IE 400: Principles of Engineering Management
Study Set 5

Summer 2020

Question 1. Consider the problem

$$
\begin{aligned}
\max\ & 5x_1 + 3x_2 \\
\text{s.t. } & 4x_1 + 2x_2 \leq 12 \\
& 4x_1 + x_2 \leq 10 \\
& x_1 + x_2 \leq 4 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

(a) Convert the problem into standard form and construct a bfs at which $(x_1, x_2) = (0, 0)$.

(b) Carry out the full tableau implementation of the simplex method, starting with the basic feasible solution of part (a).

(c) Draw a graphical representation of the problem in terms of the original variables x_1, x_2 and indicate the path taken by the simplex algorithm.

Question 2. Glueco produces three types of glue on two different production lines. Each line can be utilized by up to seven workers at a time. Workers are paid \$500 per week on production line 1, and \$900 per week on production line 2. A week of production costs \$1,000 to set up production line 1 and \$2,000 to set up production line 2. During a week on a production line, each worker produces the number of units of glue shown in the table below.

	Glue		
Production Line	1	2	3
1	20	30	40
2	50	35	45

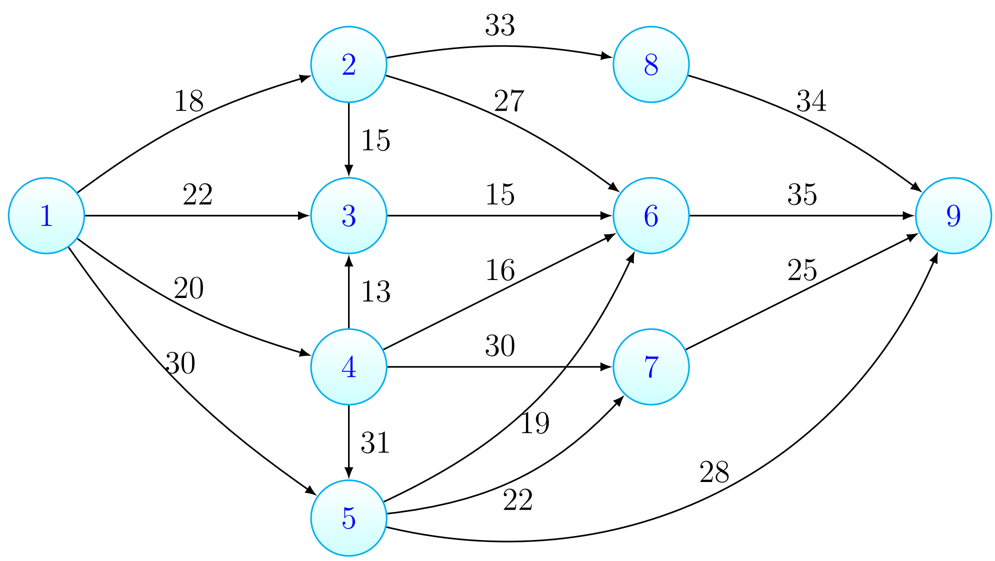
Each week, at least 120 units of glue 1, at least 150 units of glue 2, and at least 200 units of glue 3 must be produced. Formulate an IP to minimize the total cost of meeting weekly demands.

Question 3. Suppose it costs $12,000 to purchase a new car. The annual operating cost and resale value of a used car are shown in the table below.

