IE 400: Principles of Engineering Management
Study Set 5 Solutions

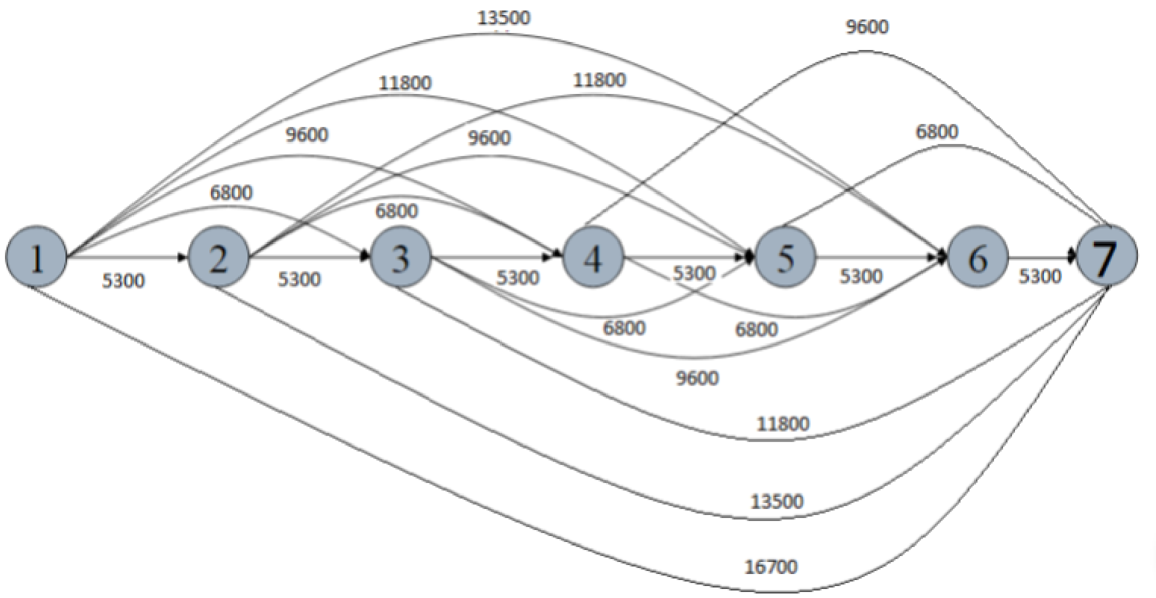
Spring 2021

Question 1.

Let c_{ij} be the total net cost incurred from year i to j.

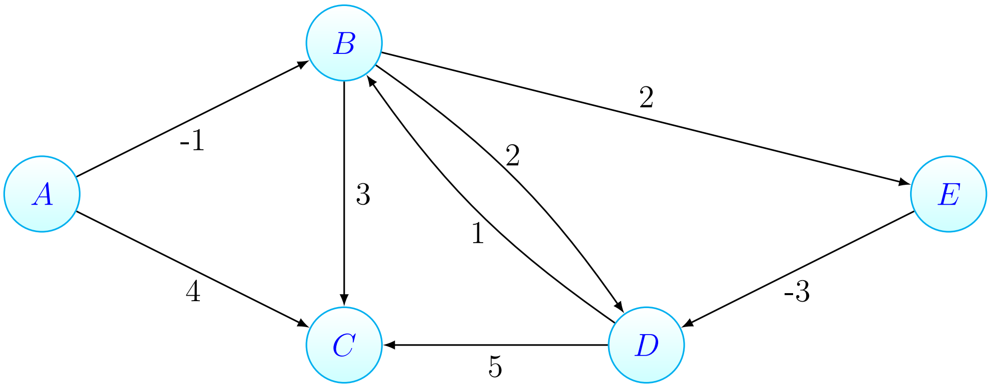
c_{ij}= maintenance cost incurred during years$i, i+1, \cdots, j-1$ +cost of purchasing a car at the beginning of year i- trade-in value received at the beginning of year j.

