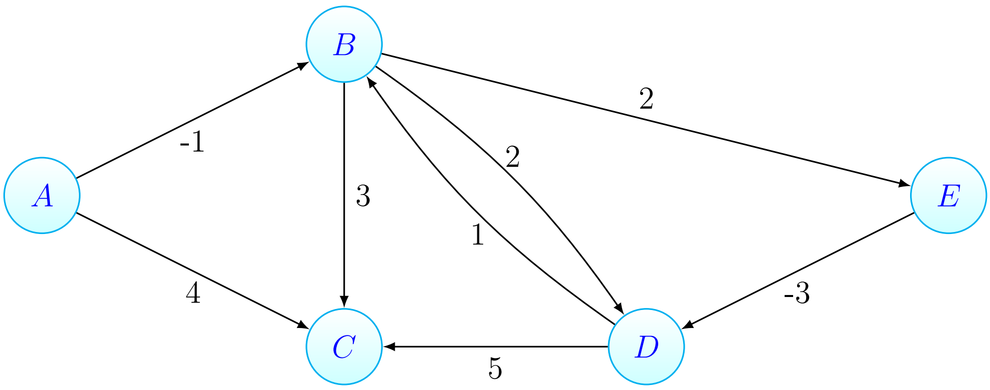
IE 400: Principles of Engineering Management
Study Set 5 Solutions

Summer 2021

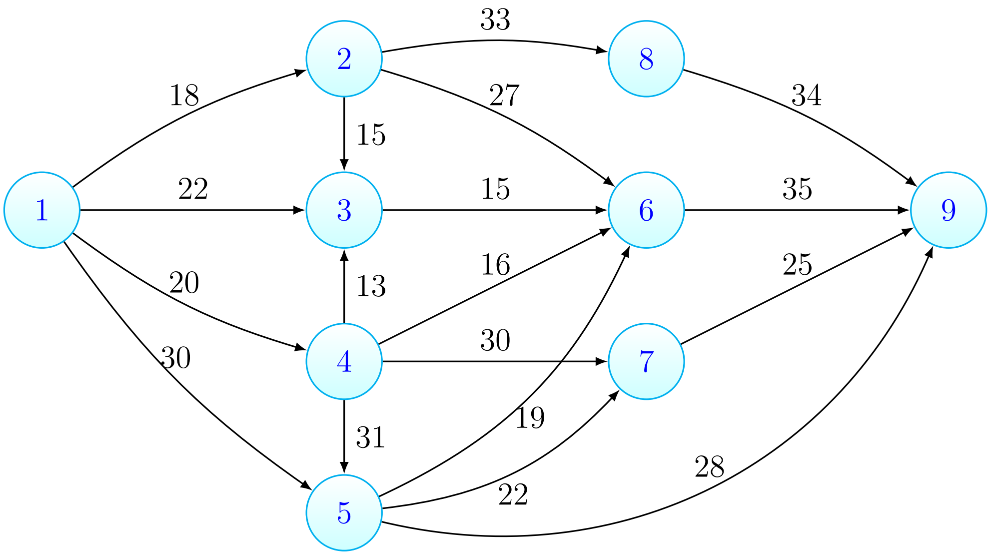
Question 1. Use Bellman Ford algorithm to find the shortest path of the following graph where A is the source node.



$$V_{[k]}^{(t)} = \min\{V_{[i]}^{t-1} + c_{ik}\}$$

t	$V_{(A)}^t$	$V_{(B)}^t$	$V_{(C)}^t$	$V_{(D)}^t$	$V_{(E)}^t$	d(A)	d(B)	d(C)	d(D)	d(E)
0	0	$+\infty$	$+\infty$	$+\infty$	$+\infty$					
1	0	-1	4	$+\infty$	$+\infty$		A	A		
2	0	-1	2	1	1			B	B	B
3	0	-1	2	-2	1				E	

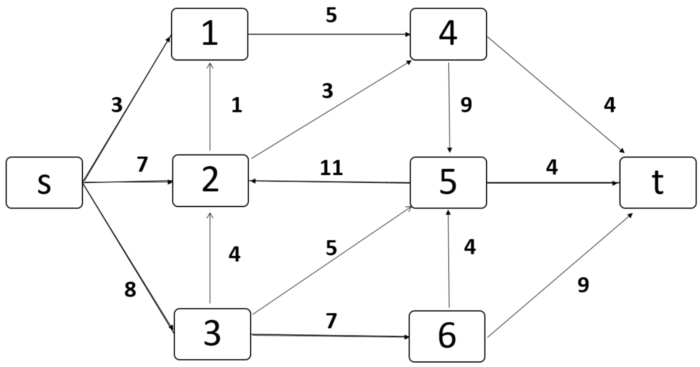
Question 2. Find shortest paths from 1 to all other nodes using Dijkstra's algorithm.



