IE 400: Principles of Engineering Management
Study Set 4

Summer 2021

Question 1. Use the branch-and-bound method to find the optimal solution to the following IP:

$$\begin{aligned} \min\ & 9x_1 + 13x_2 + 10x_3 + 8x_4 + 8x_5 \\ \text{s.t. } & 6x_1 + 3x_2 + 2x_3 + 4x_4 + 7x_5 \geq 40 \\ & x_1 \leq 1, x_2 \geq 1, x_3 \geq 2, x_4 \geq 1, x_5 \leq 3 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \\ & x_1, x_2, x_3, x_4, x_5 \text{ integer} \end{aligned}$$

Question 2. Use the branch-and-bound method to find the optimal solution to the following IP:

$$\begin{aligned} \max\ & 4x_1 + 5x_2 \\ \text{s.t. } & 3x_1 + 2x_2 \leq 10 \\ & x_1 + 4x_2 \leq 11 \\ & 3x_1 + 3x_2 \leq 13 \\ & x_1, x_2 \geq 0 \\ & x_1, x_2 \text{ integer} \end{aligned}$$

Question 3. Use the branch-and-bound method to find the optimal solution to the following IP:

$$\begin{aligned} \max\ & 40x_1 + 5x_2 + 60x_3 + 8x_4 \\ \text{s.t. } & 18x_1 + 3x_2 + 20x_3 + 5x_4 \leq 25 \\ & x_1, x_2, x_3, x_4 \in \{0, 1\} \end{aligned}$$

Question 4. Solve the following IP to optimality using the branch and bound method:

$$\begin{aligned} \max\ & 4x_1 + 10x_2 \\ \text{s.t. } & 6x_1 - 2x_2 \leq 21 \\ & \text{-}4x_1 + 6x_2 \leq 9 \\ & 4x_1 + 10x_2 \leq 31 \\ & x_1, x_2 \geq 0 \text{ and integer} \end{aligned}$$

Question 5. Solve the following IP to optimality using the branch and bound method:

$$\begin{aligned} \max\ & 15x_1 + 10x_2 + 9x_3 + 2x_4 \\ \text{s.t. } & 3x_1 + 5x_2 + 3x_3 + 2x_4 \leq 14 \\ & x_i \geq 0 \text{ and integer for } i\text{=1, 2, 3, 4} \end{aligned}$$

Question 6. Suppose we are applying branch and bound algorithm for solving a 0-1 discrete optimization problem and we have reached node 27. Assume that this node corresponds to the following problem:

$$\begin{aligned} \min\ & c_1x_6 + c_2x_7 \\ \text{s.t. } & a_1x_6 + a_2x_7 \geq 15 \\ & x_6, x_7 \in \{0, 1\} \end{aligned}$$

The current incumbent has an objective value of 34. In the following, give conditions on parameters c_1, c_2, a_1 and a_2 such that the statements become true. Assume all parameters are integer valued.

(a) Node 27 is fathomed because of infeasibility.

(b) Node 27 is fathomed because of integrality.

(c) Node 27 is fathomed because of the bound.

(d) Node 27 of the tree is not fathomed.