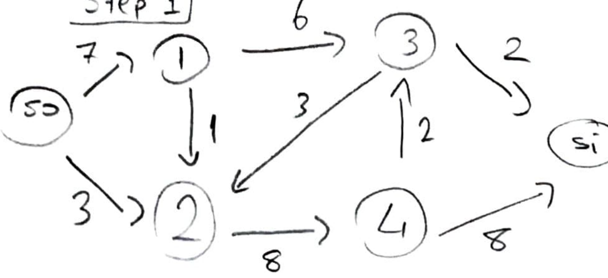
IE 400 - HW3 ANSWER KEY

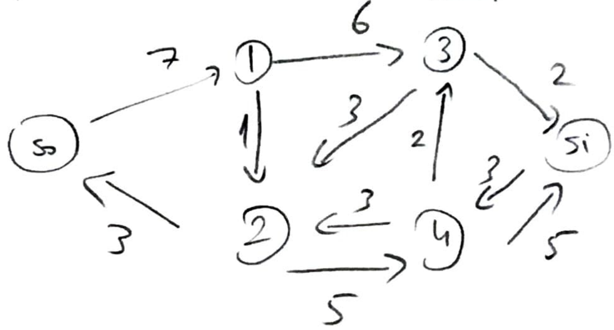
Q1)

Step 1)



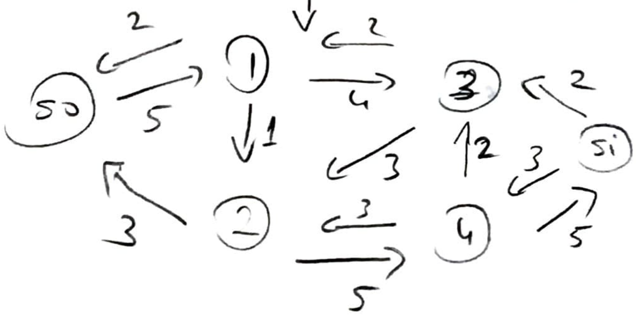
Send Flow of 3 through so=>2=>4=>si

Step 2



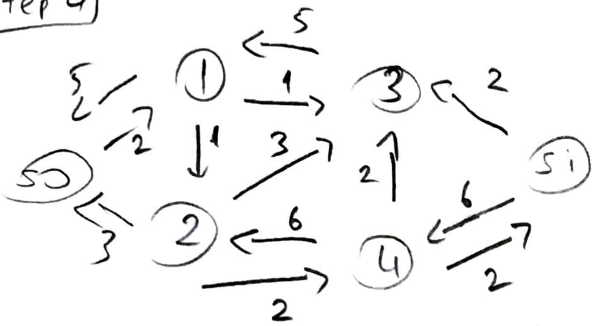
Send 2 through so=>1=>3=>si

Step 3

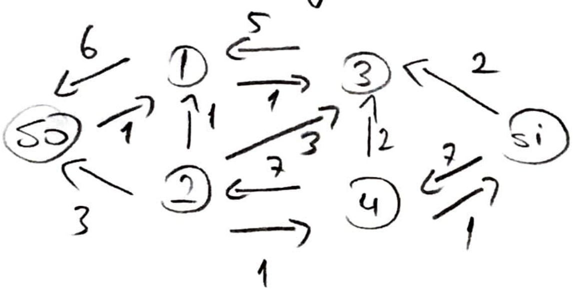


Flow of 3 so=>1->3->2->4->si

Step 4)



Flow of 1 through so->1->2->4->si



(Augmenting path)
There is no directed path from so->si. So this is our last step (We looked for directed graphs and send the minimum over this path in each step)

Total flow is 3+2+3+1 = 9

Therefore max flow is 9.

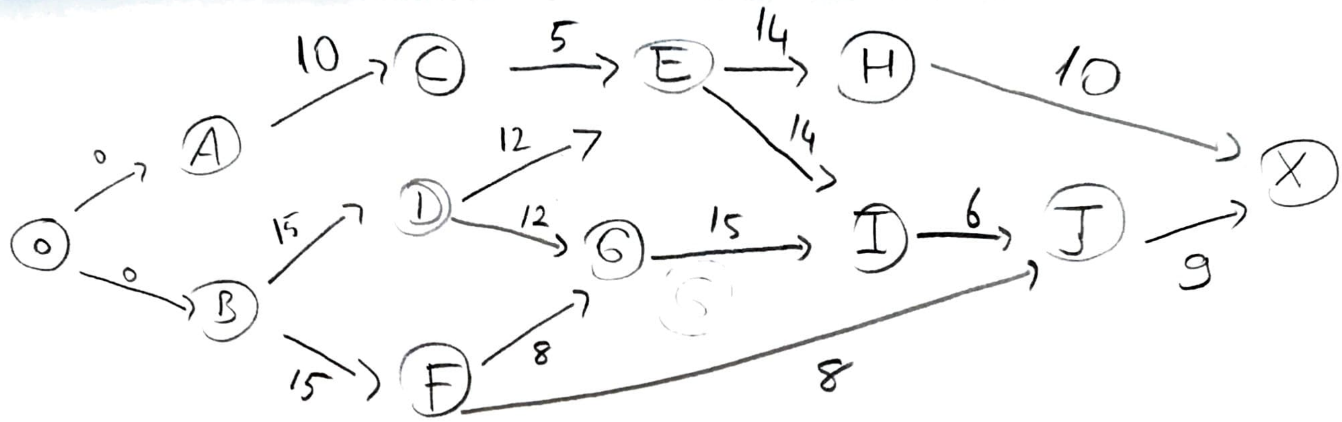
S-T cut is $S=\{so, 1, 3\}$, $T=\{si, 2, 4\}$

(S=Nodes that we can go from so, T is others).