p	**v[1]**	**v[2]**	**v[3]**	**v[4]**	**v[5]**	**v[6]**	**v[7]**	**v[8]**	**v[9]**
(init)	0	∞	∞	∞	∞	∞	∞	∞	∞
1	(perm)	18	22	20	30				
2		(perm)				45		51	
4				(perm)		36	50		
3			(perm)						
5					(perm)				58
6						(perm)			
7							(perm)		
8								(perm)	
9									(perm)
(final)	0	18	22	20	30	36	50	51	58

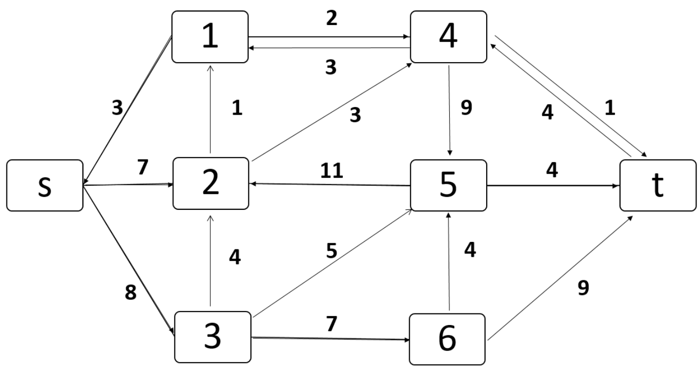
p	**d[1]**	**d[2]**	**d[3]**	**d[4]**	**d[5]**	**d[6]**	**d[7]**	**d[8]**	**d[9]**
1	(perm)	1	1	1	1				
2		(perm)				2		2	
4				(perm)		4	4		
3			(perm)						
5					(perm)				5
6						(perm)			
7							(perm)		
8								(perm)	
9									(perm)
(final)	-	1	1	1	1	4	4	2	5

Question 3. Consider costs as capacities. Using Ford - Fulkerson algorithm find max-flow.



