IE 400: Principles of Engineering Management
Study Set 2

Fall 2020

Question 1. Identify the basic solutions, basic feasible solutions and degenerate basic feasible solutions (if exists) in the following linear system.

$$x_1 - x_2 + 3x_3 + 5x_4 = 15$$
$$x_1 + x_2 + x_4 = 9$$
$$x_i \geq 0 \quad i = 1, \cdots, 4$$

Question 2. Consider the following simplex tableaus of LP problems. First two are minimization problems while the last tableau belongs to a maximization problem.

x_1	x_2	e_1	e_2	x_3	x_4	RHS
-2	-3	0	0	-M	-M	0
2	1	-1	0	1	0	4
1	-1	0	-1	0	1	1

x_1	x_2	s_1	e_1	x_3	x_4	RHS
0	0	0	0	-1	-1	0
1	1	1	0	0	0	3
2	1	0	-1	1	0	4
1	1	0	0	0	1	3

x_1	x_2	x_3	s_1	s_2	e_1	RHS
-2	-1	-1	0	0	0	0
1/3	2/3	0	1	0	1/3	1
2	1	0	0	1	0	3
2/3	1/3	1	0	0	-1/3	1

Are these tableaus in proper format? If yes, indicate the entering and leaving variables. If not, explain why and show how can you convert them into proper format.

Question 3. Use the simplex algorithm to find two optimal solutions to the following LP:

$$\begin{aligned} \max\ & 5x_1 + 3x_2 + x_3 \\ \text{s.t.}\ & x_1 + x_2 + x_3 \leq 6 \\ & 5x_1 + 3x_2 + 6x_3 \leq 15 \\ & x_i \geq 0\ i = 1, 2, 3 \end{aligned}$$

Question 4. Consider the following problem:

$$\begin{aligned} \max\ & 2x_1 + x_2 + 6x_3 - 4x_4 \\ \text{s.t.}\ & x_1 + 2x_2 + 4x_3 - x_4 \leq 6 \\ & 2x_1 + 3x_2 - x_3 + x_4 \leq 12 \\ & 2x_1 + 3x_2 - x_3 + x_4 \leq 6 \\ & x_i \geq 0\ i = 1, 2, 3, 4 \end{aligned}$$

Convert this LP to standard form. Construct the simplex tableau for the basis: x1, x2 and x4. Indicate whether the tableau is optimal, if not state the entering and the leaving variables. You do not need to perform further iterations.

Question 5. Consider the following simplex tableau of a given **minimization** LP problem.

	z	x_1	x_2	x_3	s_1	s_2	s_3	*RHS*
z	1	0	0	**A**	-10	-1	**B**	32
s_3	0	0	0	**C**	4	-2	1	**D**
x_2	0	0	1	-2	**F**	-1	0	6
x_1	0	**E**	0	-1	1	2	0	4

Give **general** conditions on each of the unknowns **A-F** such that each of the following statements is true. Even if the statement holds independently of the values of a specific variable, you should still mention that variable.

(a) The tableau is final and there exists a unique optimal solution.

(b) The simplex method determines an unbounded solution from this tableau.

(c) The current bfs is degenerate (not necessarily optimal).

(d) The current solution is optimal, there are alternative optimal solutions but no alternative optimal bfs.

Question 6. Use the two-phase method to find the optimal solution to the following LP:

$$\begin{aligned} \max\ & -3x_1 + 4x_2 \\ \text{s.t.}\ & x_1 + x_2 \leq 4 \\ & 2x_1 + 3x_2 \geq 18 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Question 7. Convert the following LP to standard form and solve with the Big-M method.

$$\begin{aligned} \max\ & -x_1 + 3x_2 + x_3 \\ \text{s.t.}\ & -x_1 + x_2 - x_3 \geq -4 \\ & 2x_1 + x_2 + x_3 \geq 10 \\ & x_1 \geq 0 \\ & x_2 \leq 0 \\ & x_3 \text{ free} \end{aligned}$$

Question 8. Ford has four automobile plants. Each is capable of producing the Taurus, Lincoln or Escort but it can only produce one of these cars. Suppose that Ford should produce all car types. The fixed cost of operating each plant for a year and the variable cost of producing a car of each type at each plant are given in the table below:

	Fixed Cost	**Variable Cost**		
		Taurus	Lincoln	Escort
Plant 1	7 billion	12000	16000	9000
Plant 2	6 billion	15000	18000	11000
Plant 3	4 billion	17000	19000	12000
Plant 4	2 billion	19000	22000	14000

(a) Ford faces the following restrictions:

(1) Each plant can produce one type of car.

(2) The total production of each type of car must be at a single plant; that is, for example, if any Tauruses are made at plant 1, then all Tauruses must be made there.

(3) If plants 3 and 4 are used, then plant 1 must also be used.

Formulate the problem in order to minimize the total cost.

(b) Suppose now that instead of the restriction (3) in part (a), Ford faces the following restriction:

- Either at most one of the plants 1 and 2 or at least one of the plants 3 and 4 should be used.

Add the necessary constraint to your model.

Question 9. NASA has 4 astronauts to be employed at a space mission for 4 posts on a space ship. The posts require different qualifications (competencies) but each astronaut can perform each one of these tasks, albeit with varying degree of proficiency. The proficiency rating of astronaut i assigned to post j is measured as p_{ij}. It is asked to find the optimal assignment of astronauts to posts which gives the highest total proficiency rating while meeting the following requirements:

- Each astronaut should be assigned to a single post.
- Each post should have a single astronaut.

Question 10. A paper manufacturer produces rolls of standard fixed width w and of standard length l. Customers order rolls of width w but varying lengths. In particular, d_k rolls with length l_k and width w are ordered for customer $k = 1, \ldots, n$ (assume $l_k \leq l$ for all $k = 1, \ldots, n$). What is the minimum number of rolls that should be cut to meet the demand?
Hint: Let M be a large number such that M number of rolls are enough to satisfy all demand (e.g. $M = \sum_{k=1}^{n} d_k$ is such value) Assume the manufacturer has M number of unmanufactured rolls in the stock. Use decision variables:

$$x_i = \begin{cases} 1 & \text{if roll i is used} \\ 0 & \text{o.w} \end{cases}$$

$$y_{ik} = \text{number of rolls of type k (of length } l_k\text{) cut from roll i}$$
$$i = 1, \ldots, M, k = 1, \ldots, n$$