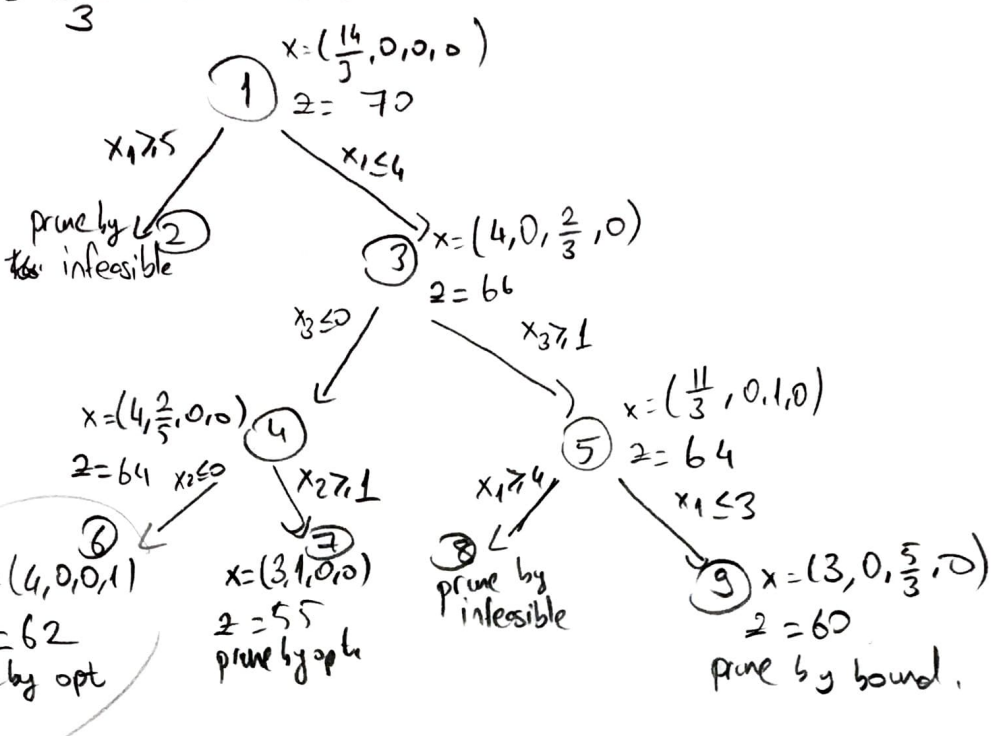
ANSWER KEY FOR HW-2

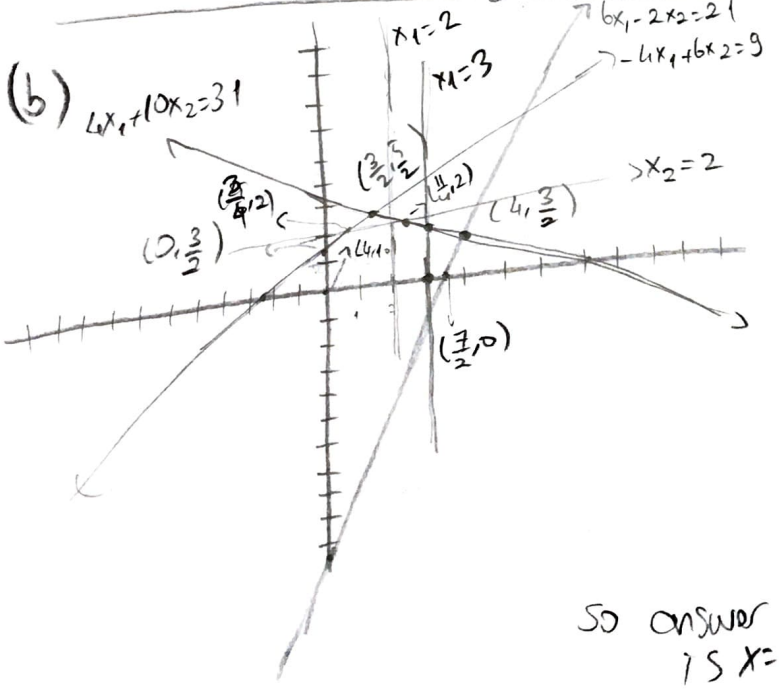
Q1/ Look for the ratios $\frac{15}{3} > \frac{9}{3} > \frac{10}{5} > \frac{2}{2}$ so $x_1 > x_3 > x_2 > x_4$

