# IE 400: Principles of Engineering Management
# Study Set 2

Summer 2021

**Question 1.** Identify the basic solutions, basic feasible solutions and degenerate basic feasible solutions (if exists) in the following linear system.

$$
\begin{aligned}
&x_1 - x_2 + 3x_3 + 5x_4 = 15 \\
&x_1 + x_2 + x_4 = 9 \\
&x_i \geq 0 \quad i = 1, \cdots, 4
\end{aligned}
$$

**Question 2.** Use the simplex algorithm to find two optimal solutions to the following LP:

$$
\begin{aligned}
\max\ & 5x_1 + 3x_2 + x_3 \\
\text{s.t.}\ & x_1 + x_2 + 3x_3 \leq 6 \\
& 5x_1 + 3x_2 + 6x_3 \leq 15 \\
& x_i \geq 0\ i = 1, 2, 3
\end{aligned}
$$

Try to solve the same question with x3 as urs.

**Question 3.** Consider the following problem:

$$
\begin{aligned}
\max\ & 2x_1 + x_2 + 6x_3 - 4x_4 \\
\text{s.t.}\ & x_1 + 2x_2 + 4x_3 - x_4 \leq 6 \\
& 2x_1 + 3x_2 - x_3 + x_4 \leq 12 \\
& x_1 + x_3 + x_4 \leq 6 \\
& x_i \geq 0\ i = 1, 2, 3, 4
\end{aligned}
$$

Convert this LP to standard form. Construct the simplex tableau for the basis: $x_1$, $x_2$ and $x_4$. Indicate whether the tableau is optimal, if not state the entering and the leaving variables. You do not need to perform further iterations.

**Question 4.** Consider the following LP:

$$\begin{aligned} \max\ & 10x_1 + 8x_2 - 3x_3 + 3x_4 \\ \text{s.t. } & 2x_1 + 4x_2 - 0.5x_3 + 0.5x_4 \leq 3 \\ & -2x_1 + 6x_2 - 4.5x_3 + 4.5x_4 \leq 7 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

Convert the LP in standard form. Then construct the tableau where $x_1$ is the basic variable in the fi

rst constraint and$x_4$ is the basic variable in the second constraint (order is important). Then, answer the following

(a) Is this a basic solution? Why?

(b) Is this a basic feasible solution? Why?

(c) Is this an optimal basic feasible solution? Why?

**Question 5.** Reatix must determine how much investment and debt to undertake during the next year. Each TL invested reduces the net present value (NPV) of the company by 10kr, and each TL of debt increases the NPV by 50kr (due to deductibility of interest payments). Reatix can invest at most 1 million TL during the coming year. Debt can be at most 40% of the investment. Reatix now has 800,000 TL in cash available. All investment must be paid from current cash or borrowed money. Set up an LP that maximizes the NPV of Reatix. Then, solve this LP by **simplex method**.

**Question 6.** Consider the following simplex tableaus of LP problems. First two are minimization problems while the last tableau belongs to a maximization problem.

| $x_1$ | $x_2$ | $e_1$ | $e_2$ | $x_3$ | $x_4$ | RHS |
|---|---|---|---|---|---|---|
| -2 | -3 | 0 | 0 | -M | -M | 0 |
| 2 | 1 | -1 | 0 | 1 | 0 | 4 |
| 1 | -1 | 0 | -1 | 0 | 1 | 1 |

| | | | | | | |
|---|---|---|---|---|---|---|
| $x_1$ | $x_2$ | $s_1$ | $e_1$ | $x_3$ | $x_4$ | RHS |
| 0 | 0 | 0 | 0 | -1 | -1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 3 |
| 2 | 1 | 0 | -1 | 1 | 0 | 4 |
| 1 | 1 | 0 | 0 | 0 | 1 | 3 |

| | | | | | | |
|---|---|---|---|---|---|---|
| $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $e_1$ | RHS |
| -2 | -1 | -1 | 0 | 0 | 0 | 0 |
| 1/3 | 2/3 | 0 | 1 | 0 | 1/3 | 1 |
| 2 | 1 | 0 | 0 | 1 | 0 | 3 |
| 2/3 | 1/3 | 1 | 0 | 0 | -1/3 | 1 |

Are these tableaus in proper format? If yes, indicate the entering and leaving variables. If not, explain why and show how can you convert them into proper format.

**Question 7.** Consider the following simplex tableau of a given **minimization** LP problem.

| | $z$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | $RHS$ |
|---|---|---|---|---|---|---|---|---|
| $z$ | 1 | 0 | 0 | **A** | -10 | -1 | **B** | 32 |
| $s_3$ | 0 | 0 | 0 | **C** | 4 | -2 | 1 | **D** |
| $x_2$ | 0 | 0 | 1 | -2 | **F** | -1 | 0 | 6 |
| $x_1$ | 0 | **E** | 0 | -1 | 1 | 2 | 0 | 4 |

Give **general** conditions on each of the unknowns **A-F** such that each of the following statements is true. Even if the statement holds independently of the values of a specific variable, you should still mention that variable.

(a) The tableau is final and there exists a unique optimal solution.

(b) The simplex method determines an unbounded solution from this tableau.

(c) The current bfs is degenerate (not necessarily optimal).

(d) The current solution is optimal, there are alternative optimal solutions but no alternative optimal bfs.

**Question 8.** Use the two-phase method to find the optimal solution to the following LP:

$$
\begin{aligned}
\max\ & -3x_1 + 4x_2 \\
\text{s.t. }\ & x_1 + x_2 \leq 4 \\
& 2x_1 + 3x_2 \geq 18 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

**Question 9.** Convert the following LP to standard form and solve with the Big-M method.

$$
\begin{aligned}
\max\ & -x_1 + 3x_2 + x_3 \\
\text{s.t. }\ & -x_1 + x_2 - x_3 \geq -4 \\
& 2x_1 + x_2 + x_3 \geq 10 \\
& x_1 \geq 0 \\
& x_2 \leq 0 \\
& x_3 \text{ free}
\end{aligned}
$$

**Question 10.** Consider the following LP:

$$
\begin{aligned}
\max\ & x_1 + 2x_2 + 4x_3 \\
\text{s.t. }\ & -x_1 + x_2 - x_3 \geq 2 \\
& -2x_1 + x_2 + x_3 \leq 1 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

Solve the LP using big M method. Could you find an optimal solution? If yes, report the optimal solution. If no, state why.

**Question 11.** Consider the following simplex tableau of a given maximization LP problem.

| | z | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | RHS |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | **A** | 0 | 0 | 0 | 12 |
| $x_3$ | 0 | **B** | -1 | 1 | 0 | 0 | 5 |
| $x_4$ | 0 | -6 | **C** | 0 | 1 | 0 | 1 |
| $x_5$ | 0 | -11 | **D** | 0 | 0 | 1 | **E** |

Give general conditions on each of the unknowns A-E such that each of the following statements is true. Even if the statement holds independently of the values of a specific variable, you should still mention that that variable can take any value. So, for each part you should present 5 conditions, one for each unknown.

(a) The tableau is final and there exists a unique optimal solution.

(b) The simplex method determines an unbounded solution from this tableau.

(c) The current bfs is degenerate (not necessarily optimal).

(d) The current solution is optimal, there are alternative optimal solutions but no alternative optimal bfs.

**Question 12.** Solve the following linear program by both the two-phase method and the big M-method:

$$\begin{aligned} \min\ & 3x_1 + 2x_2 + 4x_3 \\ \text{s.t.}\ & 2x_1 + x_2 + 3x_3 = 60 \\ & 3x_1 + 3x_2 + 5x_3 \geq 120 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$