

IE 400: Principles of Engineering Management

Study Set 2

Spring 2021

Question 1. Identify the basic solutions, basic feasible solutions and degenerate basic feasible solutions (if exists) in the following linear system.

$$\begin{aligned}x_1 - x_2 + 3x_3 + 5x_4 &= 15 \\x_1 + x_2 + x_4 &= 9 \\x_i &\geq 0 \quad i = 1, \dots, 4\end{aligned}$$

Question 2. Use the simplex algorithm to find two optimal solutions to the following LP:

$$\begin{aligned}\max \quad & 5x_1 + 3x_2 + x_3 \\ \text{s.t.} \quad & x_1 + x_2 + 3x_3 \leq 6 \\ & 5x_1 + 3x_2 + 6x_3 \leq 15 \\ & x_i \geq 0 \quad i = 1, 2, 3\end{aligned}$$

Try to solve the same question with x_3 as urs

Question 3. Corrected version

Consider the following problem:

$$\begin{aligned}\max \quad & 2x_1 + x_2 + 6x_3 - 4x_4 \\ \text{s.t.} \quad & x_1 + 2x_2 + 4x_3 - x_4 \leq 6 \\ & 2x_1 + 3x_2 - x_3 + x_4 \leq 12 \\ & x_1 + x_3 + x_4 \leq 6 \\ & x_i \geq 0 \quad i = 1, 2, 3, 4\end{aligned}$$

Convert this LP to standard form. Construct the simplex tableau for the basis: x_1 , x_2 and x_4 . Indicate whether the tableau is optimal, if not state the entering and the leaving variables. You do not need to perform further iterations.

Question 4. Consider the following LP:

$$\begin{aligned} \max \quad & 10x_1 + 8x_2 - 3x_3 + 3x_4 \\ \text{s.t.} \quad & 2x_1 + 4x_2 - 0.5x_3 + 0.5x_4 \leq 3 \\ & -2x_1 + 6x_2 - 4.5x_3 + 4.5x_4 \leq 7 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

Convert the LP in standard form. Then construct the tableau where x_1 is the basic variable in the first constraint and x_4 is the basic variable in the second constraint (order is important). Then, answer the following

- (a) Is this a basic solution? Why?
- (b) Is this a basic feasible solution? Why?
- (c) Is this an optimal basic feasible solution? Why?

Question 5. Reatix must determine how much investment and debt to undertake during the next year. Each TL invested reduces the net present value (NPV) of the company by 10kr, and each TL of debt increases the NPV by 50kr (due to deductibility of interest payments). Reatix can invest at most 1 million TL during the coming year. Debt can be at most 40% of the investment. Reatix now has 800,000 TL in cash available. All investment must be paid from current cash or borrowed money. Set up an LP that maximizes the NPV of Reatix. Then, solve this LP by **simplex method**.

Question 6. A Company produces two products (products 1 and 2) that require both metal frame parts and electrical components and wants to determine how many units of each product to produce so as to maximize profit. For each unit of product 1, 1 unit of frame parts and 2 units of electrical components are required. For each unit of product 2, 3 units of frame parts and 2 units of electrical components are required. The company has 200 units of frame parts and 300 units of electrical components. Each unit of product 1 gives a profit of 1 dollar, and each unit of product 2, up to 60 units, gives a profit of 2 dollars. Any excess over 60 units of product 2 brings no profit, so such an excess should not be allowed.

- (a) Formulate a linear programming model for this problem.
- (b) Use the graphical method to solve this model. Show feasible region and mark all the extreme points of the feasible region in the graph. What is the optimum profit?

Question 7. Consider the following simplex tableaus of LP problems. They belong to maximization problems.

x_1	x_2	s_1	e_1	x_3	x_4	RHS
0	0	0	0	-1	-1	0
1	1	1	0	0	0	3
2	1	0	-1	1	0	4
1	1	0	0	0	1	3

x_1	x_2	x_3	s_1	s_2	e_1	RHS
-2	-1	-1	0	0	0	0
1/3	2/3	0	1	0	1/3	1
2	1	0	0	1	0	3
2/3	1/3	1	0	0	-1/3	1

Are these tableaus in proper format? If yes, indicate the entering and leaving variables. If not, explain why and show how can you convert them into proper format.