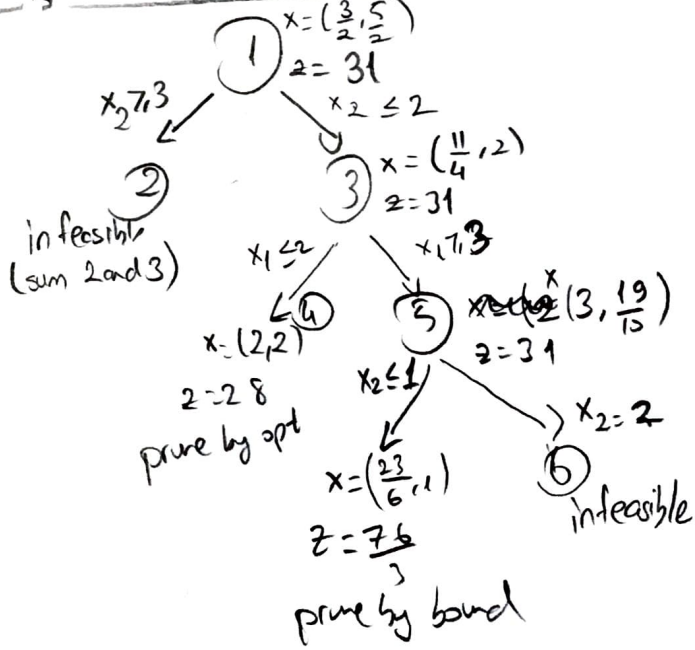
So take $x_1 = \frac{14}{3}$ others are 0.



optimal $x_1=4, x_2=0, x_3=0, x_4=1$ and $z=62$

(b)





So answer is $x=(2,2)$

$z=28$

(Q2) c_{ij} = cost of operating from i to j when we buy phone at beginning of ith year.

c_{ij} = price of phone + operating cost$_{(j-i)}$

$c_{12} = 40+20 = 60$

$c_{13} = 40+20+30 = 90$

$c_{14} = 40+20+30+40 = 130$

$c_{15} = 190$

$c_{16} = 260$

$c_{17} = 320$

$c_{23} = 60$

$c_{24} = 90$

$c_{25} = 130$

$c_{26} = 190$

$c_{27} = 260$

$c_{34} = 60$

$c_{35} = 90$

$c_{36} = 130$

$c_{37} = 190$

$c_{45} = 60$

$c_{46} = 90$

$c_{47} = 130$

$c_{56} = 60$

$c_{57} = 90$

$c_{67} = 60$

Functional equations $v(i)$ = costs at the beginning of year i. Answer is $v(7)$

$v(1) = 0$, $v(i) = \min_{j<i} \{v(j) + c_{jk}\}$

$v(2) = v(1) + 60 = 60$ $d(2) = 1$

$v(3) = \min\{v(1)+c_{13}, v(2)+c_{23}\} = 90$ $d(3) = 1$

$v(4) = \min\{v(1)+c_{14}, v(2)+c_{24}, v(3)+c_{34}\} = 130$ $d(4) = 1$

$v(5) = \min\{v(1)+c_{15}, v(2)+c_{25}, v(3)+c_{35}, v(4)+c_{45}\} = 180$ $d(5) = 3$

$v(6) = \min\{v(1)+c_{16}, v(2)+c_{26}, v(3)+c_{36}, v(4)+c_{46}, v(5)+c_{56}\} = 220$ $d(6) = 3,4$

$v(7) = \min\{v(1)+c_{17}, v(2)+c_{27}, v(3)+c_{37}, v(4)+c_{47}, v(5)+c_{57}, v(6)+c_{67}\} = 260$ $d(7) = 4$

So we will buy at year 1 then at year 4.

