IE 400: Principals of Engineering Management

SET 1- Solutions to In-Class Examples

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Question 1. (**Milk Production**) There are two machines needed to process milk to produce low-fat and regular milk. The processing times are:

	(Pasteurization) Machine 1	**(Homogenization) Machine 2**
Low-fat	3 min/lt	1 min/lt
Regular	2 min/lt	2 min/lt

In a day, machine 1 can be used for 12 hours and machine 2 can be used for 6 hours. The sales price of low-fat milk is 2.5 TL/lt and that of regular milk is 1.5 TL/lt. Find how much low-fat and regular milk should be produced in a day to maximize total revenue.

Decision Variables:

x_1 : amount in liters of low-fat milk produced in a day

x_2 : amount in liters of regular milk produced in a day

Model:

$$
\begin{aligned}
\max\ & 2.5x_1 + 1.5x_2 \\
\text{s.t.}\ & 3x_1 + 2x_2 \leq 720 \\
& x_1 + 2x_2 \leq 360 \\
& x_1 \geq 0 \\
& x_2 \geq 0
\end{aligned}
$$

Question 2. (**Blending Problem 1**)

An oil refinery distills crude oil from SaudiArabia and Venezuela into gasoline, jet fuel and lubricants. The crude differ in chemical composition and yields

- 1 ton of S.A. crude oil $\Longrightarrow$ 0.3 tons of gasoline, 0.4 tons of jet fuel, 0.2 tons of lubricants
- 1 ton of Ven. crude oil $\Longrightarrow$ 0.4 tons of gasoline, 0.2 tons of jet fuel, 0.3 tons of lubricants

Remaining 0.1 tons is lost in refining.

At most 90 tons can be purchased daily from S.A. at a cost of \$600/ton. At most 60 tons can be purchased daily from Ven. at a cost of \$550/ton

The daily demand is 20 tons of gasoline, 15 tons of jet fuel, and 5 tons of lubricants.

How should the demand be met at a **minimum total cost**?

Decision Variable:

x_i : Amount of crude oil purchased from country i,
$i = 1, 2$. (1: S.A., 2: Venezuella)

Model:

$$
\begin{aligned}
\min\ & 600x_1 + 550x_2 \\
\text{s.t.}\ & 0.3x_1 + 0.4x_2 \geq 20 \\
& 0.4x_1 + 0.2x_2 \geq 15 \\
& 0.2x_1 + 0.3x_2 \geq 5 \\
& x_1 \leq 90 \\
& x_2 \leq 60 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

Question 3. (Transportation Problem)
There are two power plants generating electricity, and three factories that need electricity in their production. power plant 1 has 1250 MWatts capacity and power plant 2 has 2000 MWatts capacity. The demands of factories 1,2 and 3 are 1500 MWatts, 750 MWatts and 750 MWatts respectively. The transmission costs are given as follows:

Table: Transmission costs per MWatt in USD

	F1	**F2**	**F3**
PP1	50	100	60
PP2	30	20	35

How to satisfy the demand of each factory from the two power plants with minimum total cost?

Decision Variable:

x_{ij} : Electricity provided in Mwatts from power plant i to factory j, $i = 1, 2,\ j = 1, 2, 3$

Model:

$$
\begin{aligned}
\min\ & 50x_{11} + 100x_{12} + 60x_{13} + 30x_{21} + 20x_{22} + 35x_{23} \\
\text{s.t.}\ & x_{11} + x_{12} + x_{13} \leq 1250 \\
& x_{21} + x_{22} + x_{23} \leq 2000 \\
& x_{11} + x_{21} \geq 1500 \\
& x_{12} + x_{22} \geq 750 \\
& x_{13} + x_{23} \geq 750 \\
& x_{ij} \geq 0 \quad i = 1, 2 \quad j = 1, 2, 3
\end{aligned}
$$

Question 4. (General Transportation Problem)
Say there are m warehouses and n stores.
Let a_i denote the amount of stock at warehouse i for $i = 1, \ldots, m$.
Let d_j denote the demand of store j for $j = 1, \ldots, n$.
Let c_{ij} denote the cost of transporting a unit of product from warehouse i to store j, for $i = 1, \ldots, m$ and $j = 1, \ldots, n$.
How should the demands of stores be met at a minimum total transportation cost while respecting the stock limitations of the warehouses?

Decision Variable:

x_{ij} : Amount of product transported from warehouse i to store j,
$i = 1, \dots m$ $j = 1, \dots, n$

Model:

$$
\begin{aligned}
\min \ & \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} & \\
\text{s.t.} \ & \sum_{j=1}^{n} x_{ij} \le a_i & i = 1, \dots m \\
& \sum_{i=1}^{m} x_{ij} \ge d_j & j = 1, \dots, n \\
& x_{ij} \ge 0 & i = 1, \dots m \ j = 1, \dots, n
\end{aligned}
$$

Question 5. (Blending Problem 2)
OJ Inc. produces orange juice from 3 different grades of Oranges. Oranges are graded 1(poor), 2(medium), and 3(good) depending on their quality.