$c_{12} = 300 + 12000 - 7000 = 5300$
$c_{13} = 300 + 500 + 12000 - 6000 = 6800$
$c_{14} = 300 + 500 + 800 + 12000 - 4000 = 9600$
$c_{15} = 300 + 500 + 800 + 1200 + 12000 - 3000 = 11800$
$c_{16} = 300 + 500 + 800 + 1200 + 1600 + 12000 - 2000 = 13500$
$c_{17} = 300 + 500 + 800 + 1200 + 1600 + 2200 + 12000 - 1000 = 16700$
$c_{23} = 300 + 12000 - 7000 = 5300$
$c_{24} = 300 + 500 + 12000 - 6000 = 6800$
$c_{25} = 300 + 500 + 800 + 12000 - 4000 = 9600$
$c_{26} = 300 + 500 + 800 + 1200 + 12000 - 3000 = 11800$
$c_{27} = 300 + 500 + 800 + 1200 + 1600 + 12000 - 2000 = 13500$
$c_{34} = 300 + 12000 - 7000 = 5300$
$c_{35} = 300 + 500 + 12000 - 6000 = 6800$
$c_{36} = 300 + 500 + 800 + 12000 - 4000 = 9600$
$c_{37} = 300 + 500 + 800 + 1200^{+}12000 - 3000 = 11800$
$c_{45} = 300 + 12000 - 7000 = 5300$
$c_{46} = 300 + 500 + 12000 - 6000 = 6800$
$c_{47} = 300 + 500 + 800 + 12000 - 4000 = 9600$
$c_{56} = 300 + 12000 - 7000 = 5300$
$c_{57} = 300 + 500 + 12000 - 6000 = 6800$
$c_{67} = 300 + 12000 - 7000 = 5300$



Replacing the car at years 2,4 and 6 yields the least costly solution.

Question 2. Use Bellman Ford algorithm to find the shortest path of the following graph where A is the source node.

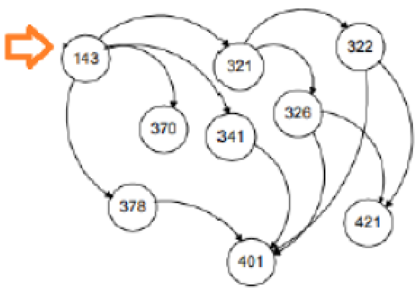


$$V_{[k]}^{(t)} = \min\{V_{[i]}^{t-1} + c_{ik}\}$$

t	$V_{(A)}^t$	$V_{(B)}^t$	$V_{(C)}^t$	$V_{(D)}^t$	$V_{(E)}^t$	d(A)	d(B)	d(C)	d(D)	d(E)
0	0	$+\infty$	$+\infty$	$+\infty$	$+\infty$					
1	0	-1	4	$+\infty$	$+\infty$		A	A		
2	0	-1	2	1	1			B	B	B
3	0	-1	2	-2	1				E	

Question 3. Pick the node without any incoming arcs:

Randomly select another node without any incoming arcs and repeat:



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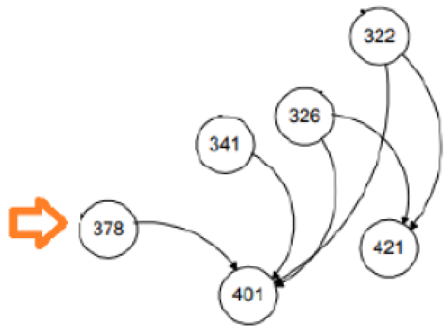

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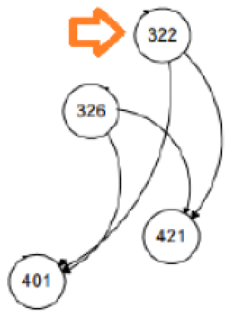

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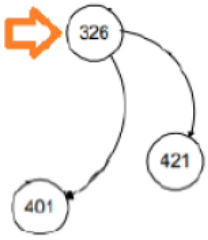

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Question 4. For the following statements, indicate whether they are True or False. Either provide a counter example or explain your answer.

(a) False. Consider nodes A,B,C, cost of edge (A,B) is 3, edge (A,C) is 1 and (B,C) is 2. There are 2 shortest paths from A to B both of length 3.

(b) True. Both algorithms are guaranteed to produce the same shortest path weight, but if there are multiple shortest paths, Dijkstra's will choose the shortest path according to the greedy strategy, and Bellman-Ford will choose the shortest path depending on the order of relaxations, and the two shortest path trees may be different.