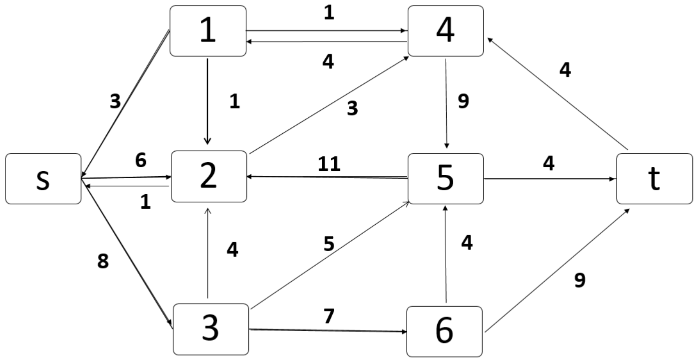
Step 1: Choose path s - 1 - 4 - t

min{3, 5, 4}= 3



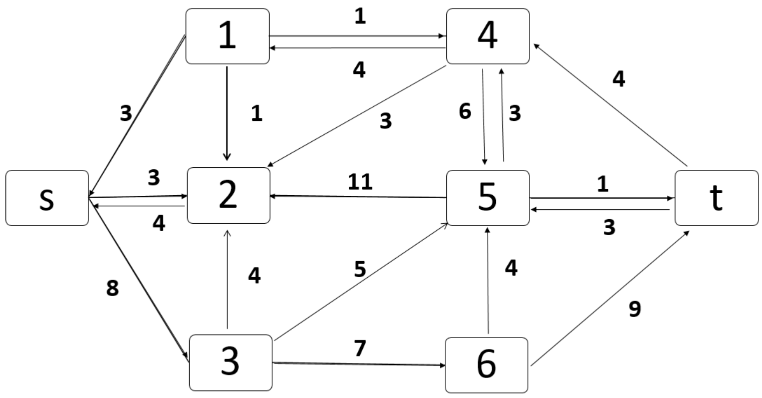
Step 2: Choose path s - 2 - 1 - 4 - t

min{7, 1, 2, 1}= 1



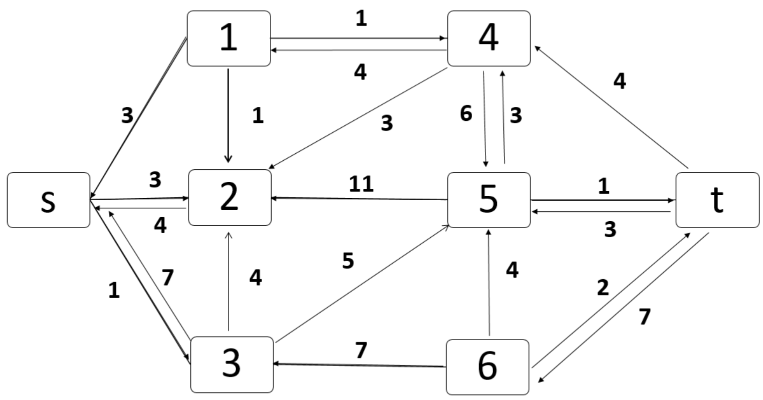
Step 3: Choose path s - 2 - 4 - 5 - t

min{6, 3, 9, 4}= 3



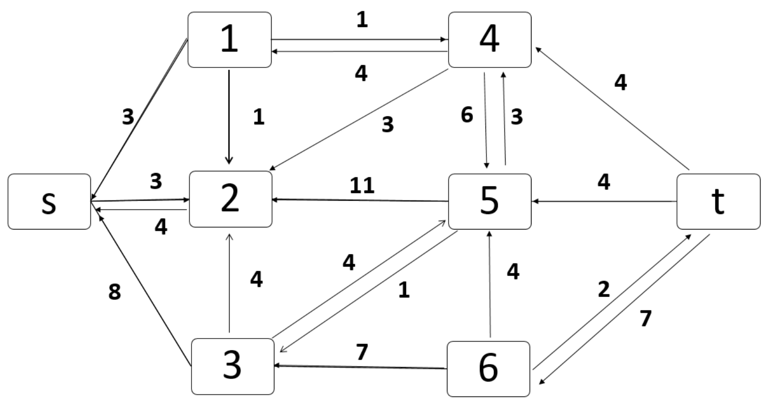
Step 4: Choose path s - 3 - 6 - t

min{8, 7, 9}= 7



Step 5: Choose path s - 3 - 5 - t

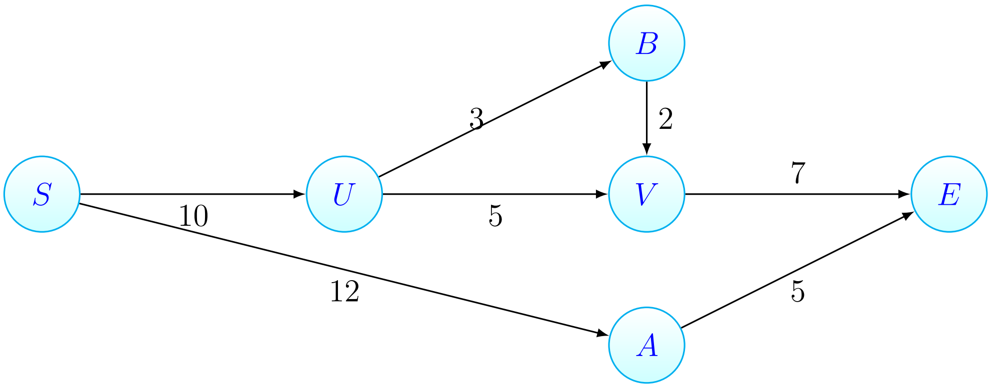
min{1, 5, 1}= 1



There is no other path from s to t.
Since max-flow = min-cut in optimal solution. The minimum cost to block flood's way to city is 15.

Question 4.

(a) The principal of optimality does not apply. See the following **counter example**:



Since we are trying to minimize the highest height encountered: Shortest path from S to E is S-U-V-E=10 (OR S-U-B-V-E =10) **but** shortest path from U to V is U-B-V=3, not U-V=5.

(b)