Grade	Yield (Juice lt/kg)
1	0.4
2	0.5
3	0.6

The company currently has

- 100 kgs of Grade 1 oranges
- 150 kgs of Grade 2 oranges
- 200 kgs of Grade 3 oranges

Question 5.

OJ Inc produces 3 types of orange juices from these oranges: Superior, premium and regular. The production requires are as follows:

Type	Minimum Average Grade	Profits per Liter	Minimum Daily Production
Superior	2.4	1.5 TL	45 lt
Premium	2.2	1 TL	60 lt
Regular	2	0.75 TL	100 lt

Assume the company may sell more than demand at the same prices. How can the company maximize total profit while satisfying production requirements?

Decision Variable:

x_{ij} : Liters of Orange Juice type i produced using oranges of grade j respectively,

$i = 1, 2, 3$ (1: superior, 2: premium, 3:regular), $j = 1, 2, 3$

Model:

$$
\begin{aligned}
\max\ & 1.5(x_{11} + x_{12} + x_{13}) + (x_{21} + x_{22} + x_{23}) + 0.75(x_{31} + x_{32} + x_{33}) \\
\text{s.t. } & x_{11} + x_{21} + x_{31} \leq 40 \quad (100 \times 0.4) \\
& x_{12} + x_{22} + x_{32} \leq 75 \quad (150 \times 0.5) \\
& x_{13} + x_{23} + x_{33} \leq 120 \quad (200 \times 0.6) \\
& x_{11} + x_{12} + x_{13} \geq 45 \\
& x_{21} + x_{22} + x_{23} \geq 60 \\
& x_{31} + x_{32} + x_{33} \geq 100 \\
& x_{11} + 2x_{12} + 3x_{13} \geq 2.4(x_{11} + x_{12} + x_{13}) \\
& x_{21} + 2x_{22} + 3x_{23} \geq 2.2(x_{21} + x_{22} + x_{23}) \\
& x_{31} + 2x_{32} + 3x_{33} \geq 2(x_{31} + x_{32} + x_{33}) \\
& x_{ij} \geq 0 \quad i = 1, 2, 3 \quad j = 1, 2, 3
\end{aligned}
$$

Question 6. (Short Term Investment Planning)
You have 12,500 TL at the beginning of year 2018 and would like to invest your money. Your goal is to maximize your total amount of money at the beginning of year 2021. You have the following options:

Option	**Duration (years)**	**Total Interest Rate**	**Available at the Beginning of**
1	2	26%	2018,2019
2	1	12%	2018,2019,2020
3	3	38%	2018
4	2	27%	2019

Decision Variables:

x_t : Amount of money invested in option 1 in the beginning of year t, $t = 0, 1$ (0: 2018,1: 2019)

y_t : Amount of money invested in option 2 in the beginning of year t, $t = 0, 1, 2$ (0: 2018,1: 2019,2: 2020)

z : Amount of money invested in option 3 in the beginning of year 0.

t : Amount of money invested in option 4 in the beginning of year 1.

Model:

$$
\begin{aligned}
\max\ & 1.26x_1 + 1.12y_2 + 1.38z + 1.27t \\
\text{s.t.}\ & x_0 + y_0 + z = 12500 \\
& x_1 + y_1 + t = 1.12y_0 \\
& y_2 = 1.12y_1 + 1.26x_0 \\
& x_t \geq 0 && t = 0, 1 \\
& y_t \geq 0 && t = 0, 1, 2 \\
& z \geq 0 \\
& t \geq 0
\end{aligned}
$$

Question 7. (**Product Mix Problem**)
Milk chocolate is produced with three ingredients. 1 kg of milk chocolate contains:

- 0.5 kg of milk
- 0.4 kg of cocoa
- 0.1 kg of sweetener

Each of these three ingredients must be processed before they can be used in the production of milk chocolate. The factory has two departments that can process these ingredients:

Ingredient	Dept 1 (hrs/kg)	Dept 2 (hrs/kg)
Milk	0.4	0.6
Cocoa	0.3	0.2
Sweetener	0.5	0.6

Both departments have 150 hours of available time per week for processing. Formulate an LP problem that maximizes the total amount of milk chocolate produced in a week.

