

IE400 Homework 1 - Spring 2020

1. Use the simplex algorithm to solve the following LP:

$$\begin{aligned} \max z &= x_1 + x_2 \\ \text{s.t. } x_1 - x_2 &\leq 1 \\ x_1 + x_2 &\leq 2 \\ x_1, x_2 &\geq 0 \end{aligned}$$

2. Use the Big-M and Two-Phase Methods to solve the following LP:

$$\begin{aligned} \min z &= x_1 + x_2 \\ \text{s.t. } 2x_1 + x_2 + x_3 &= 4 \\ x_1 + x_2 + 2x_3 &= 2 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

3. A company wishes to build various industrial plants to supply 7 cities with a certain product. The demand of these cities based on demographic factors and social characteristics is estimated. These values are shown in Table 1. A study has indicated 6 possible sites for these industrial plants. It is supposed that all the plants have the same characteristics. The maximum production capacity per plant is 6 units. Fixed cost of building a plant has been calculated in 10 monetary units for the period of study. Table 2 shows the profit achieved by selling, to city j, one unit manufactured in one plant located at site i. The decision maker needs to determine the number of plants and their locations, so that the cities' demands are satisfied, and a maximum financial benefit is obtained. The optimization problem associated with this decision process consists of maximizing the total benefit including amortization costs, subject to the constraints.

Table 1

	City 1	City 2	City 3	City 4	City 5	City 6	City 7
Demands	1.5	2	3	4	2.5	1	2

Table 2

Locations/Cities	City 1	City 2	City 3	City 4	City 5	City 6	City 7
Location 1	4	4.5	2.5	0.5	1	0.5	-3.5
Location 2	4	4.5	2.5	4.2	3.5	1.5	-0.5
Location 3	3.5	5	4	3.5	4.5	1.5	0
Location 4	1.3	3	5	3.3	5.5	1.8	1.3
Location 5	0.5	1	1.5	5	4	5.5	3
Location 6	-0.1	0	1.5	3.3	4	4.5	2

4. The manager of a factory wants to be able to manufacture five different product types. There are 10 different candidate machines. Investment cost of each machine is given by Table 3. However, each machine is able to manufacture only certain product types. If a machine is able to produce a product type it is marked as “X” in Table 4.
- Formulate an IP that determines a set of machines requiring the minimum amount of investment cost such that each product type can be manufactured.
 - Modify your formulation to also ensure that
 - if machine 3 or machine 5 is used, then machine 2 must also be used and
 - \$500 discount is given if 3 or more machines are bought.

Table 3

	1	2	3	4	5	6	7	8	9	10
Investment Cost (\$)	3000	5000	1000	2000	1000	4000	3000	1000	2000	2000

Table 4

Products/Machines	1	2	3	4	5	6	7	8	9	10
1	X	X		X	X			X	X	
2	X		X							
3		X			X		X			X
4			X			X		X		
5	X	X		X		X	X		X	X