1) i: products $i = 1,2,3$

j: machines $j = 1,2$

k: month $k = 1,2,3$

Decision variables: x_{ijk}: # of product i produced by machine j at month k

y_{ik}: # of product i held in inventory at month k

Parameters: d_{ik}: demand of product i in month k

h_i: holding cost of product i

p_{ij}: production cost of product i produced by machine j

a_{ij}: (binary parameter) whether product i can be produced by machine j or not

Model: $\min \sum_i \sum_j \sum_k x_{ijk} p_{ij} + \sum_i \sum_k y_{ik} h_i$ (3)

s.t. $y_{i0} = 0 \quad \forall i$ (1)

$y_{i,(k-1)} + \sum_j a_{ij} x_{ijk} = d_{ik} + y_{ik} \quad \forall i,k$ (3)

$\sum_j 30 a_{1j} x_{1jk} + 20 a_{2j} x_{2jk} + 15 a_{3j} x_{3jk} \leq 60 \cdot 60 \quad \forall k$ (3)

$x_{ijk}, y_{ik} \geq 0 \quad \forall i,j,k$

2) a) i: plant $i = 1,2,3,4$

t: year $t = 1,2,3,4,5$

Decision variables: $x_{it} = \begin{cases} 1 & \text{is plant } i \text{ is operated in year } t \\ 0 & \end{cases}$

$y_i = \begin{cases} 1 & \text{is plant } i \text{ is built} \\ 0 & \end{cases}$

Parameters: d_t: generating capacity requirement in year t

k_i: construction cost of plant i

c_i: annual operating cost of plant i

g_i: generating capacity of plant i

Model $\min \sum_t \sum_i x_{it} c_i + \sum_i y_i k_i$ (1.5)

s.t. $\sum_i g_i x_{it} \geq d_t \quad \forall t$ (1.5)

$x_{it} \leq y_i \quad \forall i,t$ (1)

$x_{it}, y_i \in \{0,1\} \quad \forall i,t$

b) Decision Variables : x_{it} : $\begin{cases} 1 & \text{if plant } i \text{ is operated during year } t \\ 0 & \text{ow} \end{cases}$

y_{it} : $\begin{cases} 1 & \text{if plant } i \text{ is shut down at the end of year } t \\ 0 & \text{ow} \end{cases}$

z_{it} : $\begin{cases} 1 & \text{if plant } i \text{ is reopened at beginning of year } t \\ 0 & \text{ow} \end{cases}$

Parameters added : r_i : reopening cost of plant i

s_i : shutdown cost of plant i

Model : $\min \sum_i \sum_t x_{it} c_i + y_{it} s_i + z_{it} r_i$ 1.5

s.t. $\sum_i g_i x_{it} \geq d_t \quad \forall t$ 1

$x_{it} + z_{i(t+1)} = x_{i(t+1)} + y_{it} \quad \forall i,t$ 2.5

$x_{i0} = 1 \quad \forall i$ 1

$x_{it}, y_{it}, z_{it} \in \{0,1\}$

Q3/ a) $f_i(s)$: max profit from year i to n while having s fish in the lake

~~[illegible]~~ Years are until n

Decision variable x_i: number of fish sold in year i.

$f_{n+1}(s) = 0$ for $\forall s$ (Base case)

$$f_i(s) = \max_{0 \le x_i \le s} \{ r_i(x_i) - c(x_i, s) + f_{i+1}(2(s - x_i)) \}$$

$f_1(t)$: optimal value when there is t fish in the beginning.

b) $f_4(s) = 0$ for $\forall s$ (base case) Optimal value $f_1(10)$.

$$f_3(s) = \max_{0 \le x_3 \le s} \{ 12x_3 - x_3 \} = 11s \text{ for any s.} \quad (1) \quad (\text{Take } x_3 = s)$$

$$f_1(10) = \max_{0 \le x_1 \le 10} \{ 10x_1 - 2x_1 + f_2(2(10 - x_1)) \}$$

$$f_2(20 - 2x_1) = \max_{0 \le x_2 \le 20 - 2x_1} \{ 13x_2 - 4x_2 + f_3(2(20 - 2x_1 - x_2)) \}$$

