Project Scheduling

Network Optimization can be used as an aid in the scheduling of large complex projects that consist of many activities

A project is divided into a collection of work activities (for planning and control purposes) that must all be completed to accomplish the project

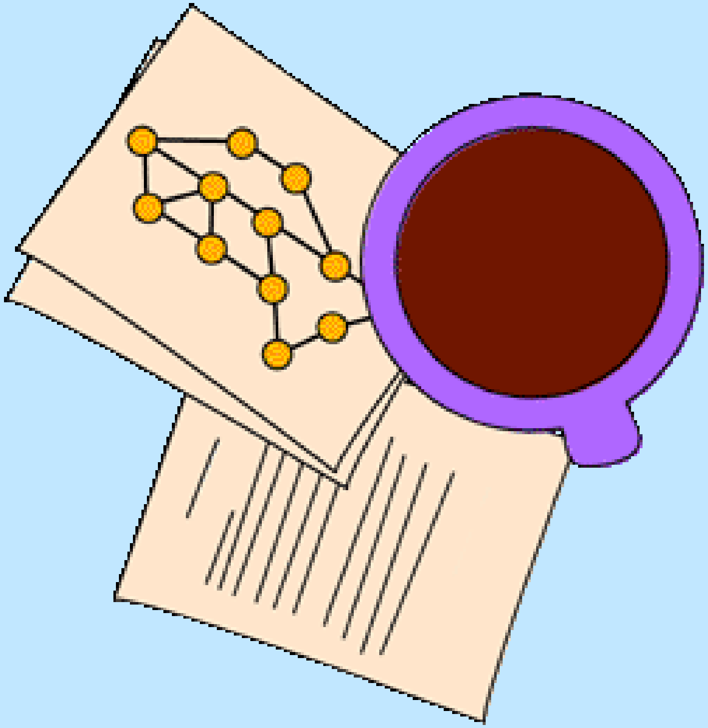
Project Scheduling

- ☑ Identifying precedence relationships
- ☑ Sequencing activities
- ☑ Determining activity times & costs
- ☑ Estimating material and worker requirements
- ☑ Determining critical activities

Purposes of Project Scheduling

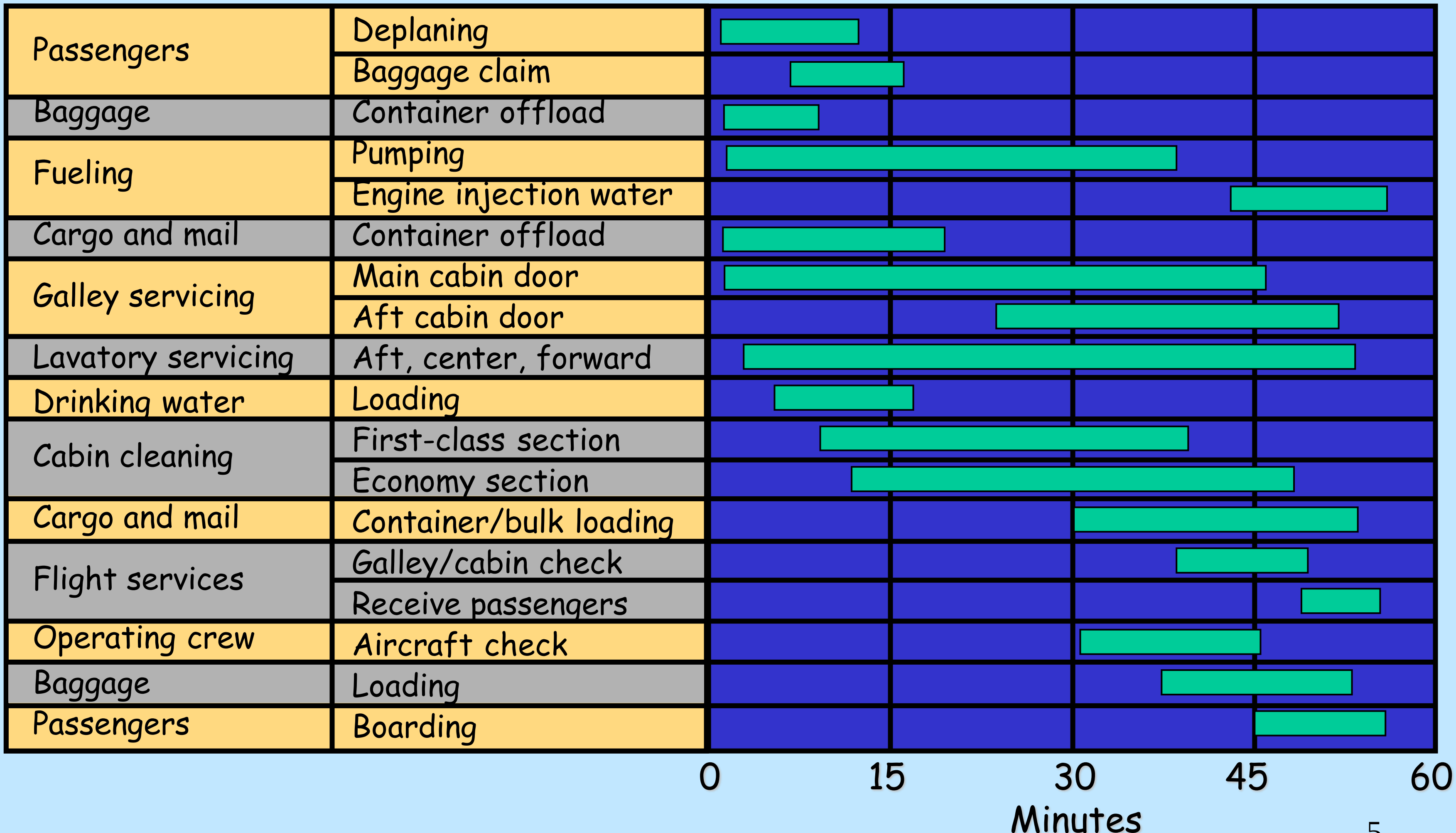
1. Shows the relationship of each activity to others and to the whole project
2. Identifies the precedence relationships among activities
3. Encourages the setting of realistic time and cost estimates for each activity
4. Helps make better use of people, money, and material resources by identifying critical bottlenecks in the project

Project Management Techniques



- ☑ Gantt chart
- ☑ Critical Path Method (CPM)
- ☑ Program Evaluation and Review Technique (PERT)

Service For A Delta Jet



CPM & PERT

- ☑ Network techniques
- ☑ Developed in 1950's
- ☑ Consider precedence relationships and interdependencies
- ☑ Each uses a different estimate of activity times

  CPM: activity durations are known with certainty

  PERT: not known with certainty

Steps CPM & PERT

1. Define the project and prepare the work breakdown structure
2. Develop relationships among the activities - decide which activities must precede and which must follow others
3. Draw the network connecting all of the activities

Six Steps CPM & PERT

4. Assign time and/or cost estimates to each activity
5. Compute the longest time path through the network – this is called the critical path
6. Use the network to help plan, schedule, monitor, and control the project

Questions CPM & PERT Can Answer

1. When will the entire project be completed?
2. What are the critical activities or tasks in the project?
3. Which are the noncritical activities?
4. Is the project on schedule, behind schedule, or ahead of schedule? (due date!)

Questions CPM & PERT Can Answer

5. Is the money spent equal to, less than, or greater than the budget?
6. Are there enough resources available to finish the project on time?
7. If the project must be finished in a shorter time, what is the way to accomplish this at least cost (crashing)?