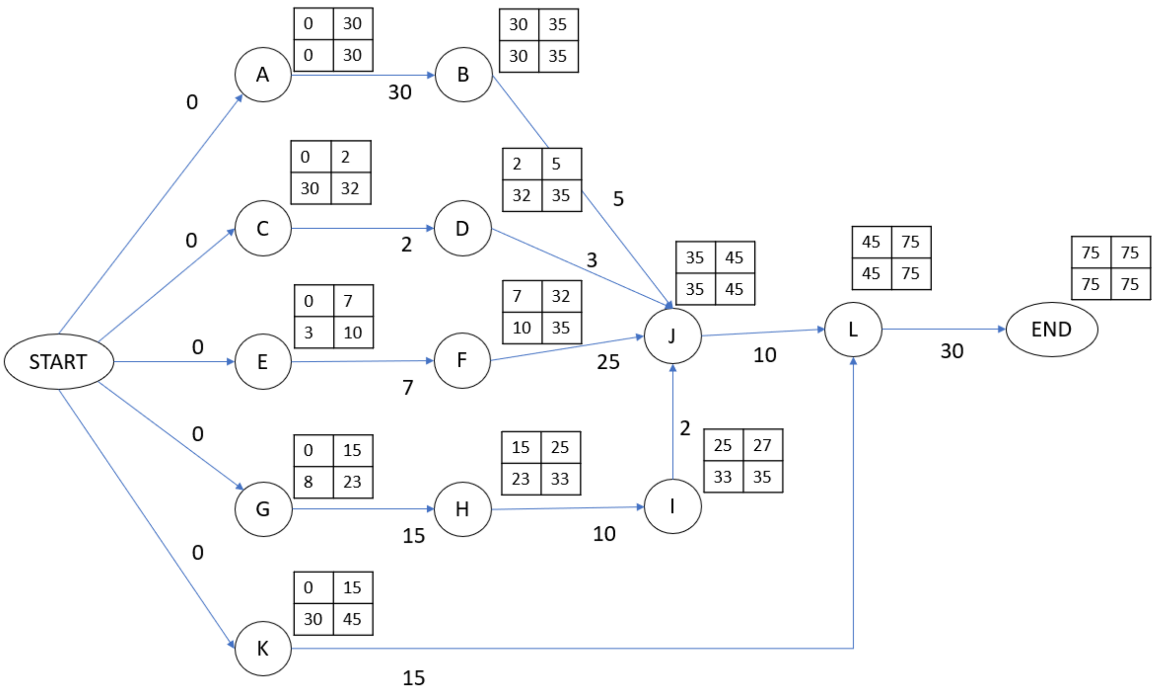
(c) False. Undirected graph on nodes A,B,C. Edges (A,B) and (B,C) have cost 1 and edge (A,C) has cost 3. Length of shortest A to C path is 2. If each edge cost is increased by k = 10 the shortest path length becomes 13. But 13 - 2 is not a multiple of 10.

Question 5.

ES: Earliest Start, EF: Earliest Finish, LS: Latest Start, LF: Latest Finish

ES	EF
LS	LF

(a)-(b)



Activity	Slack
A	0
B	0
C	30
D	30
E	3
F	3
G	8
H	8
I	8
J	0
K	30
L	0

(c) Since the slack for cutting the onions and mushrooms is 3, the dinner will be delayed 6-3=3 minutes.
If the cutting time is 2 minutes, slack is 10 (latest finish time) – 2 (task duration) = 8 minutes. Since the slack time is more than the delay time, it does not delay the dinner.

(d) Let N be the set of activities and A be the set of arc in the project network.

