IE 400: Principles of Engineering Management
Study Set 2

Summer 2020

Question 1. Identify the basic solutions, basic feasible solutions and degenerate basic feasible solutions (if exists) in the following linear system.

$$x_1 - x_2 + 3x_3 + 5x_4 = 15$$
$$x_1 + x_2 + x_4 = 9$$
$$x_i \geq 0 \quad i = 1, \cdots, 4$$

Question 2. Consider the following simplex tableaus of LP problems. First two are minimization problems while the last tableau belongs to a maximization problem.

x_1	x_2	e_1	e_2	x_3	x_4	RHS
-2	-3	0	0	-M	-M	0
2	1	-1	0	1	0	4
1	-1	0	-1	0	1	1

x_1	x_2	s_1	e_1	x_3	x_4	RHS
0	0	0	0	-1	-1	0
1	1	1	0	0	0	3
2	1	0	-1	1	0	4
1	1	0	0	0	1	3

x_1	x_2	x_3	s_1	s_2	e_1	RHS
-2	-1	-1	0	0	0	0
1/3	2/3	0	1	0	1/3	1
2	1	0	0	1	0	3
2/3	1/3	1	0	0	-1/3	1

Are these tableaus in proper format? If yes, indicate the entering and leaving variables. If not, explain why and show how can you convert them into proper format.

Question 3. Use the simplex algorithm to find two optimal solutions to the following LP:

$$
\begin{aligned}
\max\ & 5x_1 + 3x_2 + x_3 \\
\text{s.t. } & x_1 + x_2 + x_3 \leq 6 \\
& 5x_1 + 3x_2 + 6x_3 \leq 15 \\
& x_i \geq 0 \ i = 1, 2, 3
\end{aligned}
$$

Question 4. Consider the following problem:

$$
\begin{aligned}
\max\ & 2x_1 + x_2 + 6x_3 - 4x_4 \\
\text{s.t. } & x_1 + 2x_2 + 4x_3 - x_4 \leq 6 \\
& 2x_1 + 3x_2 - x_3 + x_4 \leq 12 \\
& 2x_1 + 3x_2 - x_3 + x_4 \leq 6 \\
& x_i \geq 0 \ i = 1, 2, 3, 4
\end{aligned}
$$

Convert this LP to standard form. Construct the simplex tableau for the basis: x1, x2 and x4. Indicate whether the tableau is optimal, if not state the entering and the leaving variables. You do not need to perform further iterations.

Question 5. Consider the following simplex tableau of a given **minimization** LP problem.

	z	x_1	x_2	x_3	s_1	s_2	s_3	RHS
z	1	0	0	**A**	-10	-1	**B**	32
s_3	0	0	0	**C**	4	-2	1	**D**
x_2	0	0	1	-2	**F**	-1	0	6
x_1	0	**E**	0	-1	1	2	0	4

Give **general** conditions on each of the unknowns **A-F** such that each of the following statements is true. Even if the statement holds independently of the values of a specific variable, you should still mention that variable.

(a) The tableau is final and there exists a unique optimal solution.

(b) The simplex method determines an unbounded solution from this tableau.

(c) The current bfs is degenerate (not necessarily optimal).

(d) The current solution is optimal, there are alternative optimal solutions but no alternative optimal bfs.

Question 6. Use the two-phase method to find the optimal solution to the following LP:

$$\begin{aligned} \max\ & -3x_1 + 4x_2 \\ \text{s.t.}\ & x_1 + x_2 \leq 4 \\ & 2x_1 + 3x_2 \geq 18 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Question 7. Convert the following LP to standard form and solve with the Big-M method.

$$\begin{aligned} \max\ & -x_1 + 3x_2 + x_3 \\ \text{s.t.}\ & -x_1 + x_2 - x_3 \geq -4 \\ & 2x_1 + x_2 + x_3 \geq 10 \\ & x_1 \geq 0 \\ & x_2 \leq 0 \\ & x_3 \text{ free} \end{aligned}$$

Question 8. Solve the following linear program by both the two-phase method and the big M-method:

$$
\begin{aligned}
\min\ & 3x_1 + 2x_2 + 4x_3 \\
\text{s.t.}\ & 2x_1 + x_2 + 3x_3 = 60 \\
& 3x_1 + 3x_2 + 5x_3 \geq 120 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$