Precedence Relationships and Networks

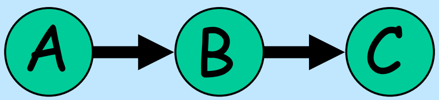
Activity j is a predecessor of activity k if activity j must be completed before activity k can begin

Project networks represent the precedence relationships between activities

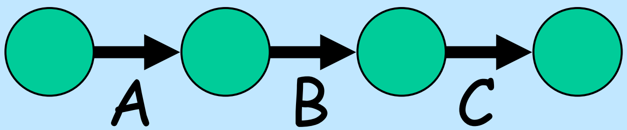
A Comparison of AON and AOA Network Conventions

Activity on Node (AON)	Activity Meaning	Activity on Arrow (AOA)

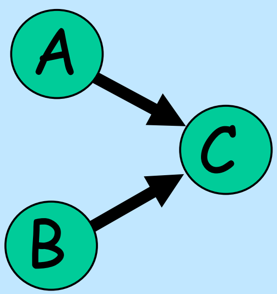
(a)



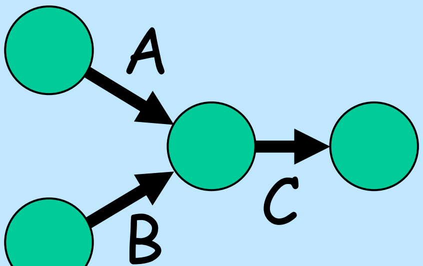
A comes before B, which comes before C



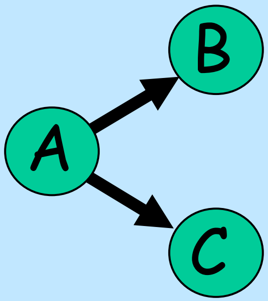
(b)



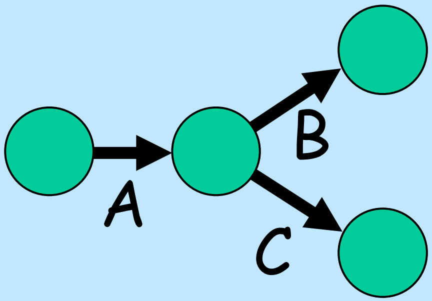
A and B must both be completed before C can start



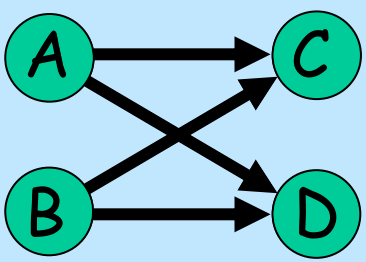
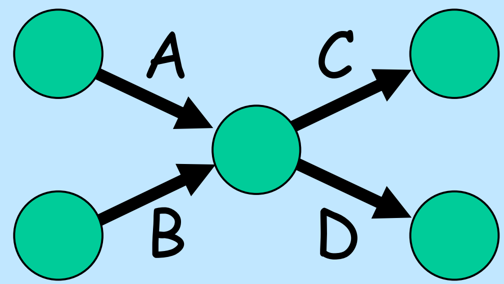
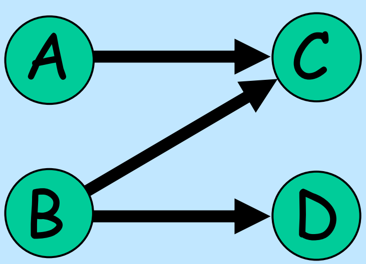
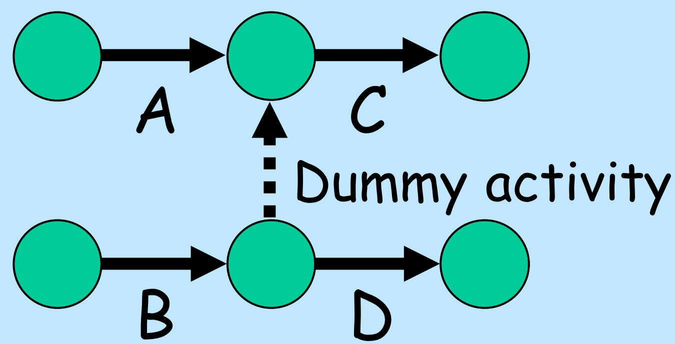
(c)



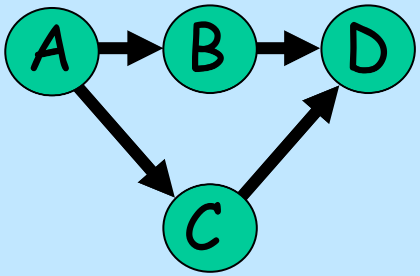
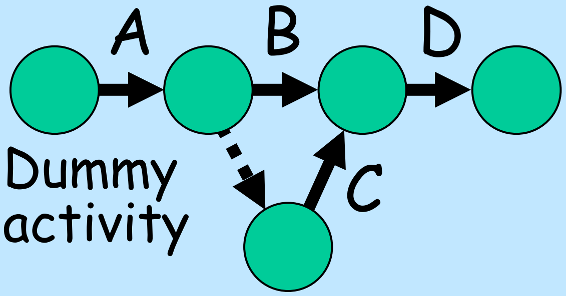
B and C cannot begin until A is completed



A Comparison of AON and AOA Network Conventions

Activity on Node (AON)	Activity Meaning	Activity on Arrow (AOA)
(d) 	C and D cannot begin until A and B have both been completed	
(e) 	C cannot begin until both A and B are completed; D cannot begin until B is completed. A dummy activity is introduced in AOA	

A Comparison of AON and AOA Network Conventions

Activity on Node (AON)	Activity Meaning	Activity on Arrow (AOA)
(f) 	B and C cannot begin until A is completed. D cannot begin until both B and C are completed. A dummy activity is again introduced in AOA.	

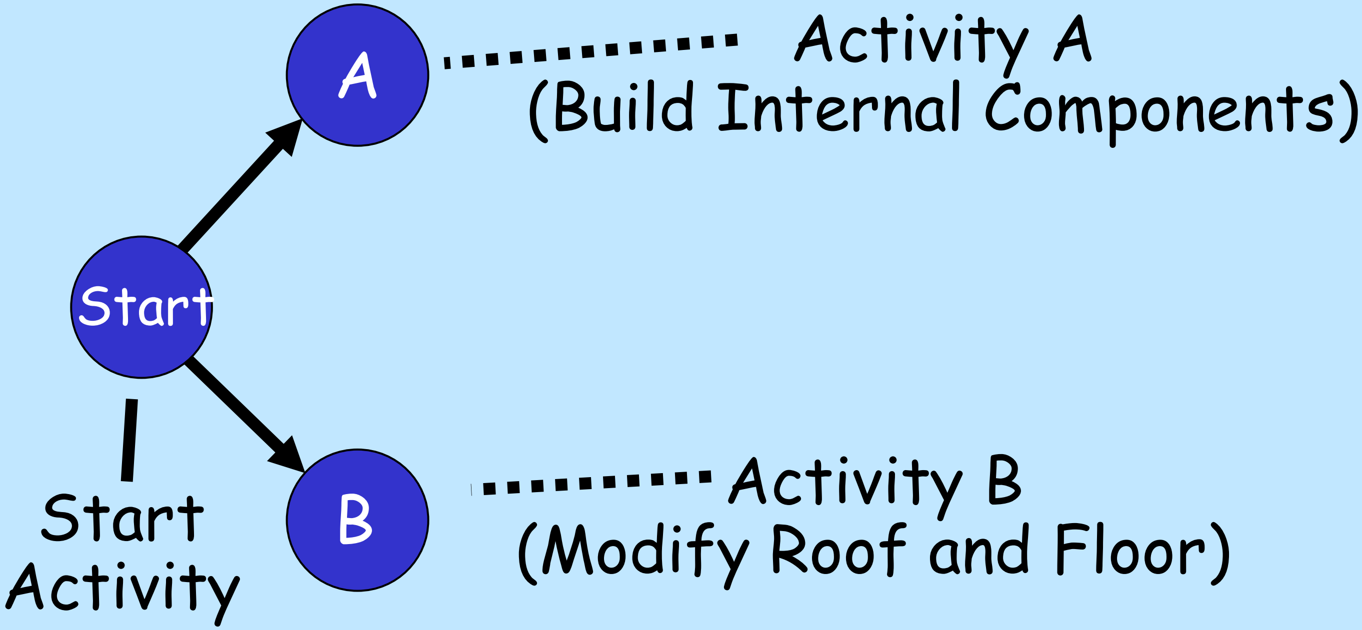
AON Example

Milwaukee Paper Manufacturing's Activities and Predecessors

Activity	Description	Immediate Predecessors
A	Build internal components	—
B	Modify roof and floor	—
C	Construct collection stack	A
D	Pour concrete and install frame	A, B
E	Build high-temperature burner	C
F	Install pollution control system	C
G	Install air pollution device	D, E
H	Inspect and test	F, G

