IE 400: Principles of Engineering Management
Study Set 4 Solutions

Fall 2020

Question 1.

First calculate the ratios:

x_1 : 9/6

x_2 : 13/3

x_3 : 10/2

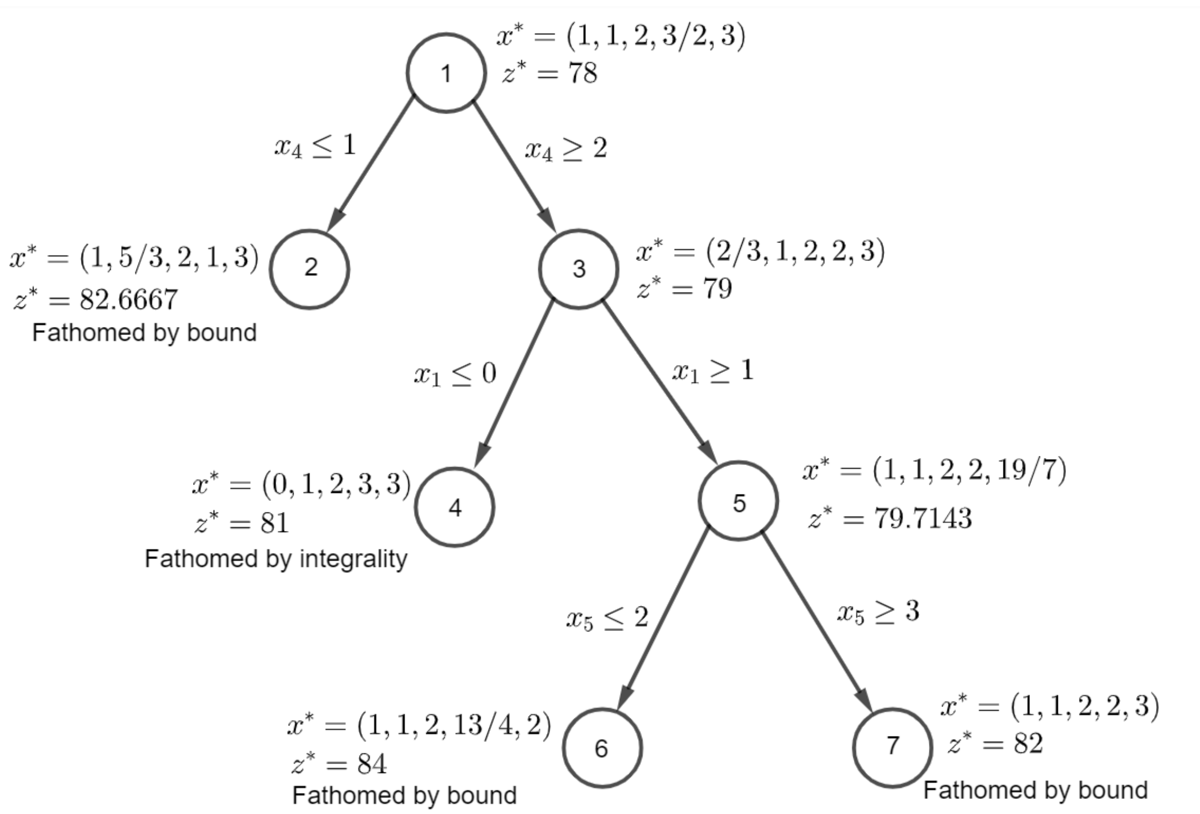
x_4 : 8/4

x_5 : 8/7

For the **minimization problem**, sort the ratios from smallest to largest:

$$x_5 < x_1 < x_4 < x_2 < x_3$$

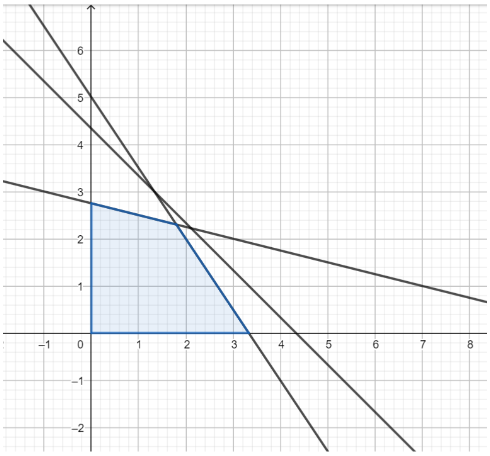
Solve the LP relaxation of the knapsack model according to the above priority list (x_5 is the most preferable one) . The solution to the LP relaxation of the given model is displayed at node 1. Since this is not an integer solution, we branch from node 1, and add new constraints as given in the Branch and Bound tree below. Note that at each node we solve the LP relaxation of the IP model along with the newly added constraints.
Note that node 2 is fathomed by bound, because we have found an integer solution at node 4 that has a better objective value than the one at node 2. (If we branch node 2, then the resulting nodes would have an objective value ≥ 82.6667 which will always result in a solution worse than node 4)



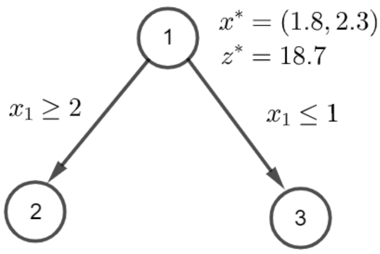
The optimal solution is $x^* = (0, 1, 2, 3, 3)$.

Question 2.

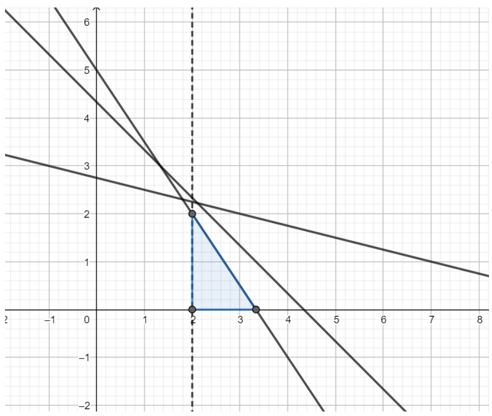
Solve the LP relaxation of the original model using graphical method:



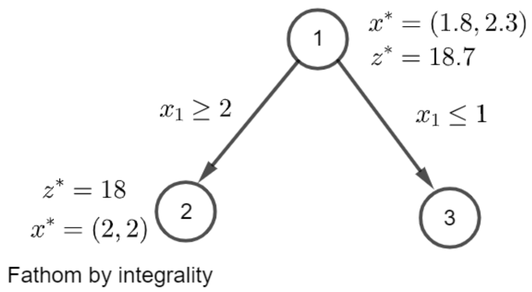
For c=(4,5), we obtain $x^* = (1.8, 2.3)$. This is not an integer solution, so start branching:



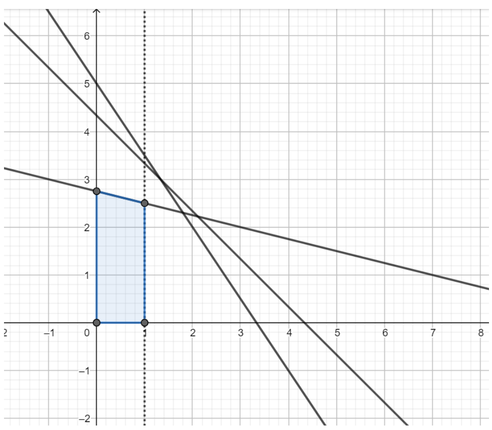
Now move to node 2. Here the constraint $x_1 \geq 2$ is added to the model:



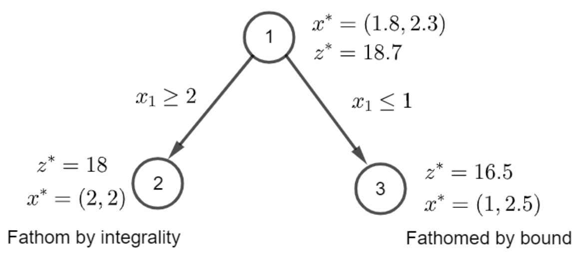
Solve the new model with graphical method and update the branch and bound tree:



We obtained an integer solution at node 2. Hence, it is fathomed by integrality. Now move to the third node. Add the constraint $x_1 \leq 1$ to the original model:



Solving via the graphical method we obtain:



Node 3 is fathomed by bound, because we have already obtained an integer solution at node 2 that has a better objective value (for maximization) than the model at node 3 . If we branch the third node, the objective value will be ≤ 16.5 !

