IE 400: Principles of Engineering Management
Study Set 3

Spring 2021

Question 1. Consider the following simplex tableaus of LP problems. First two are minimization problems while the last tableau belongs to a maximization problem.

x_1	x_2	e_1	e_2	x_3	x_4	RHS
-2	-3	0	0	-M	-M	0
2	1	-1	0	1	0	4
1	-1	0	-1	0	1	1

x_1	x_2	s_1	e_1	x_3	x_4	RHS
0	0	0	0	-1	-1	0
1	1	1	0	0	0	3
2	1	0	-1	1	0	4
1	1	0	0	0	1	3

x_1	x_2	x_3	s_1	s_2	e_1	RHS
-2	-1	-1	0	0	0	0
1/3	2/3	0	1	0	1/3	1
2	1	0	0	1	0	3
2/3	1/3	1	0	0	-1/3	1

Are these tableaus in proper format? If yes, indicate the entering and leaving variables. If not, explain why and show how can you convert them into proper format.

Question 2. Consider the following simplex tableau of a given **minimization** LP problem.

	z	x_1	x_2	x_3	s_1	s_2	s_3	*RHS*
z	1	0	0	**A**	-10	-1	**B**	32
s_3	0	0	0	**C**	4	-2	1	**D**
x_2	0	0	1	-2	**F**	-1	0	6
x_1	0	**E**	0	-1	1	2	0	4

Give **general** conditions on each of the unknowns **A-F** such that each of the following statements is true. Even if the statement holds independently of the values of a specific variable, you should still mention that variable.

(a) The tableau is final and there exists a unique optimal solution.

(b) The simplex method determines an unbounded solution from this tableau.

(c) The current bfs is degenerate (not necessarily optimal).

(d) The current solution is optimal, there are alternative optimal solutions but no alternative optimal bfs.

Question 3. Use the two-phase method to find the optimal solution to the following LP:

$$\begin{aligned} \max\ & -3x_1 + 4x_2 \\ \text{s.t. } & x_1 + x_2 \leq 4 \\ & 2x_1 + 3x_2 \geq 18 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Question 4. Convert the following LP to standard form and solve with the Big-M method.

$$
\begin{aligned}
\max\ & -x_1 + 3x_2 + x_3 \\
\text{s.t.}\ & -x_1 + x_2 - x_3 \geq -4 \\
& 2x_1 + x_2 + x_3 \geq 10 \\
& x_1 \geq 0 \\
& x_2 \leq 0 \\
& x_3 \text{ free}
\end{aligned}
$$

Question 5. Consider the following simplex tableau of a given **maximization** LP problem.

	z	x_1	x_2	x_3	x_4	x_5	RHS
1	1	2	**A**	0	0	0	12
x_3	0	**B**	-1	1	0	0	5
x_4	0	-6	**C**	0	1	0	1
x_5	0	-11	**D**	0	0	1	**E**

Give general conditions on each of the unknowns A-E such that each of the following statements is true. Even if the statement holds independently of the values of a specific variable, you should still mention that that variable can take any value. So, for each part you should present 5 conditions, one for each unknown.

(a) The tableau is final and there exists a unique optimal solution.

(b) The simplex method determines an unbounded solution from this tableau.

(c) The current bfs is degenerate (not necessarily optimal).

(d) The current solution is optimal, there are alternative optimal solutions but no alternative optimal bfs.

Question 6. Consider the following LP:

$$
\begin{aligned}
\max\ & x_1 + 2x_2 + 4x_3 \\
\text{s.t.}\ & -x_1 + x_2 - x_3 \geq 2 \\
& -2x_1 + x_2 + x_3 \leq 1 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

Solve the LP using big M method. Could you find an optimal solution? If yes, report the optimal solution. If no, state why.