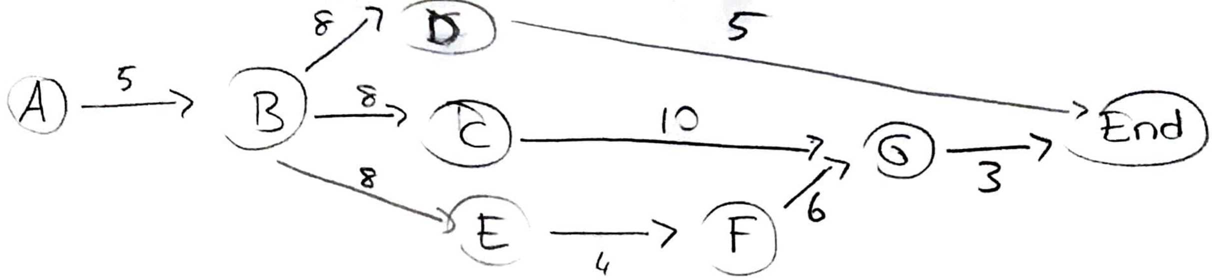
$$f_3(40 - 4x_1 - 2x_2) = 11(40 - 4x_1 - 2x_2) = 440 - 44x_1 - 22x_2 \text{ from (1). so}$$

$$f_2(20 - 2x_1) = \max_{0 \le x_2 \le 20 - 2x_1} \{ 9x_2 + f_3(40 - 4x_1 - 2x_2) \} = \max_{0 \le x_2 \le 20 - 2x_1} (440 - 44x_1 - 13x_2)$$

$$f_1(10) = \max_{0 \le x_1 \le 10} \{ 8x_1 + \max_{0 \le x_2 \le 20 - 2x_1} (440 - 44x_1 - 13x_2) \} = \max_{\substack{0 \le x_1 \le 10 \\ 0 \le x_2 + 2x_1 \le 20}} \{ 440 - 36x_1 - 13x_2 \}$$

Take x_1 and $x_2 = 0$. So $x_1^* = 0$, $x_2^* = 0$, $x_3^* = 40$ $f_1(10) = 440$.

Q4) i)



	ES	EF	LS	LF	Slack
A	0	5	0	5	0
B	5	13	5	13	0
C	13	23	13	23	0
D	13	18	21	26	8
E	13	17	13	17	0
F	17	23	17	23	0
G	23	26	23	26	0
End	26	26	26	26	0

Critical paths are

A - B - C - G - End (Found by Slacks are 0)

A - B - E - F - G - End

Only D can be delayed since only D has non zero slack

c) x_i : ES of the activity $i \in \{A, B, C, D, E, F, G, End\}$.

y_i : Reduction time of activity $i \in \{A, B, C, D, E, F, G\}$.

$$\text{Min } 30y_A + 15y_B + 20y_C + 40y_D + 20y_E + 30y_F + 40y_G$$

$y_A \le 2,\ y_B \le 3,\ y_C \le 1,\ y_D \le 2,\ y_E \le 2,\ y_F \le 3,\ y_G \le 1.$

$x_B \ge x_A + 5 - y_A$, $x_C \ge x_B + 8 - y_B$, $x_D \ge x_B + 8 - y_B$, $x_E \ge x_B + 8 - y_B$

$x_F \ge x_E + 4 - y_E$, $x_G \ge x_C + 10 - y_C$, $x_{END} \ge x_G + 3 - y_G$, $x_{END} \ge x_D + 5 - y_D$

$x_i, y_i \ge 0$ for any i

ii)

	Expected Time $(a+4m+b)/6$	Variance $\left(\frac{(b-a)}{6}\right)^2$
A	3	1/9
B	2	1/9
C	6	16/9
D	2	1/9
E	3	4/9
F	3	1/9
G	5	4/9
H	1	1/36
I	1.5	1/36
J	2	1/9

Critical Path is : A - C - G - End

Expected Duration = 3+6+5 = 14

Project variance $\sigma_p^2 = \sum$ (Variance in critical path)

$$\underset{(A)}{= \frac{1}{9}} + \underset{(C)}{\frac{16}{9}} + \underset{(G)}{\frac{4}{9}} = \frac{7}{3} \quad \text{so } \sigma_p = \sqrt{\frac{7}{3}}$$

$P\{T \le 15\} = P\{z \le 0.654\} = \%74$

$z = (\text{due date} - \text{exp. date of completion}) / \sigma_p = \dfrac{15-14}{\sqrt{\frac{7}{3}}}$