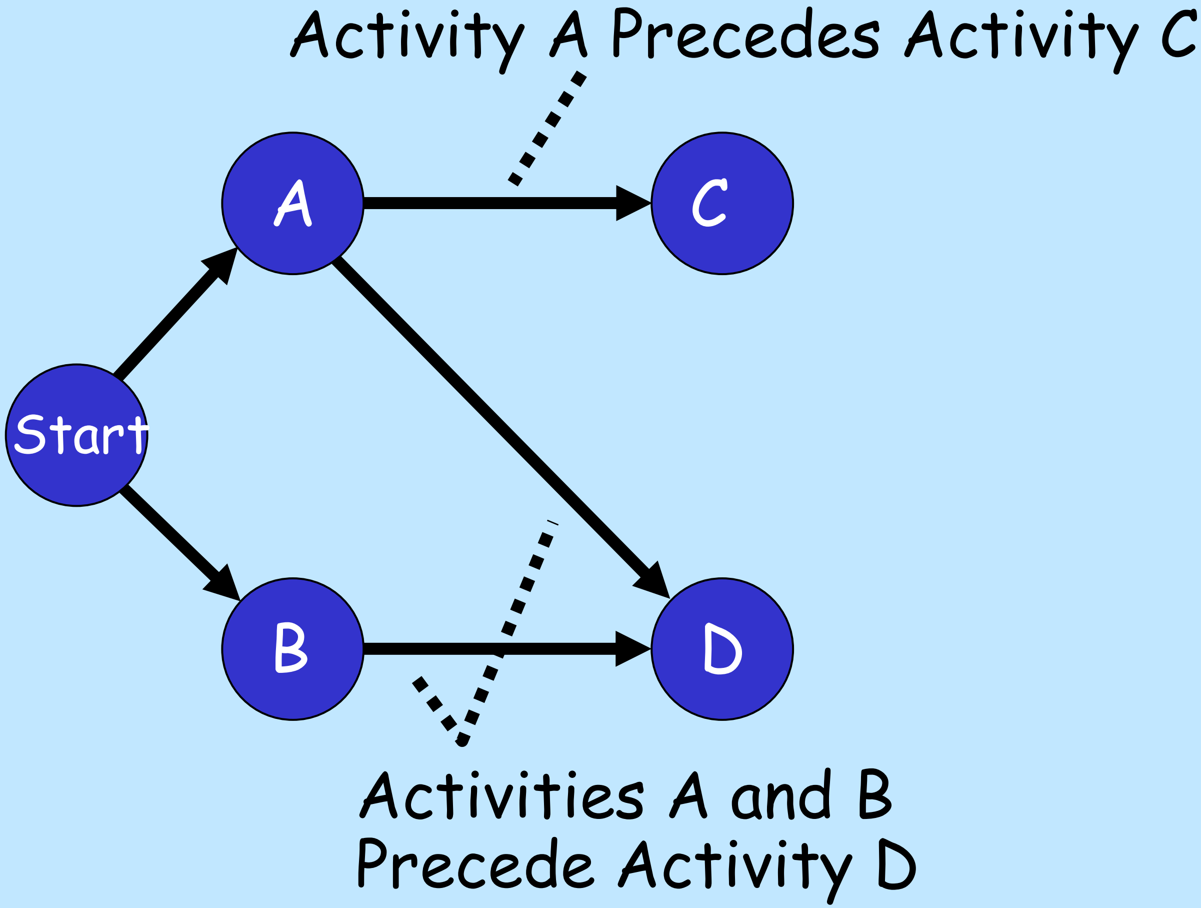
AON Network for Milwaukee Paper

Activity	Immediate Predecessors
A	—
B	—
C	A
D	A, B
E	C
F	C
G	D, E
H	F, G



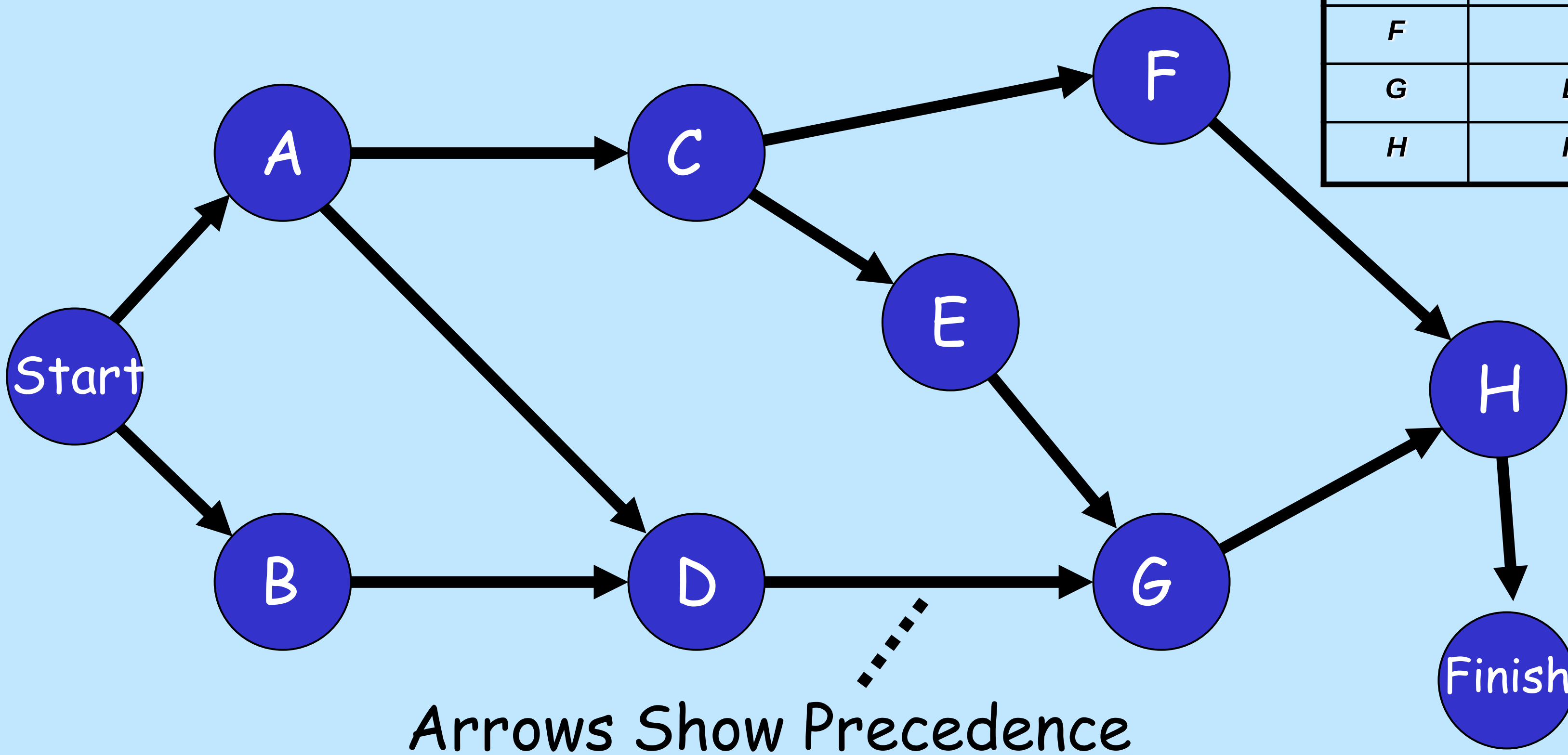
AON Network for Milwaukee Paper

Activity	Immediate Predecessors
A	—
B	—
C	A
D	A, B
E	C
F	C
G	D, E
H	F, G



AON Network for Milwaukee Paper

Activity	Immediate Predecessors
A	—
B	—
C	A
D	A, B
E	C
F	C
G	D, E
H	F, G



Arrows Show Precedence Relationships

Determining the Project Schedule

Perform a Critical Path Analysis

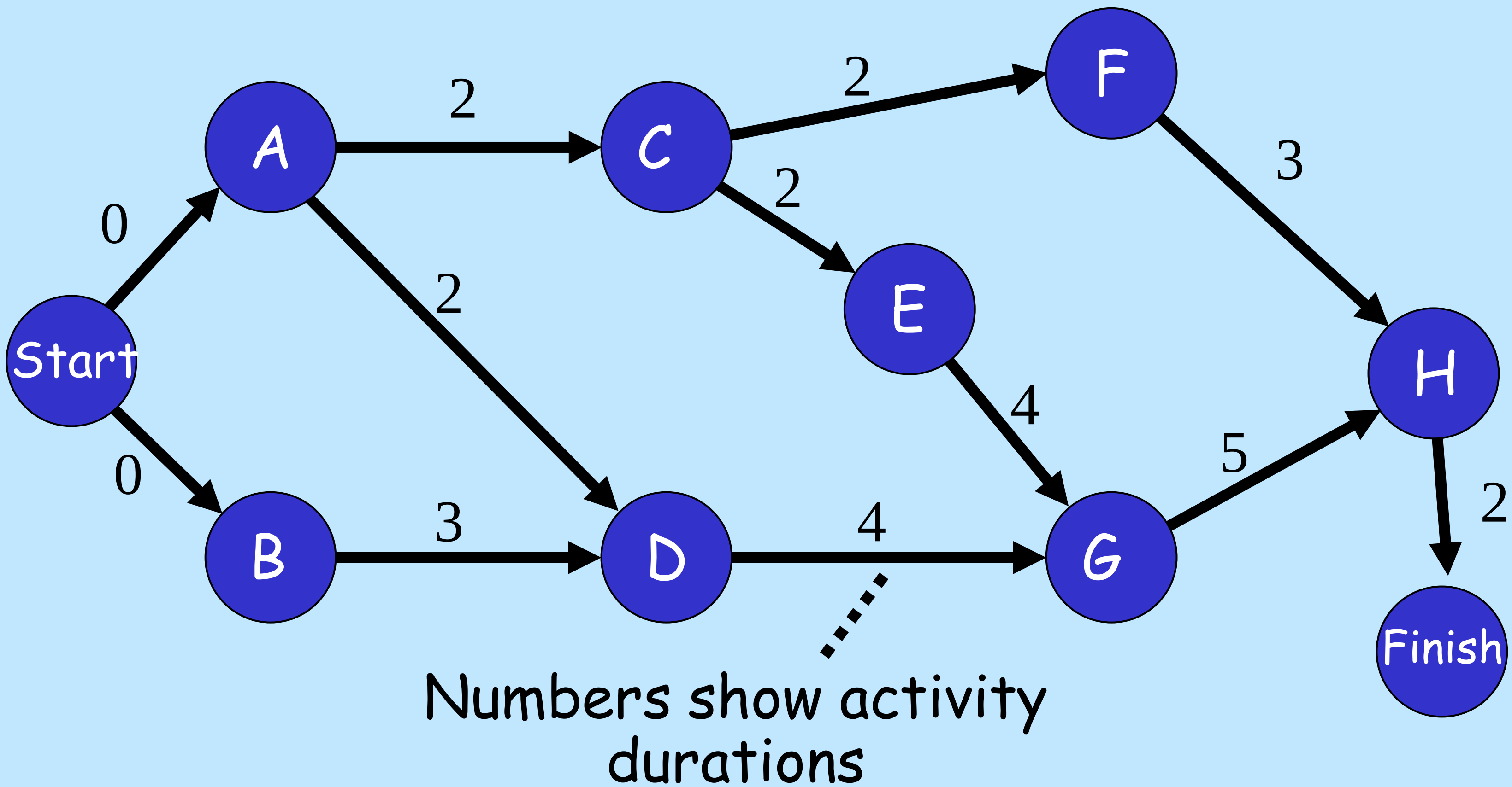
- ☑ The critical path is the longest path through the network
- ☑ The critical path is the shortest time in which the project can be completed
- ☑ Any delay in critical path activities delays the project
- ☑ Critical path activities have no slack time

Determining the Project Schedule

Perform a Critical Path Analysis

Activity	Description	Time (weeks)
A	Build internal components	2
B	Modify roof and floor	3
C	Construct collection stack	2
D	Pour concrete and install frame	4
E	Build high-temperature burner	4
F	Install pollution control system	3
G	Install air pollution device	5
H	Inspect and test	2
	Total Time (weeks)	25

AON Network for Milwaukee Paper



Longest Paths

Given $G=(N,A)$ with nonnegative lengths associated on arcs, special source node s, find the longest path from s to every other node in N.

Principle of optimality applies if no directed cycle of positive length

Project networks are acyclic!

Functional Equations

$v[s]=0$

$$v[k]=\max_{i:(i,k)\in A} \{v[i]+c_{ik}\} \qquad \forall k \in N \setminus \{s\}$$

If there is no neighbor i leading to k, then the maximum is $-\infty$

d[k]=preceeding node on the longest path from s to k

Determining the Project Schedule

Perform a Critical Path Analysis

Earliest start (ES) = earliest time at which an activity can start, assuming all predecessors have been completed

Earliest finish (EF) = earliest time at which an activity can be finished

Latest start (LS) = latest time at which an activity can start so as to not delay the completion time of the entire project

Latest finish (LF) = latest time by which an activity has to be finished so as to not delay the completion time of the entire project

