X_1+3X_2-3X_3}{2}, X_3\right\} \rightarrow \text{NOT linear, you MUST linearize in your model}$$

$$\text{Model } \max 54y - (5X_1+9X_2+X_3) - 2(2X_1+X_2) - 3(3X_1+3X_2+2X_3)$$

$$\text{s.t } (i) - (v)$$

$$y \leq 2X_1+2X_3$$

$$y \leq \frac{X_1+3X_2-3X_3}{2}$$

$$y \leq X_3$$

$$y \geq 0$$

Decision Variables

x_1 = number of netbook produced

x_2 = number of desktop

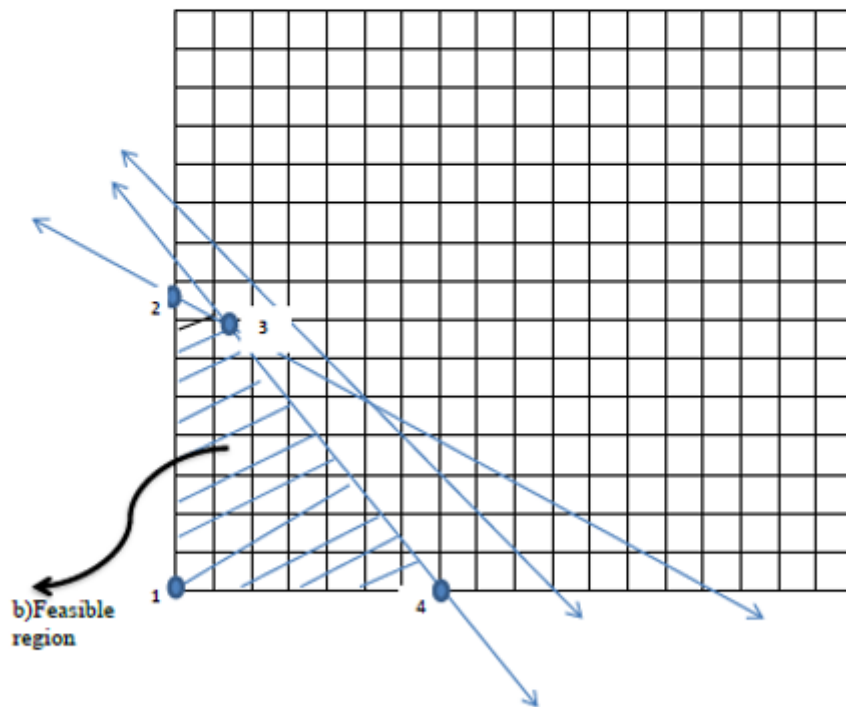
maximize $750x_1 + 1000x_2$

$$x_1 + x_2 \leq 10 \quad (\text{memory}) \quad (1)$$

$$x_1 + 2x_2 \leq 15 \quad (\text{chip}) \quad (2)$$

$$4x_1 + 3x_2 \leq 25 \quad (\text{assembly time}) \quad (3)$$

all variables ≥ 0



c) Extreme Points

1- (0,0)

2- (0, 7.5)

3- (1, 7)

4- (6.25, 0)

5)

$$\max z = 3x_1 + 2x_2$$

$$\text{s.t. } 2x_1 + x_2 \leq 4 \quad (1)$$

$$-2x_1 + x_2 \leq 2 \quad (2)$$

$$x_1 - x_2 \leq 1 \quad (3)$$

$$x_1, x_2 \geq 0$$

→

$$2x_1 + x_2 = 4$$

→

$$-2x_1 + x_2 = 2$$

→

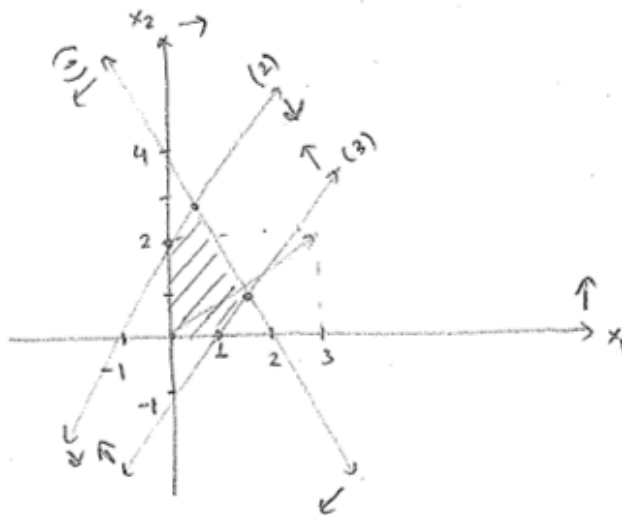
$$x_1 - x_2 = 1$$

(x_1, x_2)

$$(2, 4)$$

$$(-1, 2)$$

$$(1, -1)$$



Extreme points

Object. value

$$(0, 0)$$

$$0$$

$$(0, 2)$$

$$4$$

$$(1, 0)$$

$$3$$

$$(5/3, 2/3)$$

$$19/3$$

*

$$(1/2, 3)$$

$$15/2$$

$$z^* = 15/2$$

$$x^* = (1/2, 3)$$