IE 400 – HOMEWORK 1

1) $\max\ z = x_1 + x_2$ → converting to standard form (0.25 pts)

s.t. $x_1 - x_2 + s_1 = 1$

$x_1 + x_2 + s_2 = 2$

$x_1, x_2, s_1, s_2 \geq 0$

	z	x_1	x_2	s_1	s_2	RHS
	1	-1	-1	0	0	0
s_1	0	1	-1	1	0	1
s_2	0	1	1	0	1	2

x_1 or x_2 enters arbitrarily, I selected x_1

min. ratio test → $\min\left\{\frac{1}{1}, \frac{2}{1}\right\} = 1$

s_1 leaves

	z	x_1	x_2	s_1	s_2	RHS
	1	0	-2	1	0	1
x_1	0	1	-1	1	0	1
s_2	0	0	2	-1	1	1

x_2 enters

min. ratio test → $\min\left\{\frac{1}{2}\right\}$

s_2 leaves

	z	x_1	x_2	s_1	s_2	RHS
	1	0	0	0	1	2
x_1	0	1	0	1/2	1/2	3/2
x_2	0	0	1	-1/2	1/2	1/2

optimal solution $x^* = \begin{pmatrix} 3/2 \\ 1/2 \end{pmatrix}$ with $z = 2$ (0.25 pts)

each tableau 0.5 pts → in total 2pts

	z	x_1	x_2	s_1	s_2	RHS
	1	0	0	0	1	2
s_1	0	2	0	1	1	3
x_2	0	1	1	0	1	2

$x^* = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ with $z = 2$ (alternative optimal solution)

2) Big-M method (0.75 pts) → (0.25 pts each tableau 2-3-4)

$\max\ -x_1 - x_2 - Ma_1 - Ma_2$

s.t. $2x_1 + x_2 + x_3 + a_1 = 4$

$x_1 + x_2 + 2x_3 + a_2 = 2$

$x_1, x_2, x_3, a_1, a_2 \geq 0$

1

	z	x_1	x_2	x_3	a_1	a_2	RHS
	1	1	1	0	M	M	0
a_1	0	2	1	1	1	0	4
a_2	0	1	1	2	0	1	2

2

	z	x_1	x_2	x_3	a_1	a_2	RHS
	1	1-3M	1-2M	-3M	0	0	-6M
a_1	0	2	1	1	1	0	4
a_2	0	1	1	2	0	1	2

x_3 enters

min ratio test → $\left\{\frac{4}{1}, \frac{2}{2}\right\} = 1$

a_2 leaves

3

	z	x_1	x_2	x_3	a_1	~~a_2~~	RHS
	1	1-3/2M	1-M/2	0	0		-3M
a_1	0	3/2	1/2	0	1		3
x_3	0	1/2	1/2	1	0		1

x_1 enters

min ratio test → $\min\left\{\frac{3}{3/2}, \frac{1}{1/2}\right\} = 2$

a_1 or x_3 leaves, select a_1

④

z	x_1	x_2	x_3	$\cancel{a_1}$	$\cancel{a_2}$	RHS
1	0	2/3	0			-2
0	1	1/3	0			2
0	0	1/3	1			0

$x^* = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$ with $z = 2$ (original problem has min. objective function)

Two-phase Method ($\underbrace{0.75}_{P-I} + \underbrace{0.5}_{P-II} = 1.25$ pts)

Phase I: min $w = a_1 + a_2$

s.t. $2x_1 + x_2 + x_3 + a_1 = 4$

$x_1 + x_2 + 2x_3 + a_2 = 2$

$x_1, x_2, x_3, a_1, a_2 \geq 0$

①

	w	x_1	x_2	x_3	a_1	a_2	RHS
	1	0	0	0	-1	-1	0
a_1	0	2	1	1	1	0	4
a_2	0	1	1	2	0	1	2

②

	w	x_1	x_2	x_3	a_1	a_2	RHS
	1	3	2	3	0	0	6
a_1	0	2	1	1	1	0	4
a_2	0	1	1	2	0	1	2

x_1 or x_3 will enter

Since min. ratio test of x_3 gives a leaving variable (for x_1 it will be same for a_1 and a_2)