Forward Pass

Begin at starting event and work forward

Earliest Start Time Rule:

- ☑ If an activity has only one immediate predecessor, its ES equals the EF of the predecessor
- ☑ If an activity has multiple immediate predecessors, its ES is the maximum of all the EF values of its predecessors

ES = Max (EF of all immediate predecessors)

Forward Pass

Begin at starting event and work forward

Earliest Finish Time Rule:

- ☑ The earliest finish time (EF) of an activity is the sum of its earliest start time (ES) and its activity time

EF = ES + Activity time

Early Schedules

ES[k]=length of the longest path from start to node k in the AON project network

Project completion time
= minimum time to complete a project
= length of the longest path from start to end on AON

Computing an Early Schedule (Forward Pass)

Step 1: Order the nodes lexicographically.

ES [start] = EF [start]= 0

Step 2: Terminate if the early start time of the end node has been fixed.

Otherwise, let p be the lowest number of an unprocessed node.

Step 3: Compute:

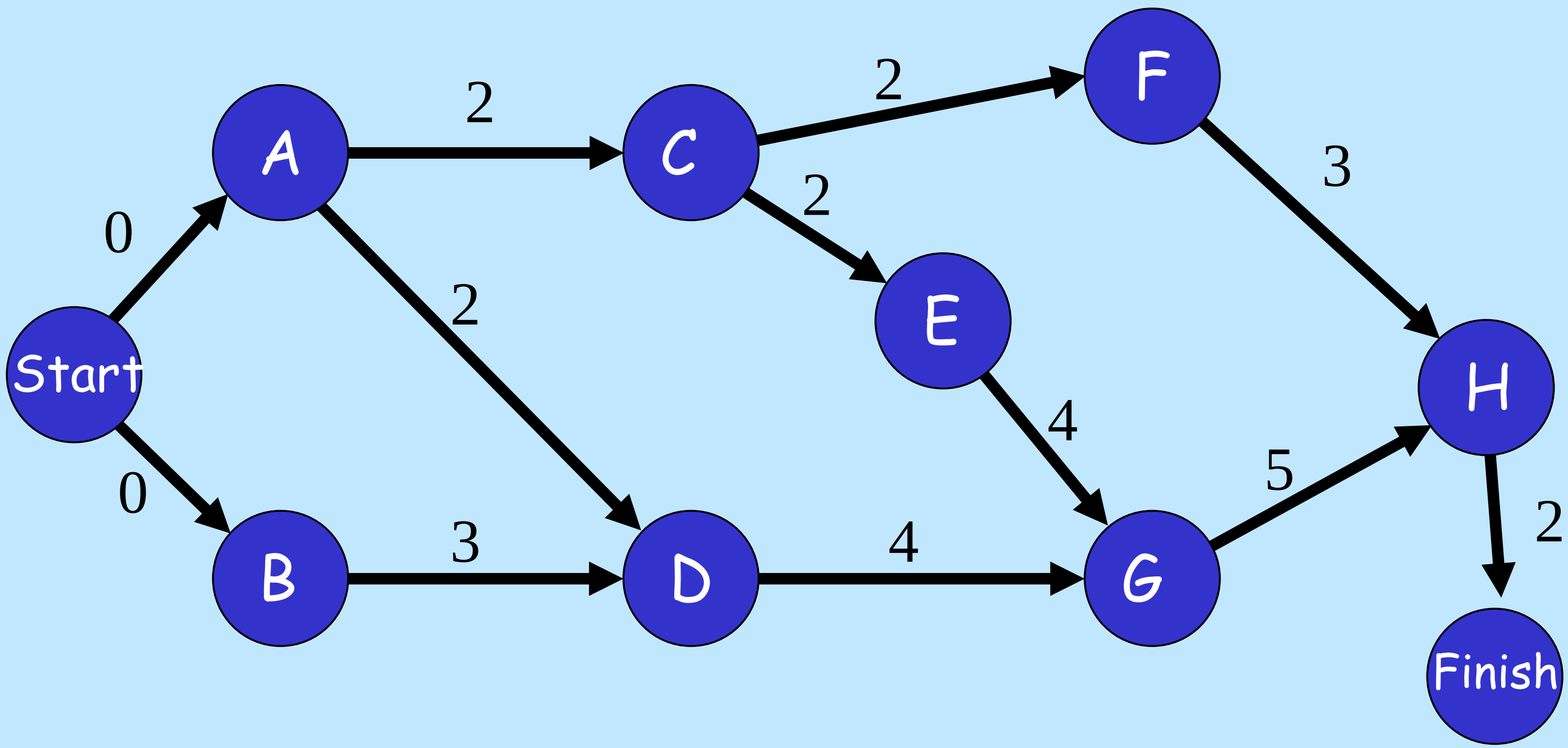
ES [p]= max{EF[i]: i is an immediate predecessor of p}

EF [p]= ES [p] + activity time (p)

d[p]= argmax of the maximum above

Return to Step 2

Early Schedule Computation



Late Schedules

Assume there is a <u>due date </u>for the project

LS[k]=due date – length of longest path from node k to node finish

Backward Pass

Begin with the last event and work backwards

Latest Finish Time Rule:

- ☑ If an activity is an immediate predecessor for just a single activity, its LF equals the LS of the activity that immediately follows it
- ☑ If an activity is an immediate predecessor to more than one activity, its LF is the minimum of all LS values of all activities that immediately follow it

LF = Min (LS of all immediate following activities)

Backward Pass

Begin with the last event and work backwards

Latest Start Time Rule:

- ☑ The latest start time (LS) of an activity is the difference of its latest finish time (LF) and its activity time

$$LS = LF - \text{Activity time}$$

Computing a Late Schedule (Backward Pass)

Step 1: Order the nodes lexicographically.