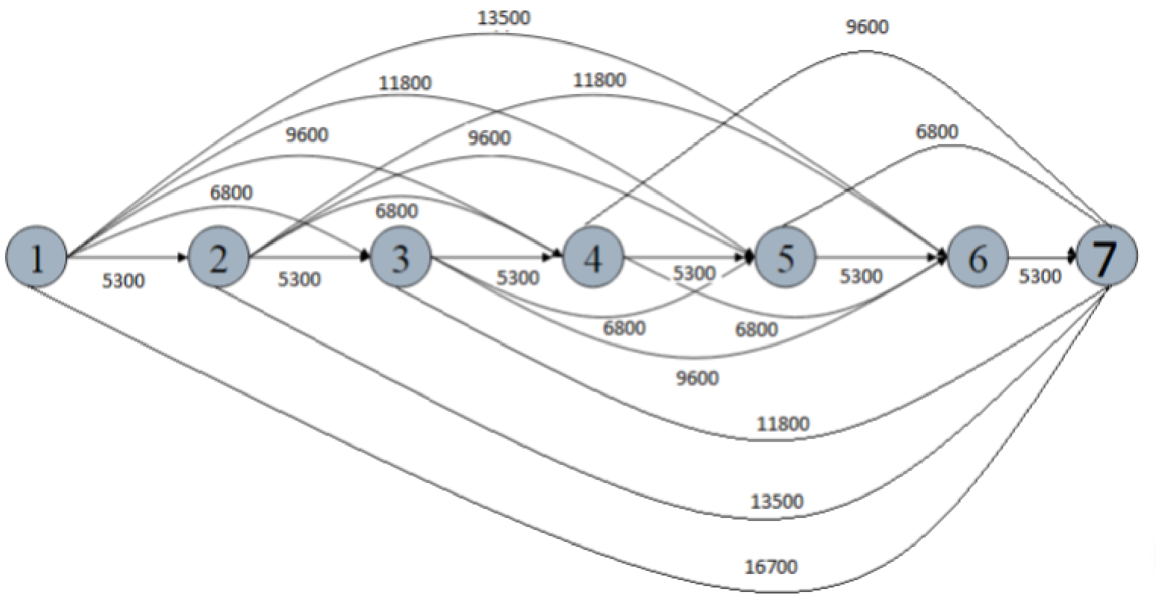
We cannot make further branches. Then, the optimal solution is $x^* = (2, 2)$ and $z^* = 18$.

Question 3.

Let c_{ij} be the total net cost incurred from year i to j.

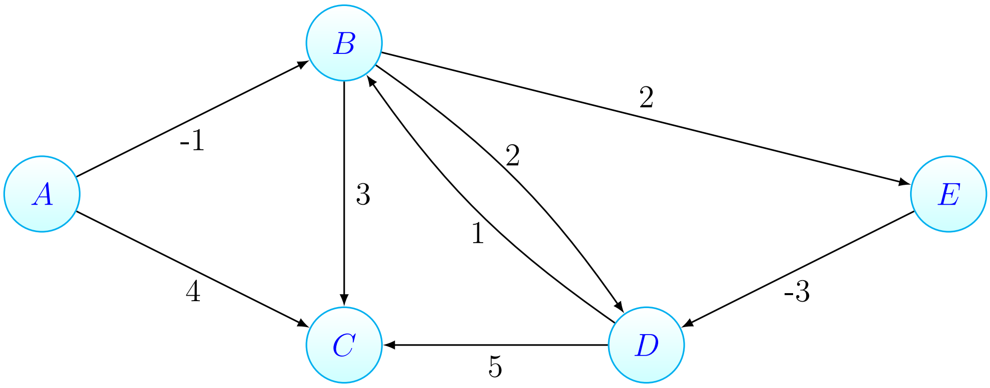
c_{ij}= maintenance cost incurred during years$i, i+1, \cdots, j-1$ +cost of purchasing a car at the beginning of year i- trade-in value received at the beginning of year j.