I select x_3.

min. ratio test $\rightarrow \min\left\{\frac{4}{1}, \frac{2}{2}\right\} = 1$

a_2 leaves

③

	w	x_1	x_2	x_3	a_1	$\cancel{a_2}$	RHS
	1	3/2	1/2	0	0		3
a_1	0	3/2	1/2	0	1		3
x_3	0	1/2	1/2	1	0		1

x_1 enters

min. ratio test $\rightarrow \min\left\{\frac{3}{3/2}, \frac{1}{1/2}\right\} = 2$

I select a_1

④

	w	x_1	x_2	x_3	$\cancel{a_1}$	$\cancel{a_2}$	RHS
	1	1	1	0			0
x_1	0	1	1/3	0			2
x_3	0	0	1/3	1			0

Phase II:

⑤

	z	x_1	x_2	x_3	RHS
	1	1	1	0	0
x_1	0	1	1/3	0	2
x_3	0	0	1/3	1	0

	z	x_1	x_2	x_3	RHS
	1	0	2/3	0	-2
x_1	0	1	1/3	0	2
x_3	0	0	1/2	1	0

$x^* = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$ with $z = 2$

3) Decision variables

x_{ij} : # of units sold to city j from location i

y_i : # of plants opened in location i $\left(\text{or } y_i = \begin{cases} 1 & \text{if a plant is opened in location } i \\ 0 & \text{ow} \end{cases}\right)$

Parameters

p_{ij} : profit obtained by selling one unit to city j from location i

d_j : demand of city j

$$\max \sum_{i=1}^{6}\sum_{j=1}^{7} x_{ij}p_{ij} - \sum_{i=1}^{6} 10\, y_i \longrightarrow 0.5 + 0.5 = 1 \text{ pt}$$

$$\text{s.t.} \sum_{j=1}^{7} x_{ij} \leq 6 y_i \quad \forall i = 1,\dots,6 \longrightarrow 0.5 \text{ pts}$$

$$\sum_{i=1}^{6} x_{ij} \geq d_j \quad \forall j = 1,\dots,7 \longrightarrow 0.5 \text{ pts}$$

$$x_{ij} \geq 0, \; y_i \geq 0 \quad y_i : \text{integer } \forall i$$

(or $y_i \in \{0,1\}$ $\forall i$) → I accepted both formulations.

4) a) Decision variables

x_i : $\begin{cases} 1 & \text{if machine } i \text{ is bought} \\ 0 & \text{ow} \end{cases}$

Parameters

c_i : investment cost of machine i

p_{ij} : $\begin{cases} 1 & \text{if machine } i \text{ can produce product } j \\ 0 & \text{ow} \end{cases}$

$$\min \sum_{i=1}^{10} c_i x_i \longrightarrow 0.25 \text{ pt.}$$

$$\text{s.t.} \sum_{i=1}^{10} x_i p_{ij} \geq 1 \quad \forall j = 1,\dots,5 \longrightarrow 0.25 \text{ pt.}$$

$$x_i \in \{0,1\} \quad \forall i$$

b)

$$\min \sum_{i=1}^{10} c_i x_i - 500 y$$

$$\text{s.t.} \sum_{i=1}^{10} x_i p_{ij} \geq 1 \quad \forall j$$

$$\left.\begin{array}{l} x_2 \geq x_3 \\ x_2 \geq x_5 \end{array}\right\} 0.75 \text{ pts}$$

$$\left.\sum_{i=1}^{10} x_i \geq 3y \right\} 0.75 \text{ pts}$$

$$x_i, y \in \{0,1\}$$