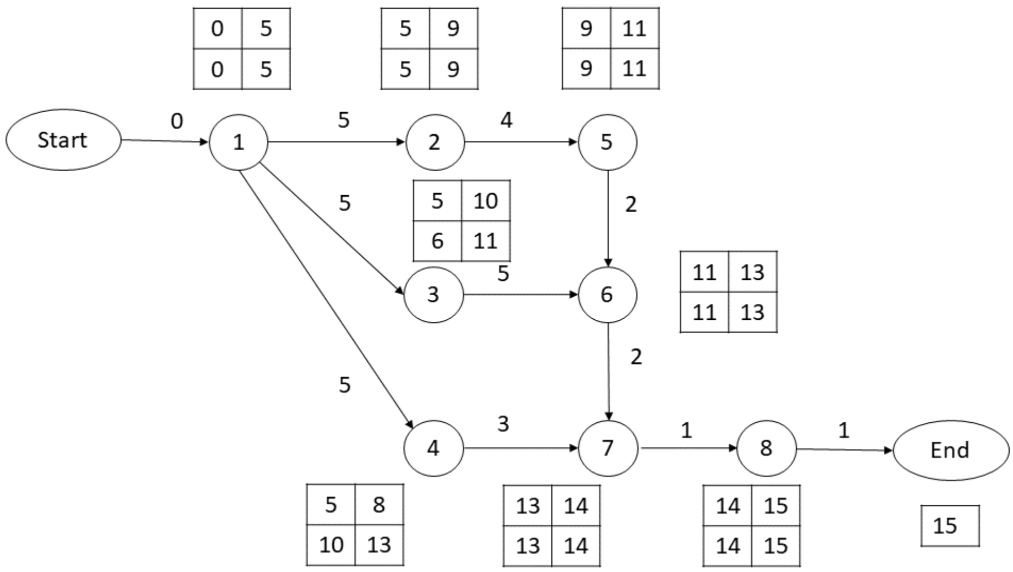
$$V[S] = 0$$

$$V[i] = \min_{(i,k)\in A} \{\max\{V[k], c_{ik}\}\}$$

(c) You can adapt Dijkstra's algorithm with the functional equations in part (b).

Question 5.

Activity	Predecessors	Duration
1 (Design)	-	5
2 (make A)	1	4
3 (make B)	1	5
4 (make C)	1	3
5 (test A)	2	2
6 (assembly A&B)	3, 5	2
7 (attach C)	6, 4	1
8 (testing)	7	1



Critical path: 1 - 2 - 5 - 6 - 7 - 8

Duration: 15

Activity	1	2	3	4	5	6	7	8
Slack	0	0	1	5	0	0	0	0

For the LP define decision variable x_j: start time of activity $j, j = 1, ..., 10,$ start, end

$$\begin{aligned}
\min \quad & x_{end} \\
\\
\text{s.t.} \quad & x_1 \geq x_{start} \\
& x_2 \geq x_1 + 5 \\
& x_3 \geq x_1 + 5 \\
& x_4 \geq x_1 + 5 \\
& x_5 \geq x_2 + 4 \\
& x_6 \geq x_3 + 5 \\
& x_6 \geq x_5 + 2 \\
& x_7 \geq x_6 + 2 \\
& x_7 \geq x_4 + 3 \\
& x_8 \geq x_7 + 1 \\
& x_{end} \geq x_8 + 1 \\
& x_{start} = 0
\end{aligned}$$

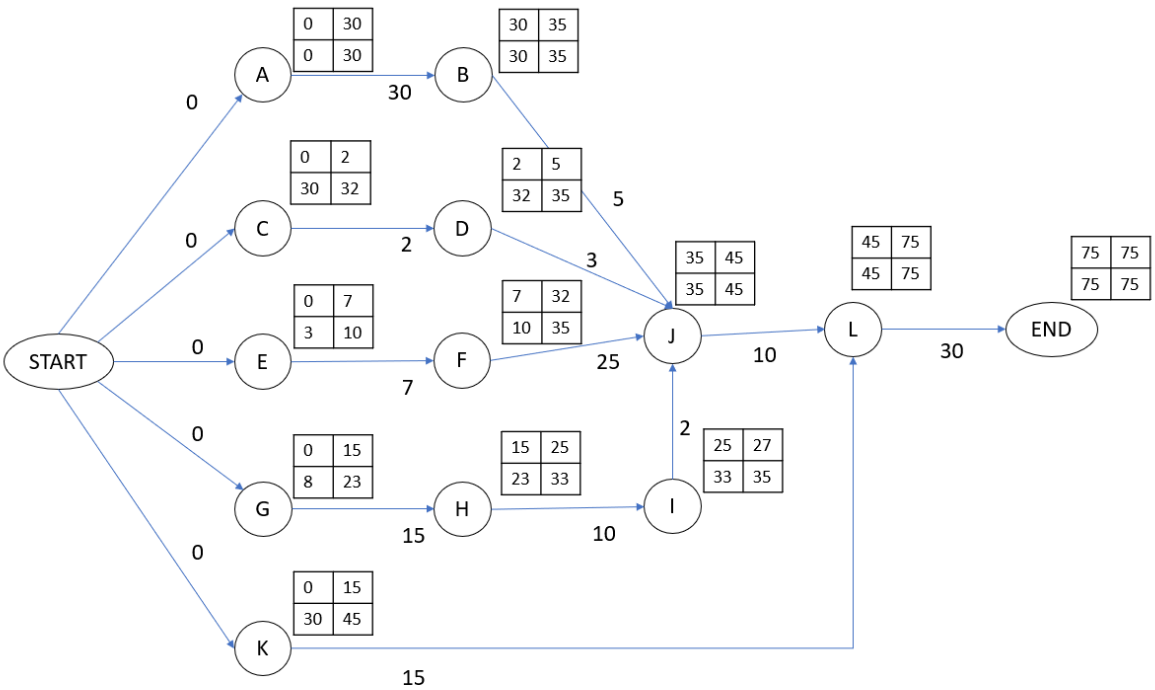
The tight constraints of the optimal solution for this LP, which hold at equality, will give you the critical path.