$c_{12} = 300 + 12000 - 7000 = 5300$
$c_{13} = 300 + 500 + 12000 - 6000 = 6800$
$c_{14} = 300 + 500 + 800 + 12000 - 4000 = 9600$
$c_{15} = 300 + 500 + 800 + 1200 + 12000 - 3000 = 11800$
$c_{16} = 300 + 500 + 800 + 1200 + 1600 + 12000 - 2000 = 13500$
$c_{17} = 300 + 500 + 800 + 1200 + 1600 + 2200 + 12000 - 1000 = 16700$
$c_{23} = 300 + 12000 - 7000 = 5300$
$c_{24} = 300 + 500 + 12000 - 6000 = 6800$
$c_{25} = 300 + 500 + 800 + 12000 - 4000 = 9600$
$c_{26} = 300 + 500 + 800 + 1200 + 12000 - 3000 = 11800$
$c_{27} = 300 + 500 + 800 + 1200 + 1600 + 12000 - 2000 = 13500$
$c_{34} = 300 + 12000 - 7000 = 5300$
$c_{35} = 300 + 500 + 12000 - 6000 = 6800$
$c_{36} = 300 + 500 + 800 + 12000 - 4000 = 9600$
$c_{37} = 300 + 500 + 800 + 1200^{+}12000 - 3000 = 11800$
$c_{45} = 300 + 12000 - 7000 = 5300$
$c_{46} = 300 + 500 + 12000 - 6000 = 6800$
$c_{47} = 300 + 500 + 800 + 12000 - 4000 = 9600$
$c_{56} = 300 + 12000 - 7000 = 5300$
$c_{57} = 300 + 500 + 12000 - 6000 = 6800$
$c_{67} = 300 + 12000 - 7000 = 5300$



Replacing the car at years 2,4 and 6 yields the least costly solution.

Question 4. Use Bellman Ford algorithm to find the shortest path of the following graph where A is the source node.

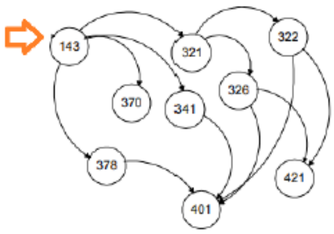


$$V_{[k]}^{(t)} = \min\{V_{[i]}^{t-1} + c_{ik}\}$$

t	$V_{(A)}^t$	$V_{(B)}^t$	$V_{(C)}^t$	$V_{(D)}^t$	$V_{(E)}^t$	d(A)	d(B)	d(C)	d(D)	d(E)
0	0	$+\infty$	$+\infty$	$+\infty$	$+\infty$					
1	0	-1	4	$+\infty$	$+\infty$		A	A		
2	0	-1	2	1	1			B	B	B
3	0	-1	2	-2	1				E	

Question 5. Pick the node without any incoming arcs:

Randomly select another node without any incoming arcs and repeat:



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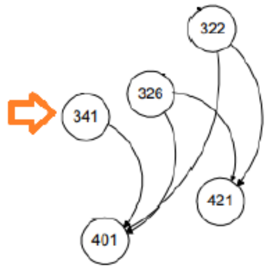

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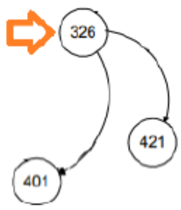

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Question 6. For the following statements, indicate whether they are True or False. Either provide a counter example or explain your answer.

(a) False. Consider nodes A,B,C, cost of edge (A,B) is 3, edge (A,C) is 1 and (B,C) is 2. There are 2 shortest paths from A to B both of length 3.

(b) True. Both algorithms are guaranteed to produce the same shortest path weight, but if there are multiple shortest paths, Dijkstra's will choose the shortest path according to the greedy strategy, and Bellman-Ford will choose the shortest path depending on the order of relaxations, and the two shortest path trees may be different.

(c) False. Undirected graph on nodes A,B,C. Edges (A,B) and (B,C) have cost 1 and edge (A,C) has cost 3. Length of shortest A to C path is 2. If each edge cost is increased by k = 10 the shortest path length becomes 13. But $13 - 2$ is not a multiple of 10.