LF [finish] = LS [finish]= due date

Step 2: Terminate if the late start time of the start node has been fixed.

Otherwise, let p be the highest number of an unprocessed node.

Step 3: Compute:

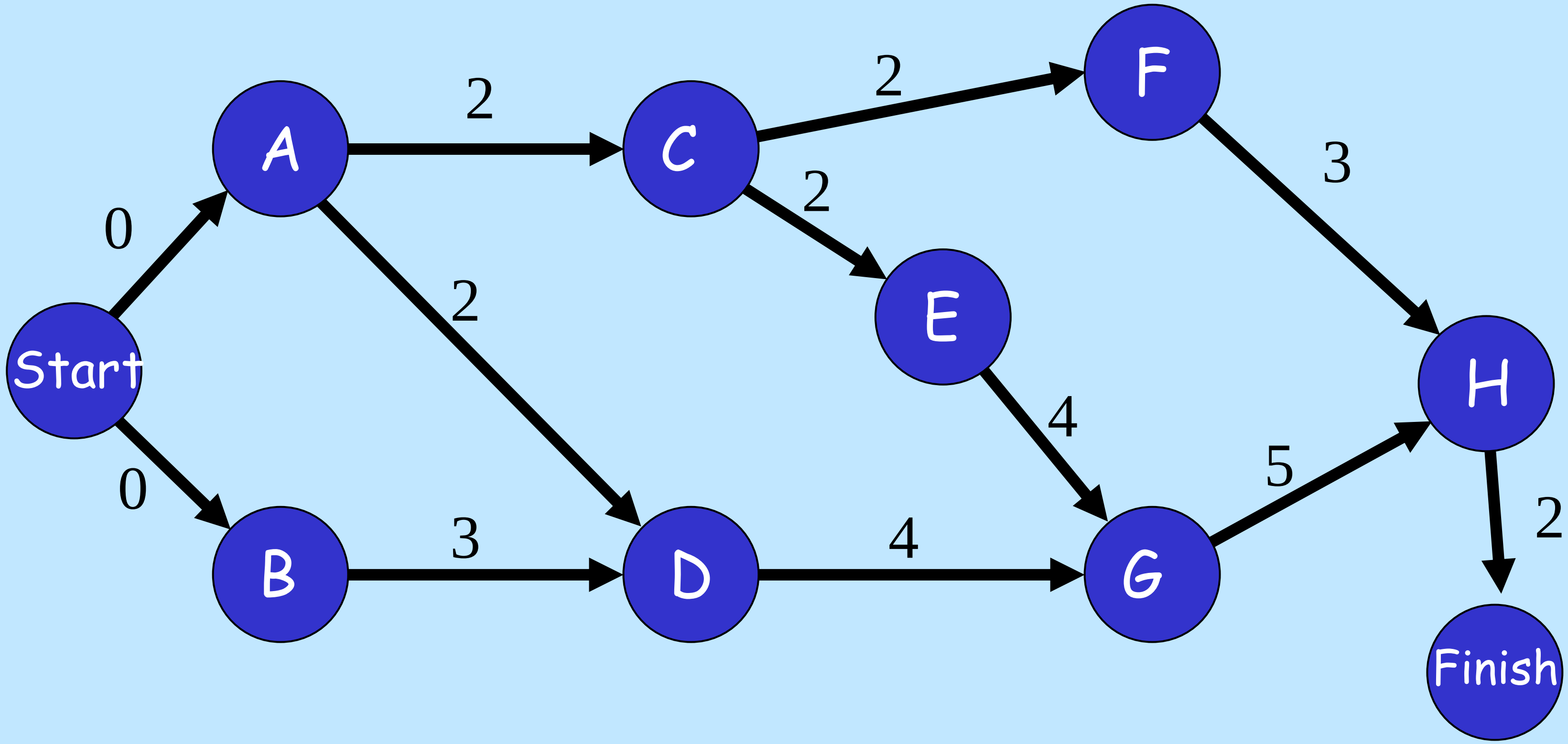
LF [p]= min{LS[i]: i is an immediate successor of p}

LS [p]= LF [p] - activity time (p)

d[p]= argmin of the minimum above

Return to Step 2

Late Schedule Computation
Due Date=15



Computing Slack Time

After computing the ES, EF, LS, and LF times for all activities, compute the slack or free time for each activity

- ☑ Slack is the length of time an activity can be delayed without delaying the entire project

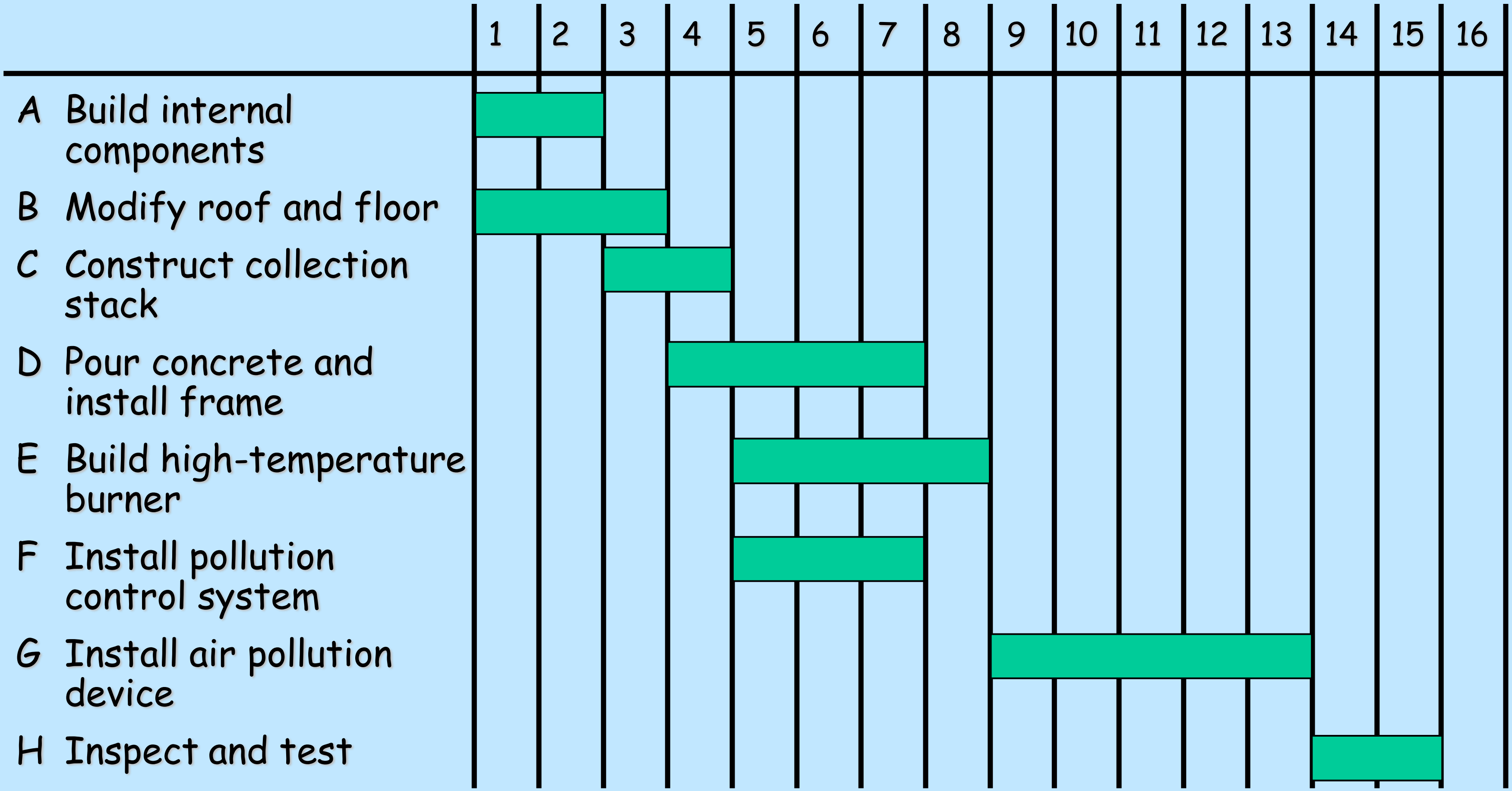
$$\text{Slack} = LS - ES \qquad \text{or} \qquad \text{Slack} = LF - EF$$

