Q. 1. [20 pts] Consider the following simplex tableau of a given **maximization** LP problem.

	z	x_1	x_2	x_3	x_4	x_5	RHS
1	1	2	**A**	0	0	0	12
x_3	0	**B**	-1	1	0	0	5
x_4	0	-6	**C**	0	1	0	1
x_5	0	-11	**D**	0	0	1	**E**

Give general conditions on each of the unknowns A-E such that each of the following statements is true. Even if the statement holds independently of the values of a specific variable, you should still mention that that variable can take any value. So, for each part you should present 5 conditions, one for each unknown.

(a) (*5 pts*) The tableau is final and there exists a unique optimal solution.

(b) (*5 pts*) The simplex method determines an unbounded solution from this tableau.

(c) (*5 pts*) The current bfs is degenerate (not necessarily optimal).

(d) (*5 pts*) The current solution is optimal, there are alternative optimal solutions but no alternative optimal bfs.

Q. 2. [20 pts] Consider the following LP:

$$\begin{aligned} \max\ & 10x_1 + 8x_2 - 3x_3 + 3x_4 \\ \text{s.t. } & 2x_1 + 4x_2 - 0.5x_3 + 0.5x_4 \leq 3 \\ & -2x_1 + 6x_2 - 4.5x_3 + 4.5x_4 \leq 7 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

(*14 pts*) Convert the LP in standard form. Then construct the tableau where x_1 is the basic variable in the first constraint and x_4 is the basic variable in the second constraint (order is important). Then, answer the following (The answers without proper reasoning will not be accepted):

(a) (*2 pts*) Is this a basic solution? Why?

(b) (*2 pts*) Is this a basic feasible solution? Why?

(c) (*2 pts*) Is this an optimal basic feasible solution? Why?

Q. 3. [20 pts] Consider the following LP:

$$\begin{aligned} \max\ & x_1 + 2x_2 + 4x_3 \\ \text{s.t.}\ & -x_1 + x_2 - x_3 \geq 2 \\ & -2x_1 + x_2 + x_3 \leq 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Solve the LP using big M method. Could you find an optimal solution? If yes, report the optimal solution. If no, state why.

Q. 4. [20 pts] Reatix must determine how much investment and debt to undertake during the next year. Each TL invested reduces the net present value (NPV) of the company by 10kr, and each TL of debt increases the NPV by 50kr (due to deductibility of interest payments). Reatix can invest at most 1 million TL during the coming year. Debt can be at most 40% of the investment. Reatix now has 800,000 TL in cash available. All investment must be paid from current cash or borrowed money. Set up an LP that maximizes the NPV of Reatix. Then, solve this LP by **simplex method**.

Q. 5. [20 pts] An employee of a manufacturing company wants to create a temporary inventory in his office so that he can access five different items. These items are scattered among ten boxes. For a given box, the employee either brings the entire box or does not bring the box to his office; he cannot select the items from the box. The table below shows which items are present in which boxes.

	Box									
Item	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	•			•			•			
2		•		•		•			•	
3			•		•			•		•
4	•	•					•			•
5			•			•			•	

The amount of space that each box occupies is given below:

Box	**Volume** (m^3)
1	3
2	5
3	3
4	1
5	2
6	1
7	3
8	2
9	4
10	2

The employee wants to minimize the space occupied by the boxes in his office. Formulate an IP that determines a set of boxes requiring the minimum total space such that each item is in at least one of these boxes.