Q3 a)

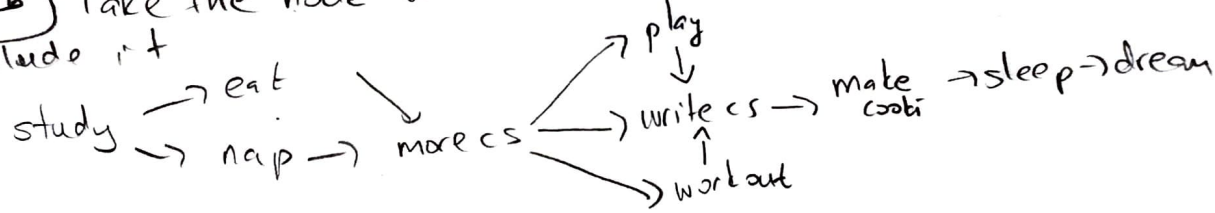
t	$v^{(t)}(A)$	$v^{(t)}(B)$	$v^{(t)}(C)$	$v^{(t)}(D)$	$v^{(t)}(E)$	d(A)	d(B)	d(C)	d(D)	d(E)
0	0	∞	∞	∞	∞					
1	0	-1	4	∞	∞	A	A			
2	0	-1	2	1	1	A	B	B	B	
3	0	-1	2	-2	1	A	B	E	B	
4	0	-1	2	-2	1	A	B	E	B	

$v^{(t)}(i)$ = cost of node i at time t ; d(i): predecessor node for shortest path

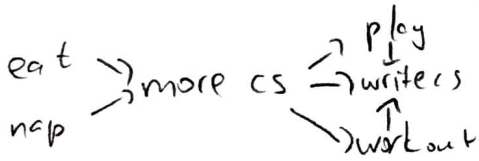
$v^{(t)}(k) = \min\{v^{(t-1)}(i) + c_{ik}\}$

Q3 b) Take the node without incoming arc so wake up = 1

Exclude it



Take without incoming arc study = 2.



Take eat, then nap eat=3 nap=4 then more cs = 5, play=6, work out = 7

write cs = 8, cookies = 9, sleep = 10, dream = 11

Q4) v(i) = shortest path to i. d(i) = previous node v(1) = 0

Permanent = {1}, Temporary others

v(2) = v(1)+2, d(2)=1 v(3) = v(1)+4, d(3)=1 v(4) = v(1)+3, d(4)=1 Smallest is 2 so make 2 permanent.

P = {1,2}.

v(1)=0, v(2)=2 d(2)=1, v(3)=4 d(3)=1, v(4)=3, d(4)=1

v(5)=9 d(5)=2, v(6)=6 d(6)=2, v(7)=8 d(7)=2

Smallest one 4.

P={1,2,4} v(5) = min{9, v(4)+4} = 7 d(5)=4

v(6) = min{6, v(4)+1} = 4 d(6)=4 v(7) = 8 not changes

Smallest one is 3 and 6 select 3

P={1,2,4,3} nothing changes. Smallest one is 6

P={1,2,4,3,6} v(8)=10 d(8)=6 ; v(9)=7 d(9)=6

Smallest one is 5 and 9 select 5

P={1,2,4,3,6,5} v(8) = min{10, v(5)+1} = 8 d(8)=5

Smallest one is 9

P={1,2,4,3,6,5,9} v(10)=11 d(10)=9

Smallest one is 7 and 8. Select 7

P={1,2,4,3,6,5,9,7} nothig changes

P={1,2,4,3,6,5,9,7,8} v(10)=11 d(10)=9,8 both ok.

P={1,2,4,...,10} Done.

So paths are

1 ← 4 ← 6 ← 9 ← 10

or

1 ← 4 ← 5 ← 8 ← 10

or

1 ← 3 ← 5 ← 8 ← 10