Parameters:
a_i : Duration of activity i, for all i $\in N$
d : Project duration (75)

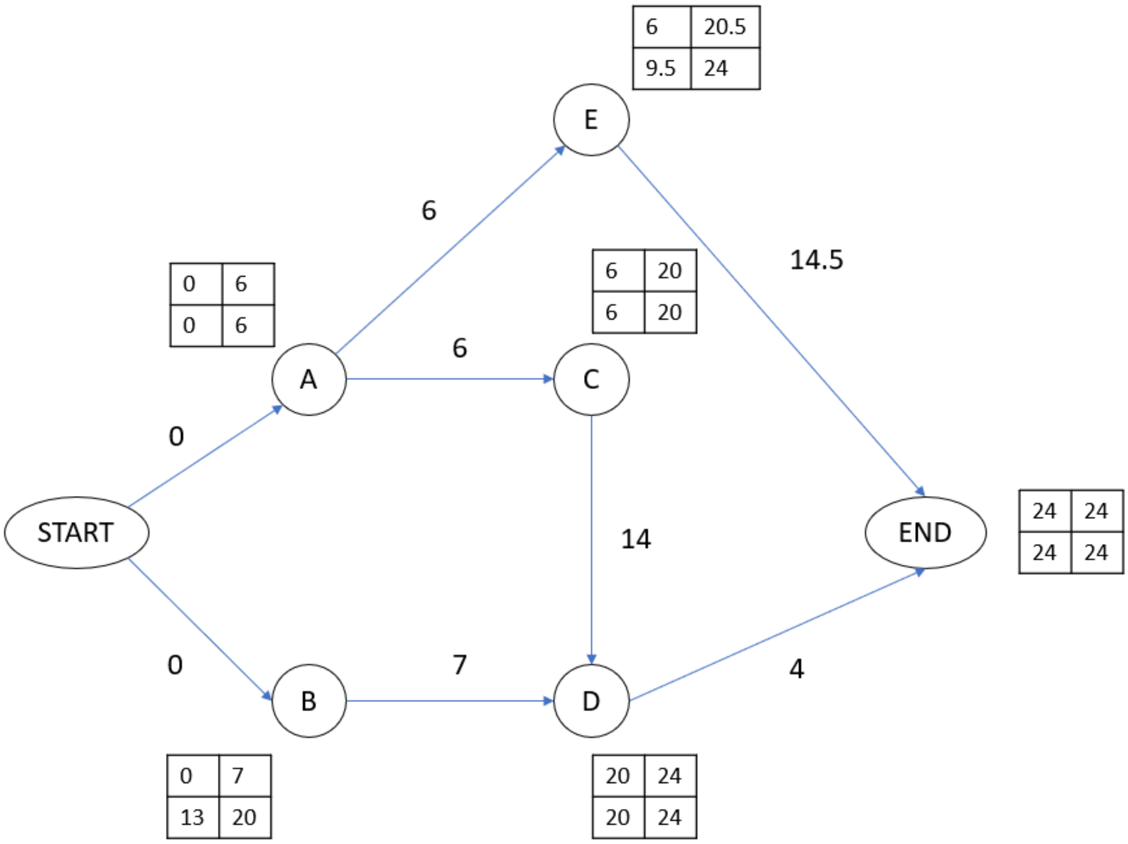
Decision Variables:
x_i : Start time of activity i, for all $i \in N$

Model:

$$
\begin{aligned}
\max \sum_{i \in N} x_i & \\
\text{s.t. } x_j \geq x_i + a_i & \qquad (i,j) \in A \\
x_{end} \leq d & \\
x_i \geq 0 & \qquad i \in N \\
x_{start} = 0 &
\end{aligned}
$$

Question 6. (a)

Activity	Mean	Variance
A	6	4/36
B	7	196/36
C	14	256/36
D	4	4/36
E	14.5	169/36



The critical path is A-C-D.

(b) Note that

$$\mathbb{P}(18 \leq t \leq 26) = \mathbb{P}(t \leq 26) - \mathbb{P}(t \leq 18)$$

We have, $\sigma_p = 2.708$

Note that $Z_{18} = \frac{18-24}{2.708} = -2.2156$

Then, $\mathbb{P}(t \leq 18) = \mathbb{P}(Z \leq -2.2156) = 0.013$

$Z_{26} = \frac{26-24}{2.708} = 0.739$

$\mathbb{P}(t \leq 26) = \mathbb{P}(Z \leq 0.739) = 0.770$

Hence,

$$\mathbb{P}(18 \leq t \leq 26) = \mathbb{P}(t \leq 26) - \mathbb{P}(t \leq 18) = 0.757$$