Capacity of this S-T cut is (so,2) + (1,2) + (3,2) + (3, si) = 3+1+3+2 = 9.

We cannot find a smaller S-T cut since max flow = min cut.

Q21



	ES	EF	LS	LF	Slack
A	0	10	13	23	13
B	0	15	0	15	0
C	10	15	23	28	13
D	15	27	15	27	0
E	27	41	28	42	1
F	15	23	19	27	4
G	27	42	27	42	0
H	41	51	47	57	6
I	42	48	42	48	0
J	48	57	48	57	0
X	57	57	57	57	0

ES of a point = max of EF its predecessors

EF = ES + activity time

LS = LF − activity time

LF = Min of LS of its later dependent points

Slack = LS − ES = LF − EF

Critical activities will be the ones that slack = 0

So they are B, D, G, I, J.

Duration is 57

e) Anything can be that has slack variable > 0. (A, C, E, F, H)

f) a_i = activity time activity i, x_i = start time of activity i

min x_X subject to $x_0 = 0$, $x_A \geq 0$, $x_B \geq 0$, $x_C \geq x_A + 10$, $x_D \geq x_B + 15$

$x_E \geq x_D + 12$, $x_E \geq x_C + 5$, $x_F \geq x_B + 15$, $x_G \geq x_D + 12$, $x_G \geq x_F + 8$, $x_H \geq x_E + 14$

$x_I \geq x_E + 14$, $x_I \geq x_G + 15$, $x_J \geq x_F + 8$, $x_J \geq x_I + 6$; $x_X \geq x_J + 9$, $x_X \geq x_H + 10$

$x_A, x_B, x_C, x_D, x_E, x_F, x_G, x_H, x_I, x_J, x_X \geq 0$.

or shortly min x_X subject to $x_j \geq x_i + a_i$ for all $(i,j) \in \hat{A}$ (set of arcs)

$x_i \geq 0$ for all $i \in N$ (set of nodes) $x_0 = 0$

Q3 | $f_k(d)$ = max reliability with components k and available d dollar.

r_{ij}: reliability of component i with j parallel units

c_{ij}: cost of using 〃 〃 〃 〃

$f_0(d) = 1$ for $d = 0, 1, \dots, 10$ and $f_i(0) = 0$ for $i = 1, 2, 3$.

$$f_i(d) = \max_{\substack{j=1,2,3 \\ c_{ij} \le x}} \{ r_{ij} \, f_{i-1}(x - c_{ij}) \}$$

So $f_1(1) = 0,6$; $f_1(2) = 0,8$; $f_1(3) = f_1(4) = \dots f_1(10) = 0.9$

$f_2(1) = 0 . f_1(1) = 0$; $f_2(2) = 0 . f_1(2) = 0$; $f_2(3) = 0.7 . f_1(0) = 0$

$f_2(4) = 0.7 . 0.6 = 0.42$; $f_2(5) = \max\{0.8 f_1(0), 0.7 f_1(2)\} = 0,56$

$f_2(6) = \max\{0.8 f_1(1) = 0.48, 0.7 f_1(3) = 0,63\} = 0,63$

$f_2(7) = \max\{(0.9) f_1(1), 0.8 f_1(2), 0,7 f_1(4)\} = 0,64$

$f_2(8) = \max\{0,9 f_1(2), 0.8 f_1(3), 0,7 f_1(5)\} = 0,72$

$f_2(9) = \max\{0,9 f_1(3), 0,8 f_1(4), 0,7 f_1(6)\} = 0,81$

$f_2(10) = \max\{0,9 f_1(4), 0,8 f_1(5), 0,7 f_1(7)\} = 0.81$

$f_3(10) = \max\{(0,9) f_2(5), (0,7) f_2(6), (0,5) f_2(8)\} = 0,504$

So, trace back

Component 3 → 3 parallel, Component 2 → 1 parallel, Component 1 → 2 parallel.

Q4 a) $280\cdot(1{,}09)^5 + 210\cdot(1{,}09)^4 + 140\cdot(1{,}09)^3 + 140(1{,}09)^2 + 210(1{,}09) + 280 =$

$X\cdot\left((1{,}09)^4 + (1{,}09)^3 + (1{,}09)^2 + (1{,}09)^1 + (1{,}09)^0\right)$

$= 430.815 + 296.432 + 181.304 + 166.334 + 228.9 + 280 = 1583.785$

$X\cdot\left(\dfrac{(1.09)^5 - 1}{1.09 - 1}\right) = 1583.785 \Rightarrow X = 264.639$

b) Start of 1st yeer $2000\cdot 1{,}1 + 2000 = 4200$

start of 2nd year $4200\cdot 1{,}1 - 2850 = 1770$

Start of 3rd yeer $1770\cdot 1{,}1 + 3500 = 5447$

Start of 4th yeer $5447\cdot 1{,}1 = 5991{,}7.$