Q. 1. A hospital prepares a diet program that requires food from four groups. For now, the following groups are available for consumption: chocolate cake, ice cream, juice, and tart. Each chocolate cake costs 50¢, each scoop of ice cream costs 20¢, each bottle of juice costs 30¢, and each piece of tart costs 80¢. Each day, at least 500 calories, 6 oz of chocolate, 10 oz of sugar, and 8 oz of fat should be consumed. The nutritional content of each food (per unit) is shown in Table below. Continuous values are allowed for each consumption. Formulate a linear programming model to satisfy daily nutritional requirements with minimum cost.

Type of Food	Calories	Chocolate (Ounces)	Sugar (Ounces)	Fat(Ounces)
Chocolate cake	400	3	2	2
Ice cream (1 scoop)	200	2	2	4
Juice (1 bottle)	150	0	4	1
Tart (1 piece)	500	0	4	5

Q. 2. An automobile tire company has the ability to produce both nylon and fiberglass tires. The company has two presses, a Wheeling machine and a Regal machine and appropriate moulds that can be used to produce these tires. The production hours available in the next three months are given in the table below. Also provided below are the number of tires they have agreed to deliver during the next three months.

MONTH	**AVAILABLE HOURS**		**DELIVERY REQUIRED**	
	Wheeling	**Regal**	**Nylon**	**Fibreglass**
JUNE	700	1,500	4,000	1,000
JULY	300	400	8,000	5,000
AUGUST	1,000	300	3,000	5,000

Each machine can make the two types of tires independently. The production times for each machine-tire combination is as follows:

TYRE	**Wheeling hr./Tire**	**Regal hr/Tire**
NYLON	0.15	0.16
FIBREGLASS	0.12	0.14

The variable cost of producing tires are 330 TL per operating hour, regardless of which machine is used or which tire is produced. The inventory carrying costs are 3 TL per month per each unit of nylon and fiberglass tires and initial inventory is zero. Material costs for the nylon and fiberglass tyres are 160 TL and 200 TL respectively. Prices have been set at 350 TL per nylon tire and 450 TL per fiberglass tire. What should be the production schedule for the three months so as to maximize the profit? Develop an appropriate linear programming model that when solved will provide the optimal production schedule. Define the decision variables needed to develop your model precisely as well as state the meaning or purpose of each constraint.

Q. 3. Recall the Blending (1) problem that was formulated in class. In this problem, a refinery distills crude petroleum obtained from two sources, Saudi Arabia and Venezuela, into three main products: gasoline, jet fuel and lubricants. The two crudes differ in chemical composition and, therefore, yield different product mixes. Each ton of Saudi crude yields 0.3 ton of gasoline, 0.4 ton of jet fuel and 0.2 ton of lubricants. Each ton of Venezuelan crude yields 0.4 ton of gasoline, 0.2 ton of jet fuel and 0.3 ton of lubricants. The remaining 10% of each ton is lost in refining. The crudes also differ in cost and availability. The refinery can purchase up to 90 tons per day from Saudi Arabia at $600 per ton. Up to 60 tons per day of Venezuelan crude is available at $550 per ton. The refinery's contracts with independent distributors require that it produce at least 20 tons per day of gasoline, 15 tons per day of jet fuel, and 5 tons per day of lubricants. How can these requirements be fulfilled most efficiently? The refinery's decision problem is formulated as a linear programming problem as follows: Let, x_1 and x_2 denote the tons of Saudi and Venezuelan crude refined per day.

The model:

$$
\begin{aligned}
\text{min } & 600x_1 + 550x_2 \\
\text{s.t. } & 0.3x_1 + 0.4x_2 \geq 20 \\
& 0.4x_1 + 0.2x_2 \geq 15 \\
& 0.2x_1 + 0.3x_2 \geq 5 \\
& x_1 \leq 90 \\
& x_2 \leq 60 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

(a) Use the graphical method and show all the feasible solutions to the refinery's decision problem. Mark all the extreme points clearly.

(b) Identify the optimal solution, i.e., the optimal value of x_1 and x_2. What is the optimal cost?

(c) What is the minimum amount by which the cost of Venezuela crude should decrease for the optimal solution in b) to move to a new extreme point? Show all your calculations.

(d) What is the minimum amount by which the daily requirement of lubricant (which is currently 5 tons) must increase to for the optimal solution to change? Show your calculations.

Q. 4. A Company produces two products (products 1 and 2) that require both metal frame parts and electrical components and wants to determine how many units of each product to produce so as to maximize profit. For each unit of product 1, 1 unit of frame parts and 2 units of electrical components are required. For each unit of product 2, 3 units of frame parts and 2 units of electrical components are required. The company has 200 units of frame parts and 300 units of electrical components. Each unit of product 1 gives a profit of $1, and each unit of product 2, up to 60 units, gives a profit of $2. Any excess over 60 units of product 2 brings no profit, so such an excess should not be allowed.

(a) Formulate a linear programming model for this problem.

(b) Use the graphical method to solve this model. Show feasible region and mark all the extreme points of the feasible region in the graph. What is the optimum profit?