IE 400: Principles of Engineering Management
Study Set 3

Summer 2021

Question 1. Ford has four automobile plants. Each is capable of producing the Taurus, Lincoln or Escort but it can only produce one of these cars. Suppose that Ford should produce all car types. The fixed cost of operating each plant for a year and the variable cost of producing a car of each type at each plant are given in the table below:

	Fixed Cost	**Variable Cost**		
		Taurus	Lincoln	Escort
Plant 1	7 billion	12000	16000	9000
Plant 2	6 billion	15000	18000	11000
Plant 3	4 billion	17000	19000	12000
Plant 4	2 billion	19000	22000	14000

(a) Ford faces the following restrictions:

(1) Each plant can produce one type of car.

(2) The total production of each type of car must be at a single plant; that is, for example, if any Tauruses are made at plant 1, then all Tauruses must be made there.

(3) If plants 3 and 4 are used, then plant 1 must also be used.

Formulate the problem in order to minimize the total cost.

(b) Suppose now that instead of the restriction (3) in part (a), Ford faces the following restriction:

- Either at most one of the plants 1 and 2 or at least one of the plants 3 and 4 should be used.

Add the necessary constraint to your model.

Question 2. NASA has 4 astronauts to be employed at a space mission for 4 posts on a space ship. The posts require different qualifications (competencies) but each astronaut can perform each one of these tasks, albeit with varying degree of proficiency. The proficiency rating of astronaut i assigned to post j is measured as p_{ij}. It is asked to find the optimal assignment of astronauts to posts which gives the highest total proficiency rating while meeting the following requirements:

- Each astronaut should be assigned to a single post.
- Each post should have a single astronaut.

Question 3. A paper manufacturer produces rolls of standard fixed width w and of standard length l. Customers order rolls of width w but varying lengths. In particular, d_k rolls with length l_k and width w are ordered for customer $k = 1, \ldots, n$ (assume $l_k \leq l$ for all $k = 1, \ldots, n$). What is the minimum number of rolls that should be cut to meet the demand?
Hint: Let M be a large number such that M number of rolls are enough to satisfy all demand (e.g. $M = \sum_{k=1}^{n} d_k$ is such value) Assume the manufacturer has M number of unmanufactured rolls in the stock. Use decision variables:

$$x_i = \begin{cases} 1 & \text{if roll i is used} \\ 0 & \text{o.w} \end{cases}$$

y_{ik} =number of rolls of type k (of length l_k) cut from roll i
$i = 1, \ldots, M, k = 1, \ldots, n$

Question 4. A cargo plane has three compartments for storing cargo: front, center and rear. These compartments have the following limits on both weight and space:

Compartment	**Weight Capacity (tons)**	**Space Capacity (cubic meters)**
Front	10	6800
Center	16	8700
Rear	8	5300

The following four cargoes are available for shipment on the next flight:

Cargo	Weight (tons)	Volume (cubic meters/ton)	Profit (TL/ton)
C1	18	480	310
C2	15	650	380
C3	23	580	350
C4	12	390	285

We assume

- that each cargo can be split into whatever proportions/fractions we desire for shipment
- that each cargo can be split between 2 or more compartments if we so desire

The aim is to determine how much (if any) of each cargo C1, C2, C3 and C4 should be accepted and how to distribute each among the compartments so that the total profit for the flight is maximized.

(a) Let us define the utilization rate of a cargo compartment as the ratio of the total cargo weight carried in that compartment to the weight capacity of that compartment (for example, if a total of 8 tons of cargo is carried in the front compartment, the utilization rate will be 8/10=0.8). Formulate this problem as a linear programming problem under the constraint that the utilization rate of each compartment must be the same to maintain the balance of the plane.

(b) Formulate this problem as a linear programming problem under the constraint that the differences between the utilization rates of any two compartments should not be more than 0.1 to maintain the balance of the plane.

(c) Consider the two LPs in parts a) and b). Which one is more likely to have a larger optimal value? Please justify your answer.

(d) If cargo can be delivered only in integer amounts (in tons) and if it is not possible to split the cargo between compartments, how would your model in part b) change? Your new model should still be a linear one.

Question 5. Kocalar Delivery Company must make deliveries to 20 customers whose respective demands are d_j for j = 1,. . . ,20. Each customer can be visited by more than one truck to satisfy the demands. The company has seven trucks available with capacities C_k and daily operating costs q_k for k = 1,. . . ,7.

(a) Formulate the model to determine which trucks to use so as to minimize the cost of delivering to all the customers.

(b) Add the following requirements to your model:

- A single truck cannot deliver to more than six customers.
- Customer 1 and customer 7 cannot be visited by the same truck.
- Both customer 2 and customer 12 should be visited by the same truck(s).
- If a truck visits customer 4 then it has to visit customer 19 or customer 20. (Assume demands of these customers do not exceed truck capacities.)
- Either at least two of the customers 5, 6 and 7 or at most three of customers 11, 12, 13 and 14 should be visited by truck 4.

Question 6. Consider a set of 20 linear inequalities $\sum_{j=1}^{n} a_{ij}x_j \leq b_i$ for i= 1,..., 20. Formulate an integer program which represents that a point x satisfies the following:

- Point x should satisfy at least 10 of the 20 constraints.
- If point x satisfies constraint 1, then it should also satisfy constraint 4.
- If point x satisfies constraint 2, then it should not satisfy constraint 3.
- If point x satisfies either both of the constraints 5 and 6 or neither of them.
- Point x should satisfy either at least one of the constraints 7,8,9 or at least two of constraints 10,11,12,13.

Question 7. Glueco produces three types of glue on two different production lines. Each line can be utilized by up to seven workers at a time. Workers are paid \$500 per week on production line 1, and \$900 per week on production line 2. A week of production costs \$1,000 to set up production line 1 and \$2,000 to set up production line 2. The weekly demand of glue 1 is 120, glue 2 is 150 and glue 3 is 200. During a week on a production line, each worker produces the number of units of glue shown in the table below.

	Glue		
Production Line	1	2	3
1	20	30	40
2	50	35	45

Formulate the problem in order to minimize the total cost.

Question 8. Use the branch-and-bound method to find the optimal solution to the following IP:

$$\begin{aligned} \min\ & 9x_1 + 13x_2 + 10x_3 + 8x_4 + 8x_5 \\ \text{s.t.}\ & 6x_1 + 3x_2 + 2x_3 + 4x_4 + 7x_5 \geq 40 \\ & x_1 \leq 1, x_2 \geq 1, x_3 \geq 2, x_4 \geq 1, x_5 \leq 3 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \\ & x_1, x_2, x_3, x_4, x_5 \text{ integer} \end{aligned}$$

Question 9. Use the branch-and-bound method to find the optimal solution to the following IP:

$$\begin{aligned} \max\ & 4x_1 + 5x_2 \\ \text{s.t.}\ & 3x_1 + 2x_2 \leq 10 \\ & x_1 + 4x_2 \leq 11 \\ & 3x_1 + 3x_2 \leq 13 \\ & x_1, x_2 \geq 0 \\ & x_1, x_2 \text{ integer} \end{aligned}$$

Question 10. Use the branch-and-bound method to find the optimal solution to the following IP:

$$\begin{aligned} \max\ & 40x_1 + 5x_2 + 60x_3 + 8x_4 \\ \text{s.t. } & 18x_1 + 3x_2 + 20x_3 + 5x_4 \leq 25 \\ & x_1, x_2, x_3, x_4 \in \{0, 1\} \end{aligned}$$