IE 400: Principles of Engineering Management

SET 7- SET 8 Solutions to In-Class Exercises

Bilkent University

November 29, 2020

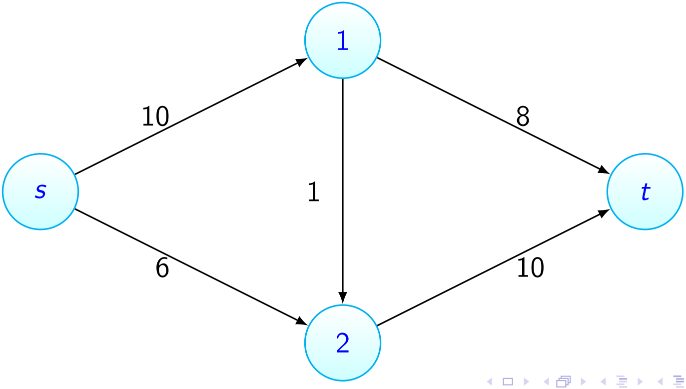
Maximum Flow: (Set 7-Slide 43)

G = (N,A)

u_{ij} = capacity of flow in arc (i,j)

s = source node

t = sink node



Maximum Flow: (Set 7-Slide 43)

Decision Variables:

x_{ij} : flow on arc (i,j)

v : net flow out of s (into t)

Model:

$$
\begin{aligned}
\max\ & v & & \\
\text{s.t. } & x_{s,1} \leq 10 & & (1)\\
& x_{s,2} \leq 6 & & (2)\\
& x_{1,2} \leq 1 & & (3)\\
& x_{1,t} \leq 8 & & (4)\\
& x_{2,t} \leq 10 & & (5)\\
& v = x_{s,1} + x_{s,2} & & (6)\\
& x_{s,1} = x_{1,2} + x_{1,t} & & (7)\\
& x_{1,2} + x_{s,2} = x_{2,t} & & (8)\\
& v = x_{1,t} + x_{2,t} & & (9)\\
& x_{i,j} \geq 0 & (i,j) \in A \quad & (10)
\end{aligned}
$$

Maximum Flow: (Set 7-Slide 43)

(1)-(2)-(3)-(4)-(5): arc capacity constraints

(6): Node s flow constraint

(7): Node 1 flow constraint

(8): Node 2 flow constraint

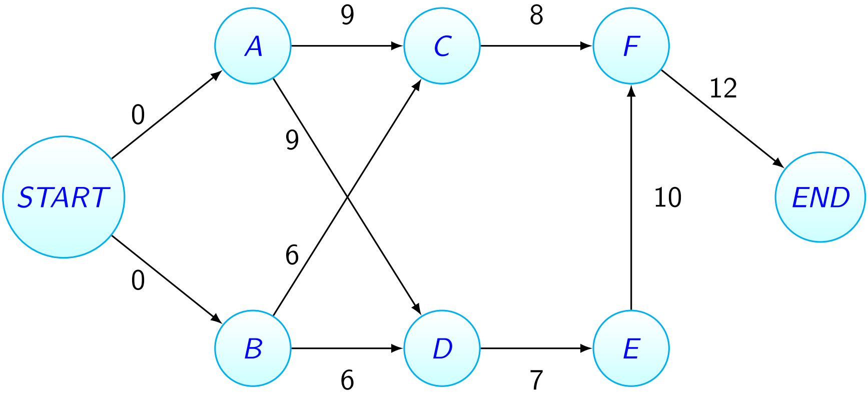
(9): Node t flow constraint

CPM Example (Set 8- Slide 38)

Activity	Descriptiom	Duration	Immediate Predecessor
A	Purchase Raw Material	9	-
B	Train workers	6	-
C	Produce part 1	8	A,B
D	Produce part 2	7	A,B
E	Test part 2	10	D
F	Assemble parts 1 and 2	12	C,E

CPM Example (Set 8- Slide 38)

AON Network



CPM Example (Set 8- Slide 38)

Activity	Earliest Start (ES)	Earliest Finish (EF)	Latest Start (LS)	Latest Finish (LF)	Slack (LS-ES)	On Critical Path
A	0	9	0	9	0	Yes
B	0	6	3	9	3	No
C	9	17	18	26	9	No
D	9	16	9	16	0	Yes
E	16	26	16	26	0	Yes
F	26	38	26	38	0	Yes

Critical path is **A-D-E-F**

LP to find the project completion time (Set 8- Slide 53)

G=(N,A): AON project network

Parameters:

d_i : Duration of activity i, $i \in N$

Decision Variables:

x_i : Starting time of activity i , $i \in N$

$$\begin{aligned} \min\ & x_{end} & \\ \text{s.t}\ & x_j \geq x_i + d_i & (i,j) \in A \\ & x_{start} = 0 & \\ & x_i \geq 0 & i \in N \end{aligned}$$

Crashing a Project (Set 8-Slide 54)

Parameters:

c_i : cost of reducing the duration of activity i by one unit, $i \in N$

u_i : maximum possible reduction in the duration of activity i, $i \in N$

d_i : Duration of activity i, $i \in N$

D : Due date

Decision Variables:

x_i : Starting time of activity i , $i \in N$

y_i: Number of days that the duration of activity i is reduced, $i \in N$

Crashing a Project (Set 8-Slide 54)

Model:

$$
\begin{aligned}
\min \ & \sum_{i \in N} c_i y_i \\
\text{s.t.} \ & y_i \leq u_i & i \in N \\
& x_j \geq x_i + d_i - y_i & (i,j) \in A \\
& x_{start} = 0 \\
& x_{end} \leq D \\
& x_i \geq 0 & i \in N \\
& y_i \geq 0 & i \in N \\
& y_i \text{ integer} & i \in N
\end{aligned}
$$

Least possible project duration with a budget of K units (Set 8-Slide 58)

Parameters:

c_i : cost of reducing the duration of activity i by one unit, $i \in N$

u_i : maximum possible reduction in the duration of activity i, $i \in N$

d_i : Duration of activity i, $i \in N$

K : Total budget

Decision Variables:

x_i : Starting time of activity i, $i \in N$

y_i: Number of days that the duration of activity i is reduced, $i \in N$

Least possible project duration with a budget of K units (Set 8-Slide 58)

Model:

$$
\begin{aligned}
\min\ & x_{end} & \\
\text{s.t. } & \sum_{i \in N} c_i y_i \leq K & \\
& y_i \leq u_i & i \in N \\
& x_j \geq x_i + d_i - y_i & (i,j) \in A \\
& x_{start} = 0 & \\
& x_i \geq 0 & i \in N \\
& y_i \geq 0 & i \in N \\
& y_i \text{ integer} & i \in N
\end{aligned}
$$