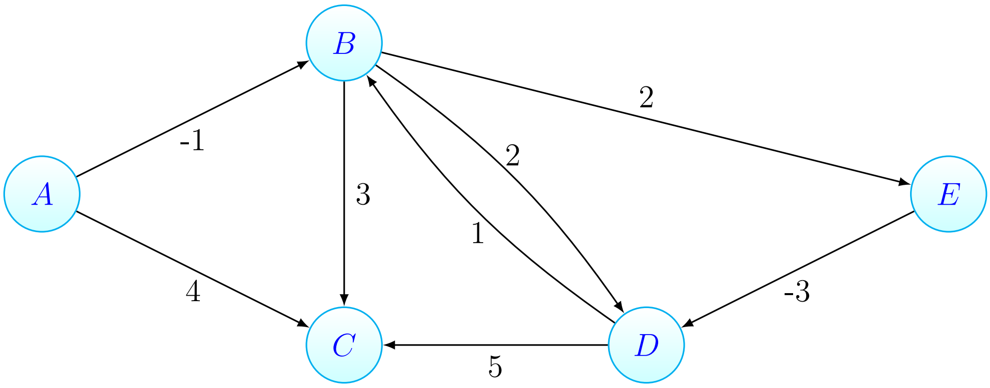
Age of Car (Years)	Resale Value ($)	Operating Cost ($)
1	7000	300 (year 1)
2	6000	500 (year 2)
3	4000	800 (year 3)
4	3000	1200 (year 4)
5	2000	1600 (year 5)
6	1000	2200 (year 6)

Assuming that one now has a new car, model and solve replacement problem that minimizes the net costs of owning and operating a car for the next six years using shortest path.

Question 4. Find shortest paths from 1 to all other nodes using Dijkstra's algorithm.



Question 5. Use Bellman Ford algorithm to find the shortest path of the following graph where A is the source node.



Question 6. Given a network, a special source node s and weights on arcs corresponding to heights, find from s to every other node a path on which the highest height encountered is as few as possible.

(a) Does principle of optimality apply for this problem? Why or why not?

(b) Write down the functional equations corresponding to this variant of the shortest path problem.

(c) Solve the above functional equation by adapting an algorithm (state which one) designed for solving the single source shortest path problem.

Question 7. Consider the list of activities for an agricultural field:

Activity			Completion Time (Hours)		
	Description	Predecessor	Optimistic	Most Likely	Pessimistic
A	Excavation	-	5	6	7
B	Sprinkler installation	-	4	5	18
C	Planting	A	4	15	20
D	Irrigation	B,C	3	4	5
E	Fertilizer Utilization	A	5	16	18

(a) Determine the expected value and the variance of the completion time for each activity and find the critical path.

(b) Assuming that the normal distribution applies, determine the probability that the critical path will take between 18 and 26 days to complete.

Question 8. A new sander costs \$3600 and has an annual maintenance cost of \$400. The salvage value after 7 years is \$600. Assuming an annual interest rate of 10%, what is the present worth?

Question 9. Solve the following linear programming problem using DP. Define stages, states and recursive function clearly.

$$
\begin{aligned}
\max\ & 2x_1 + 5x_2 \\
\text{s.t.}\ & 2x_1 + x_2 \leq 430 \\
& x_2 \leq 230 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

Question 10. An electric power utility forecasts that r_t kilowatt-hours (kwh) of generating capacity will be needed during year t (the current year is year 1). At the beginning of each year, the utility must decide by how much generating capacity should be expanded. It costs $c_t(x)$ dollars to change generating capacity by x kwh during year t (x can be both positive and negative). During each year, 10% of the old generating capacity becomes obsolete and unusable (capacity does not become obsolete during its first year of operation). It costs the utility $m_t(i)$ dollars to maintain i units of capacity during year t. At the beginning of year 1, 100,000 kwh of generating capacity are available. Formulate a dynamic programming recursion that will enable the utility to minimize the total cost of meeting power requirements for the next T years.

Question 11. A company needs 15, 30, 10, 30 and 20 workers during each of the next five years. At present, the company has 20 workers. Each worker is paid \$30,000 per year. At the beginning of each year, workers may be hired or fired. It costs \$10,000 to hire a worker and \$20,000 to fire a worker. A newly hired worker can be used to meet the current years worker requirement. During each year, 10% of all workers quit (workers who quit do not incur any firing cost). With dynamic programming, formulate a recursion that can be used to minimize the total cost incurred in meeting the worker requirements of the next five years.