Decision Variables:

x_{ij} : Amount of ingredient i processed by department j, $i = 1, 2, 3$
$j = 1, 2$

z : Total amount of milk chocolate produced in a week

Model:

$$
\begin{aligned}
\max\ & z \\
\text{s.t. } & 0.4x_{11} + 0.3x_{21} + 0.5x_{31} \leq 150 \\
& 0.6x_{12} + 0.2x_{22} + 0.6x_{32} \leq 150 \\
& x_{11} + x_{12} \geq 0.5z && (3) \\
& x_{21} + x_{22} \geq 0.4z && (4) \\
& x_{31} + x_{32} \geq 0.1z && (5) \\
& x_{ij} \geq 0 \quad i = 1, 2, 3 \quad j = 1, 2
\end{aligned}
$$

Note that constraints (3)-(4)-(5) follow as:

$$z = \min\left\{\frac{x_{11}+x_{12}}{0.5}, \frac{x_{21}+x_{22}}{0.4}, \frac{x_{31}+x_{32}}{0.1}\right\} \qquad (\star)$$

which implies:

$$\begin{aligned} z &\leq \frac{x_{11}+x_{12}}{0.5} \\ z &\leq \frac{x_{21}+x_{22}}{0.4} \\ z &\leq \frac{x_{31}+x_{32}}{0.1} \end{aligned}$$

(($\star$) cannot be added to the model as a constraint, because it not in linear form.)

Question 8. (**Error Minimization Problem**)

You estimate that there is a linear relationship between the stock price of a beverage company and the number of beverages sold (in thousands) in a day:

Day	Stock Price	Number of Beverages Sold
1	2	10
2	3	12
3	2	8
4	3	11
5	4	14

Question 8.

You predict the relationship as $s_i = an_i + b$, where s is the stock price and n_i is the number of beverages sold (in thousands) on day i, $i = 1, \dots, 5$.

For a given choice of a and b, your estimation error on day i is $|s_i - an_i - b|$, $i = 1, \dots, 5$. Choose the unknown coefficients a and b such that the largest estimation error in any one of the 5 days is as small as possible.

Decision Variables:

z : largest estimation error among days $1, \ldots, 5$

a : coefficient 1

b : coefficient 2

Parameters:

s_i : Stock price at day $i = 1, \ldots, 5$

n_i : number of beverages sold at day $i = 1, \ldots, 5$

Our objective is

$$\min \max_{i=1,\dots,5} \{|s_i - an_i - b|\} = \min z$$

Model:

$$\begin{aligned} &\min\ z \\ &\text{s.t. } z \geq s_i - an_i - b && i = 1, \dots, 5 \\ &\quad\ \ z \geq -s_i + an_i + b && i = 1, \dots, 5 \end{aligned}$$

By plugging in the values of the parameters, we can rewrite the model as:

$$
\begin{aligned}
\min\ & z \\
\text{s.t.}\ & z \geq 2 - 10a - b \\
& z \geq 3 - 12a - b \\
& z \geq 2 - 8a - b \\
& z \geq 3 - 11a - b \\
& z \geq 4 - 14a - b \\
& z \geq 10a + b - 2 \\
& z \geq 12a + b - 3 \\
& z \geq 8a + b - 2 \\
& z \geq 11a + b - 3 \\
& z \geq 14a + b - 4
\end{aligned}
$$

Question 9. XYZ Company produces three products A, B, and C and has to meet the demand for the next four weeks.

	Weekly Demand			
Product	**1**	**2**	**3**	**4**
A	25	30	15	22
B	15	16	25	18
C	10	17	12	12

Each unit of

- Product A requires 3 hours of labor
- Product B requires 2 hours of labor
- Product C requires 4 hours of labor

Question 9.

During a week, 120 hours of regular-time labor is available at a cost of \$10/hour. In addition 25 hours of overtime labor is available weekly at a cost of \$15/hour. No shortages are allowed and XYZ Company has 18A, 13 B, and 15 C initial stocks for these products. Provide a Linear Programming formulation to determine a production schedule that minimizes the total labor costs for the next four weeks.

Decision Variables:

x_{ij} : Amount of product i produced during week j, $i \in \{A, B, C\}$ and $j = 1, 2, 3, 4$.

I_{ij} : the amount of inventory of product i at the end of week j after meeting demand. $i \in \{A, B, C\}$ and $j = 0, 1, 2, 3, 4$.

O_j : The amount of overtime hours used in week j, $j = 1, 2, 3, 4$.