- ☑ Critical Activities: Activities with Zero Slack

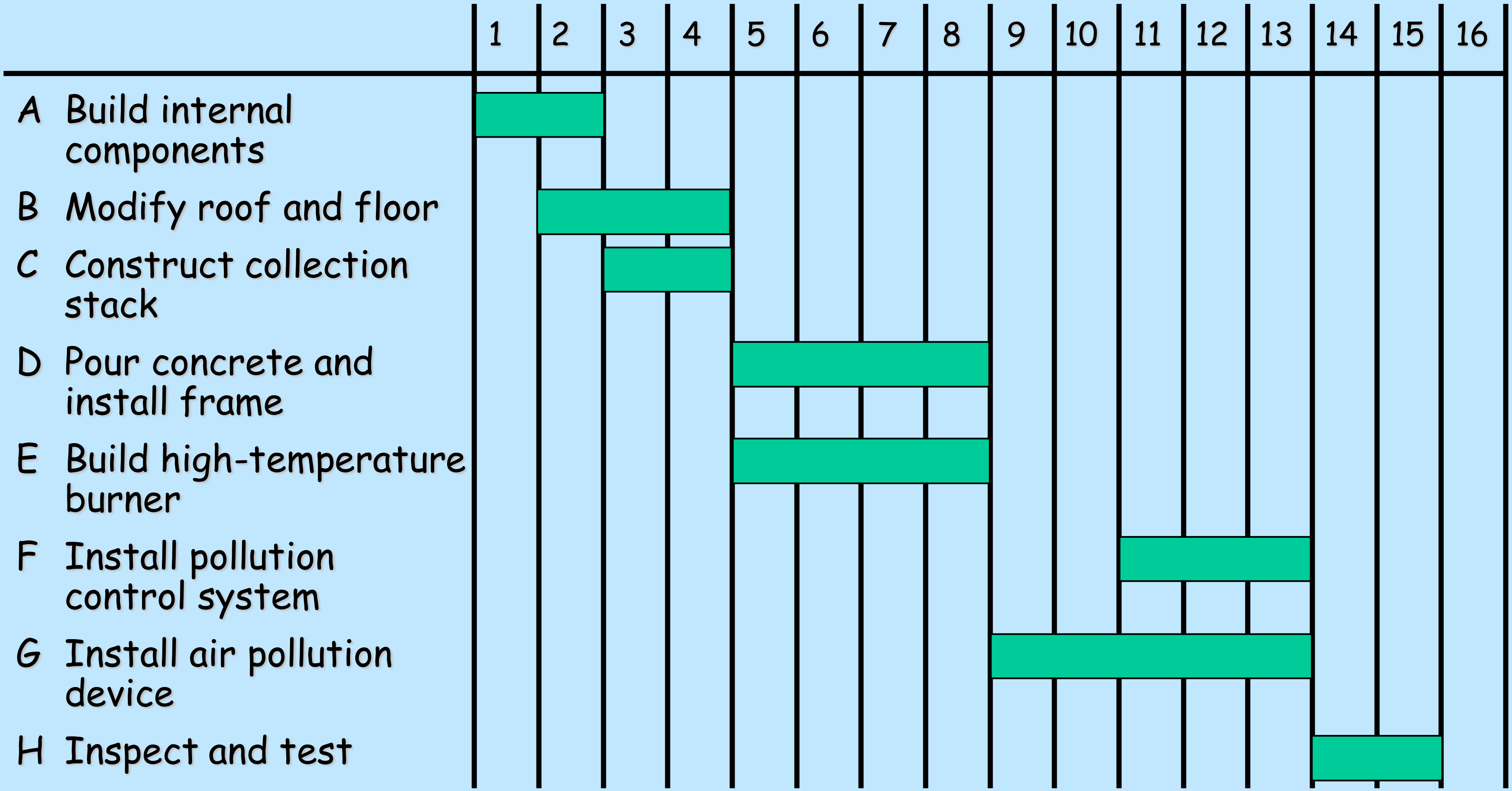
Computing Slack Time

Activity	Earliest Start ES	Earliest Finish EF	Latest Start LS	Latest Finish LF	Slack LS – ES	On Critical Path
A	0	2	0	2	0	Yes
B	0	3	1	4	1	No
C	2	4	2	4	0	Yes
D	3	7	4	8	1	No
E	4	8	4	8	0	Yes
F	4	7	10	13	6	No
G	8	13	8	13	0	Yes
H	13	15	13	15	0	Yes

ES - EF Gantt Chart for Milwaukee Paper



LS – LF Gantt Chart for Milwaukee Paper



Example:

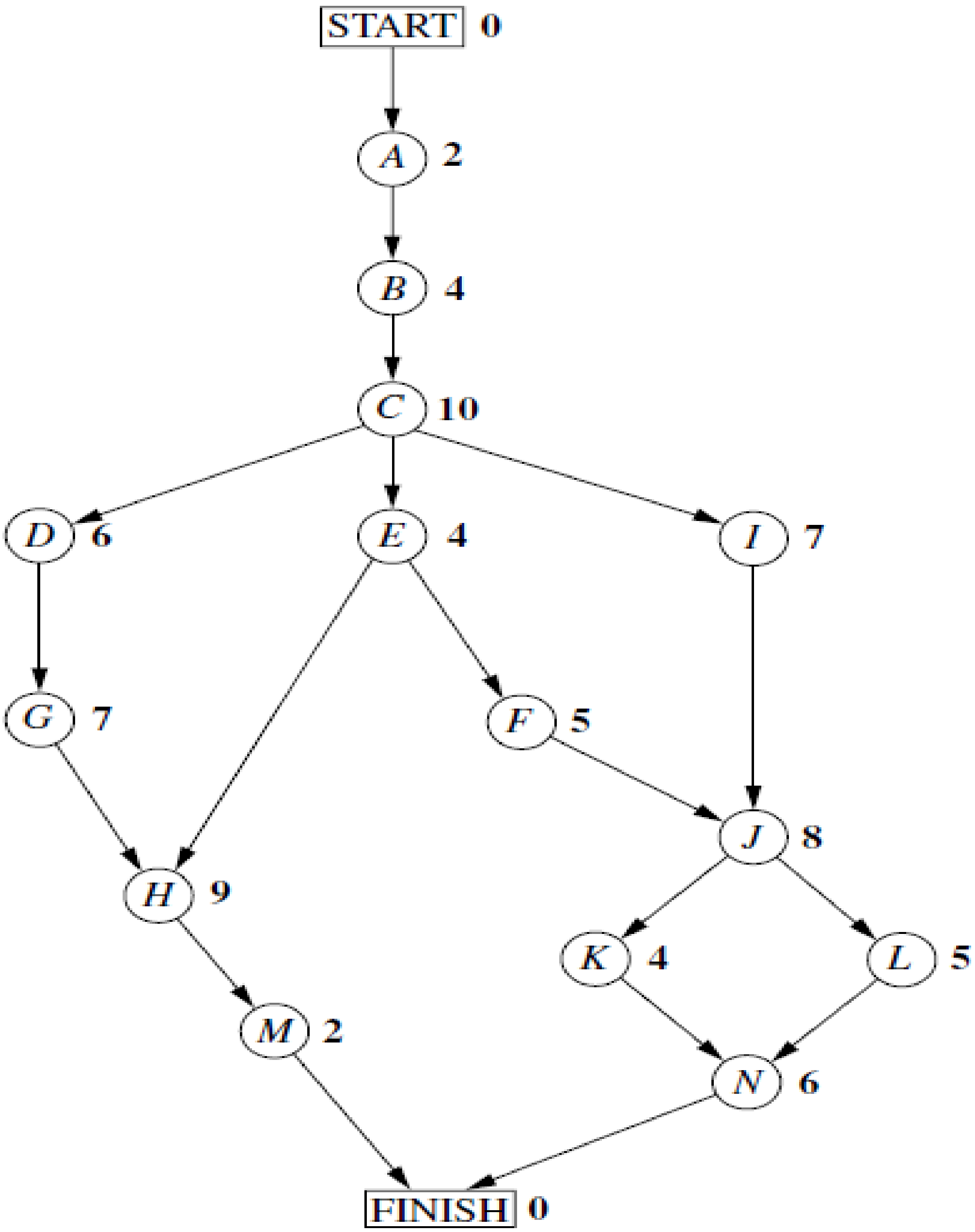
Activities	Duration	Immediate Predecessor
A: Purchase Raw Material	9	-
B: Train workers	6	-
C: Produce part 1	8	A,B
D: Produce Part 2	7	A, B
E: Test part 2	10	D
F: Assemble parts 1 and 2	12	C,E

