Q. 1. UPS must make deliveries to 20 customers whose respective demands are given with d_j for j = 1,...,20. Each customer can be visited by more than one truck to satisfy the demands. The company has seven trucks available with capacities C_k and daily operating costs q_k for k = 1,...,7.

(a) Formulate the model to determine which trucks to use so as to minimize the cost of delivering to all the customers.

(b) Add the following requirements to your model:

- A single truck cannot deliver to more than six customers.
- Customer 1 and customer 7 cannot be visited by the same truck.
- Both customer 2 and customer 12 should be visited by the same truck(s).
- If a truck visits customer 4 then it has to visit customer 19 or customer 20. (Assume demands of these customers do not exceed truck capacities.)
- Either at least two of the customers 5, 6 and 7 or at most three of customers 11, 12, 13 and 14 should be visited by truck 4.

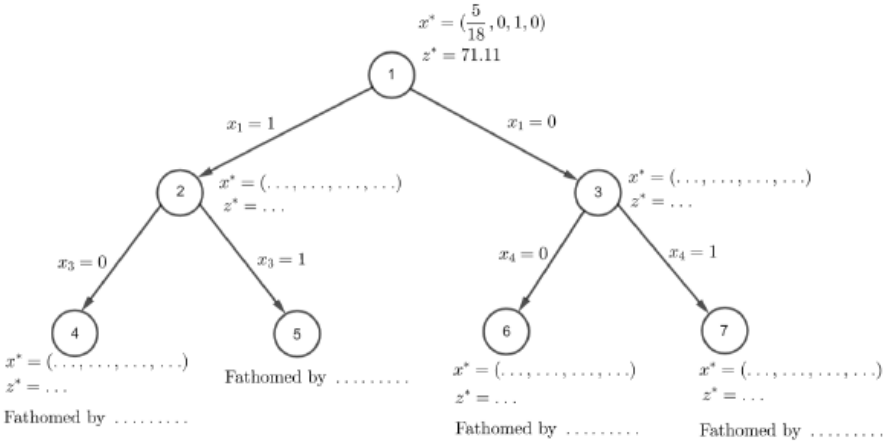
Q. 2. Consider the following IP:

$$\begin{aligned} \max\ & 40x_1 + 5x_2 + 60x_3 + 8x_4 \\ \text{s.t. } & 18x_1 + 3x_2 + 20x_3 + 5x_4 \leq 25 \\ & x_1, x_2, x_3, x_4 \in \{0, 1\} \end{aligned}$$

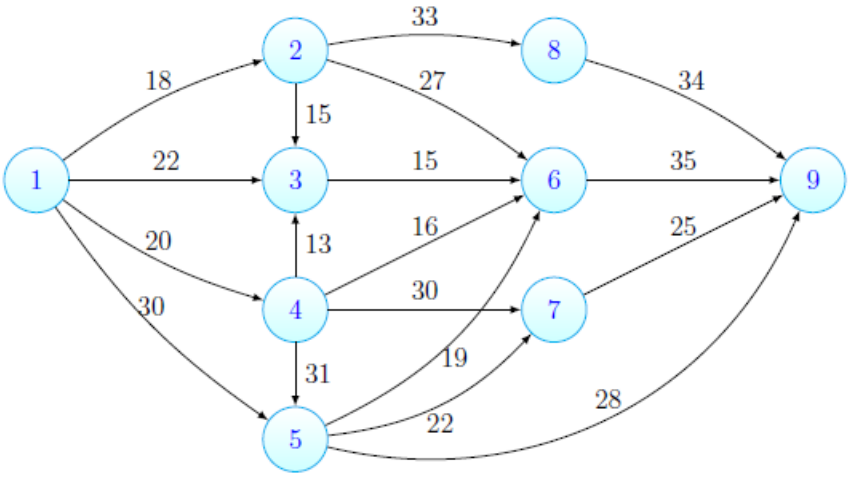
Complete the **Branch and Bound tree** below for the given IP. (Hint: be careful about order of the nodes. You should follow the guidance in the given branch and bound tree. Do not make complete enumeration!) Find the optimal solution.

Order of the variables:

.... > .... > .... > ....



Q. 3. Find shortest paths from 1 to all other nodes using Dijkstra's algorithm.



Q. 4. For the following statements, indicate whether they are True or False. Either provide a counter example or explain your answer.

(a) Among all shortest paths in a network, Dijkstra's Algorithm always finds a shortest path with the least number of arcs.

(b) Consider a network where the shortest path distance from the source node to end node is c units. If each arc length is multiplied by a positive constant k, shortest path distance becomes ck units.