$$
\begin{aligned}
\min\ & 10\left(\sum_{j=1}^{4}(3x_{A,1} + 2x_{B,1} + 4x_{C,1})\right) + 5\sum_{j=1}^{4} O_j \\
\text{s.t.}\ & 18 + x_{A,1} - 25 = I_{A,1} \\
& I_{A,1} + x_{A,2} - 30 = I_{A,2} \\
& I_{A,2} + x_{A,3} - 15 = I_{A,3} \\
& I_{A,3} + x_{A,4} - 22 = I_{A,4} \\
\\
& 13 + x_{B,1} - 15 = I_{B,1} \\
& I_{B,1} + x_{B,2} - 16 = I_{B,2} \\
& I_{B,2} + x_{B,3} - 25 = I_{B,3} \\
& I_{B,3} + x_{B,4} - 18 = I_{B,4}
\end{aligned}
$$

$$15 + x_{C,1} - 10 = I_{C,1}$$
$$I_{C,1} + x_{C,2} - 17 = I_{C,2}$$
$$I_{C,2} + x_{C,3} - 12 = I_{C,3}$$
$$I_{C,3} + x_{C,4} - 12 = I_{C,4}$$

$$3x_{A,1} + 2x_{B,1} + 4x_{C,1} \leq 120 + O_1$$
$$3x_{A,2} + 2x_{B,2} + 4x_{C,2} \leq 120 + O_2$$
$$3x_{A,3} + 2x_{B,3} + 4x_{C,3} \leq 120 + O_3$$
$$3x_{A,4} + 2x_{B,4} + 4x_{C,4} \leq 120 + O_4$$
$$O_j \leq 25 \quad j = 1, 2, 3, 4$$

$$I_{ij} \geq 0 \quad i \in \{A, B, C\} \quad j = 1, 2, 3, 4$$
$$x_{ij} \geq 0 \quad i \in \{A, B, C\} \quad j = 1, 2, 3, 4$$
$$O_j \geq 0 \quad j = 1, 2, 3, 4$$

Question 10. (**Rylon Corporation**)

Rylon Corporation manufactures Brute and Channelle perfumes from a raw material costing \$3/pound.

- Processing 1 pound of the material takes 1 hour and yields 3 ounces of Regular Brute and 4 ounces of Regular Chanelle. This perfume can be sold directly or can be further processed to make luxury versions of the perfumes.
- One ounce of Regular Brute can be sold for \$7, but 3 hours of processing and \$4 of cost will convert it into one ounce of Luxury Brute, which sells for \$18.
- One ounce of Regular Chanelle can be sold for \$6, but 2 hours of processing and \$4 of cost will convert it into one ounce of Luxury Chanelle, which sells for \$14.
- Rylon has available 4000 pounds of raw material, 6000 hours of processing time, and will sell everything it produces.

Determine how Rylon can maximize its profit.

Decision Variables:

x_1 : Amount of Regular Brute sold in ounces

x_2 : Amount of Luxury Brute sold in ounces

y_1 : Amount of Regular Chanelle sold in ounces

y_2 : Amount of Luxury Chanelle sold in ounces

z : Amount of raw material purchased

Model:

$$
\begin{aligned}
\max\ & 7x_1 + 18x_2 + 6y_1 + 14y_2 - 4x_2 - 4y_2 - 3z \\
\text{s.t. } & 3z = x_1 + x_2 \\
& 4z = y_1 + y_2 \\
& z + 3x_2 + 2y_2 \leq 6000 \\
& z \leq 4000 \\
& x_1, x_2, y_1, y_2, z \geq 0
\end{aligned}
$$

Question 11. (**Bel Aire Company**)
Bel Aire Pillow Company needs to manufacture 12,000 pillows in their factory next week. They have two kinds of employees: line workers (who do the actual work) and supervisors (who oversee the line workers). Bel Aire currently has 120 line workers and 20 supervisors. Each line worker can make 300 pillows per week. Supervisors do not engage in actual pillow-making. A recent study done on Bel Aire's operation concluded that, for proper supervision, there must be at least 1 supervisor for every 5 line workers. The study also suggested that Bel Aire might be able to cut its costs by doing some or all of the following:

Question 11.

- hiring new employees as line workers (the training cost is \$100/worker)
- firing some current line workers (the fired worker is given \$240 severance pay)
- promoting some current line workers to supervisors (the training cost is \$200/worker)
- demoting some current supervisors to line workers (no cost associated with this)

Pay for a line worker is \$400/week. Pay for a supervisor is \$700/week. All hiring and firing, and training occur before the upcoming work week, and the costs of those activities will be charged against the upcoming week. Bel Aire wishes to perform the appropriate hiring, firing, promotions and demotions that will minimize its operations cost for next week.

Decision Variables:

h : Number of employees hired as line workers

f : Number of line workers fired

p : Number of line workers promoted to supervisors

d : Number of supervisors demoted to line workers

Model:

$$
\begin{aligned}
\min\ & 400(120+h-f+d-p)+700(20+p-d)+100h+240f+200p \\
\text{s.t. } & 300(120+h-f+d-p) \geq 12000 \\
& 5(20+p-d) \geq (120+h-f+d-p) \\
& h,f,d,p \geq 0 \\
& h,f,d,p \text{ integer}
\end{aligned} \tag{4}
$$

What is different about this mathematical model? Is it a linear programming problem?

No, the number of people hired, fired, promoted or demoted can only take integer values. Therefore, contraint (4) is added to the model. This makes the model an integer programming (IP) problem.