CPM-Example

TABLE 10.1 Activity list for the Reliable Construction Co. project

Activity	Activity Description	Immediate Predecessors	Estimated Duration
A	Excavate	—	2 weeks
B	Lay the foundation	*A*	4 weeks
C	Put up the rough wall	*B*	10 weeks
D	Put up the roof	*C*	6 weeks
E	Install the exterior plumbing	*C*	4 weeks
F	Install the interior plumbing	*E*	5 weeks
G	Put up the exterior siding	*D*	7 weeks
H	Do the exterior painting	*E, G*	9 weeks
I	Do the electrical work	*C*	7 weeks
J	Put up the wallboard	*F, I*	8 weeks
K	Install the flooring	*J*	4 weeks
L	Do the interior painting	*J*	5 weeks
M	Install the exterior fixtures	*H*	2 weeks
N	Install the interior fixtures	*K, L*	6 weeks

CPM-Example

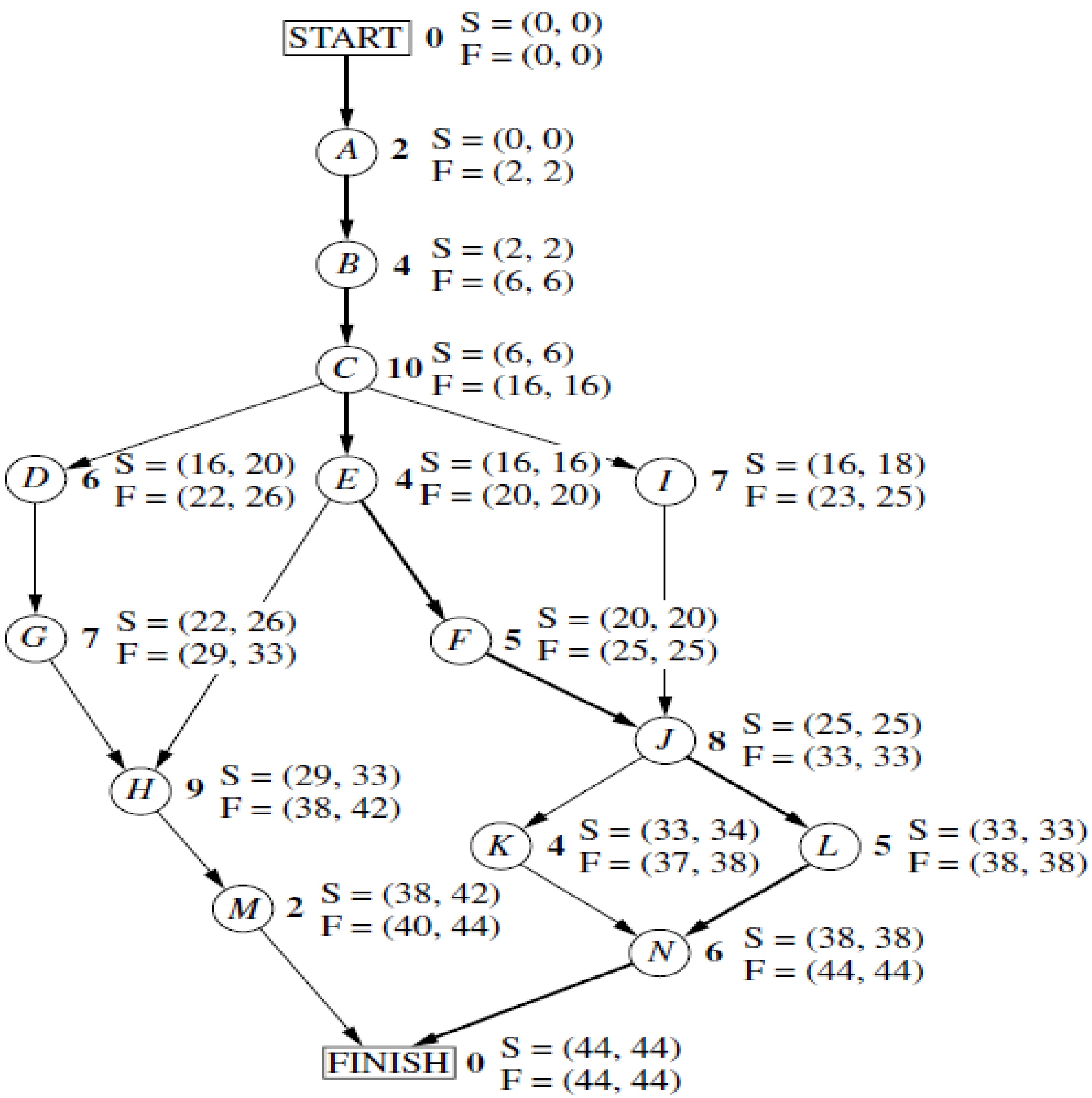


Activity Code

A. Excavate
B. Foundation
C. Rough wall
D. Roof
E. Exterior plumbing
F. Interior plumbing
G. Exterior siding
H. Exterior painting
I. Electrical work
J. Wallboard
K. Flooring
L. Interior painting
M. Exterior fixtures
N. Interior fixtures

CPM-Example

The complete project network showing ES and LS (in parentheses above the node) and EF and LF (in parentheses below the node) for each activity of the Reliable Construction Co. project. The darker arrows show the critical path through the project network.



PERT: Variability in Activity Times

- ☑ CPM assumes we know a fixed time estimate for each activity and there is no variability in activity times
- ☑ PERT uses a probability distribution for activity times to allow for variability

PERT: Variability in Activity Times

- ☑ Three time estimates are required
  - ☑ Optimistic time (a) – if everything goes according to plan
  - ☑ Pessimistic time (b) – assuming very unfavorable conditions
  - ☑ Most likely time (m) – most realistic estimate

PERT: Variability in Activity Times

Estimate follows beta distribution

Expected time:

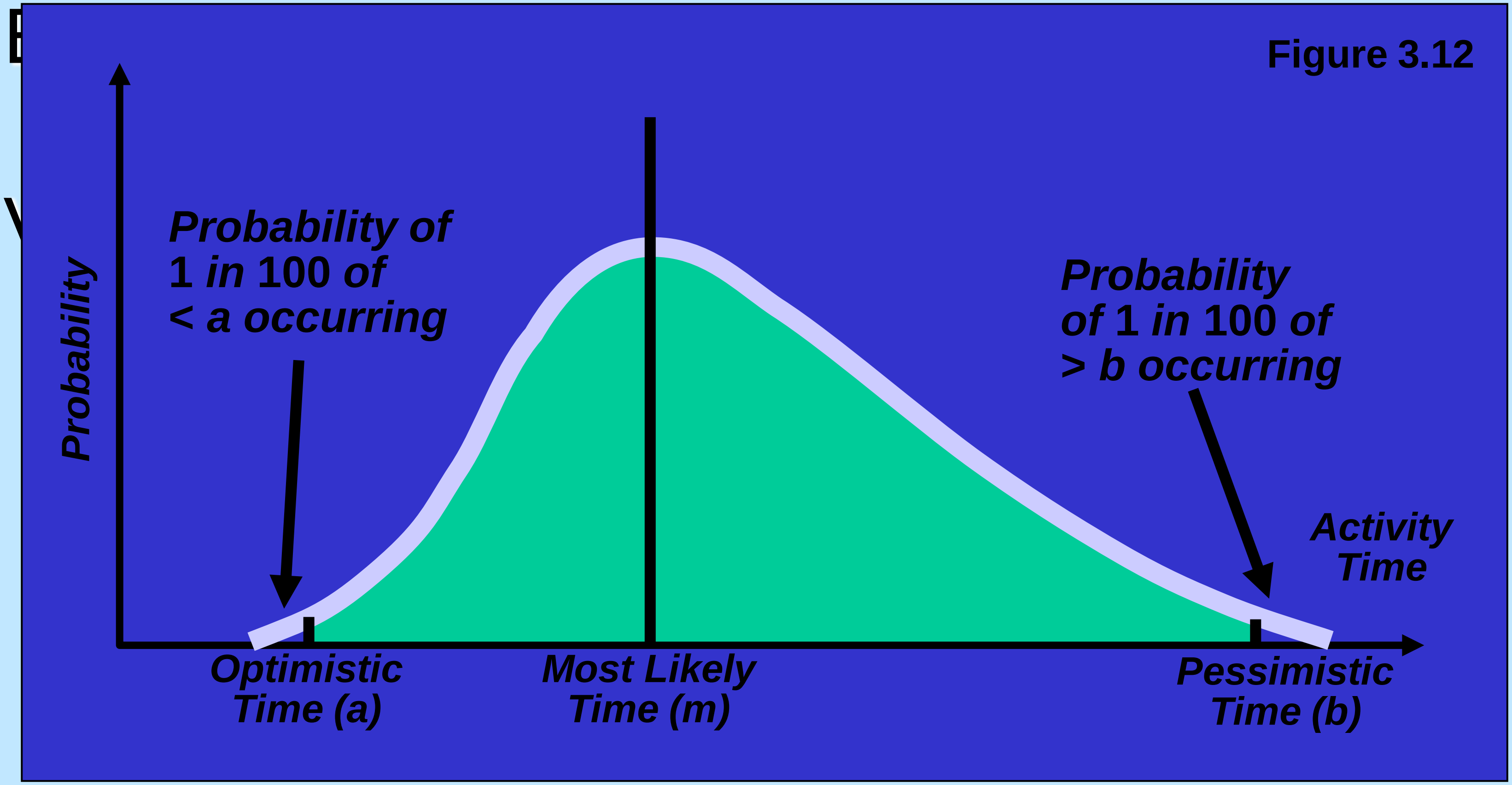
$$t = (a + 4m + b)/6$$

Variance of times:

$$v = [(b - a)/6]^2$$

Variability in Activity Times

Estimate follows beta distribution



Computing Variance

Activity	*Optimistic* *a*	*Most Likely* *m*	*Pessimistic* *b*	*Expected Time* $t = (a + 4m + b)/6$	*Variance* $[(b - a)/6]^2$
A	1	2	3	2	.11
B	2	3	4	3	.11
C	1	2	3	2	.11
D	2	4	6	4	.44
E	1	4	7	4	1.00
F	1	2	9	3	1.78
G	3	4	11	5	1.78
H	1	2	3	2	.11

Probability of Project Completion

Project variance is computed by summing the variances of critical activities

$$\sigma_p^2 = \text{Project variance}$$

$$= \sum(\text{variances of activities on critical path})$$

Probability of Project Completion

Project variance is computed by

Project variance

$$\sigma_p^2 = .11 + .11 + 1.00 + 1.78 + .11 = 3.11$$

Project standard deviation

$$\sigma_p = \sqrt{\text{Project variance}}$$

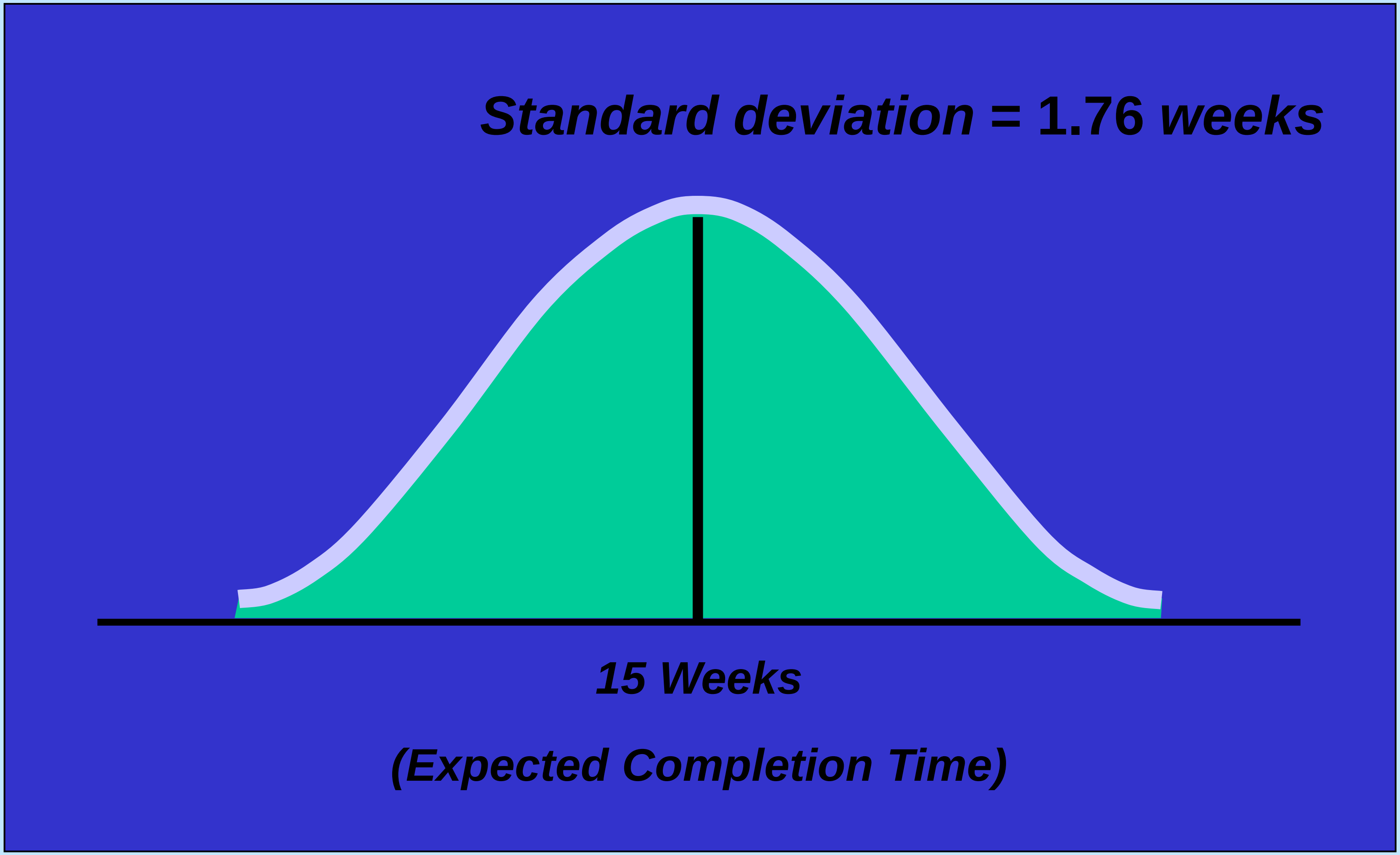
$$= \sqrt{3.11} = 1.76 \text{ weeks}$$

Probability of Project Completion

PERT makes two more assumptions:

- ☑ Total project completion times follow a normal probability distribution
- ☑ Activity times are statistically independent

Probability of Project Completion



Probability of Project Completion

What is the probability this project can be completed on or before the 16 week deadline?

***P(T* ≤ 16 *)=P(Z* ≤ 0.57) where**

$$Z = \left(\begin{matrix} \text{due} \\ \text{date} \end{matrix} - \begin{matrix} \text{expected date} \\ \text{of completion} \end{matrix} \right) / \sigma_p$$

$$= (16 \text{ wks} - 15 \text{ wks})/1.76$$

$$= 0.57$$

LP to find the project completion time

G=(N,A): AON project network

Parameters:

Decision Variables:

Model:

Crashing a Project

What if

due date < project completion time?

Crashing the project:

Reducing the project duration below its normal value by accomplishing some activities in a shorter time at a larger cost

Factors to Consider When Crashing A Project

- ☑ The amount by which an activity is crashed is, in fact, permissible
- ☑ Taken together, the shortened activity durations will enable us to finish the project by the due date
- ☑ The total cost of crashing is as small as possible

Crashing a Project

Let:

Cost[i]=cost of reducing the duration of activity i by one unit

u[i]=maximum possible reduction in the duration of activity i

LP to find the min cost crashing

Decision Variables:

Model:

Least possible project duration with a budget of K units?

Decision Variables:

Model:

Advantages of PERT/CPM

1. Especially useful when scheduling and controlling large projects
2. Straightforward concept and not mathematically complex
3. Network depiction help highlight relationships among project activities
4. Critical path and slack time analyses help pinpoint activities that need to be closely watched

Advantages of PERT/CPM

5. Project documentation and graphics point out who is responsible for various activities
6. Applicable to a wide variety of projects
7. Useful in monitoring not only schedules but costs as well

Limitations of PERT/CPM

1. Project activities have to be clearly defined, independent, and stable in their relationships
2. Precedence relationships must be specified and networked together
3. Time estimates tend to be subjective and are subject to fudging by managers
4. There is an inherent danger of too much emphasis being placed on the longest, or critical, path