Question 6.

ES: Earliest Start, EF: Earliest Finish, LS: Latest Start, LF: Latest Finish

ES	EF
LS	LF

(a)-(b)



Activity	Slack
A	0
B	0
C	30
D	30
E	3
F	3
G	8
H	8
I	8
J	0
K	30
L	0

(c) Since the slack for cutting the onions and mushrooms is 3, the dinner will be delayed 6-3=3 minutes.

If the cutting time is 2 minutes, slack is 10 (latest finish time) – 2 (task duration) = 8 minutes. Since the slack time is more than the delay time, it does not delay the dinner.

(d) Let N be the set of activities and A be the set of arc in the project network.

Parameters:

a_i : Duration of activity i, for all i $\in N$

d : Project duration (75)

Decision Variables:

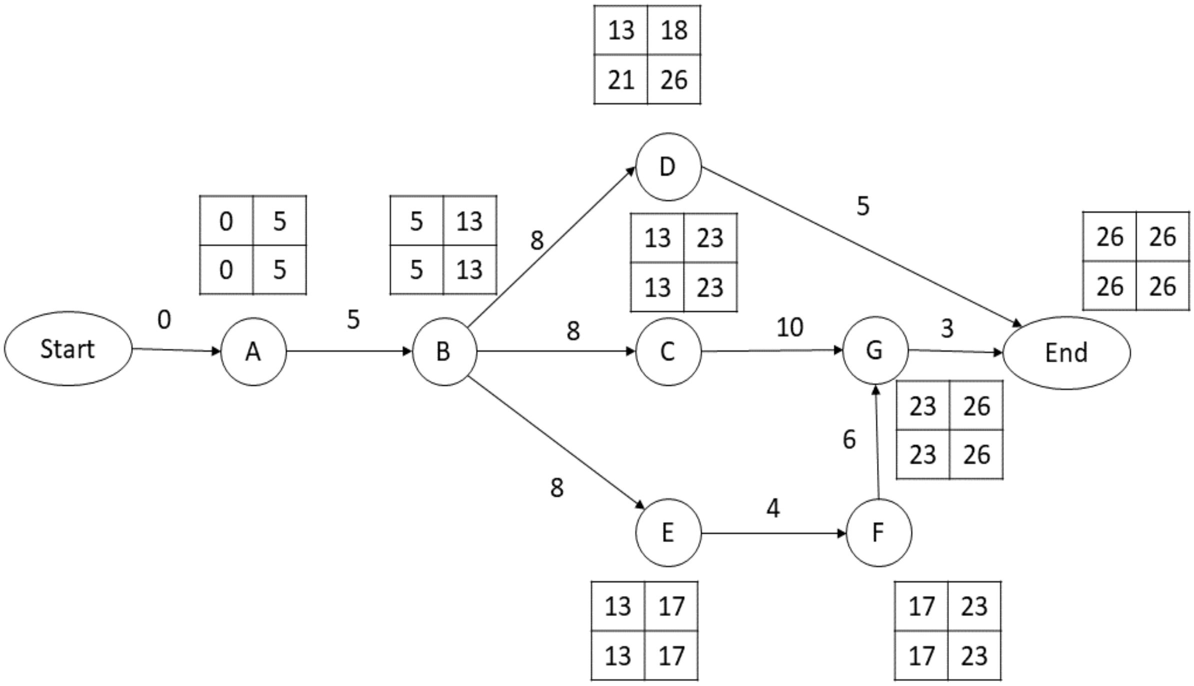
x_i : Start time of activity i, for all $i \in N$

Model:

$$
\begin{aligned}
\max \ & \sum_{i \in N} x_i \\
\text{s.t. } & x_j \geq x_i + a_i && (i,j) \in A \\
& x_{end} \leq d \\
& x_i \geq 0 && i \in N \\
& x_{start} = 0
\end{aligned}
$$

Question 7.

a)



Critical path: A - B - C - G and A - B - E - F - G
Duration: 26

b) D can be delayed for 8 days.

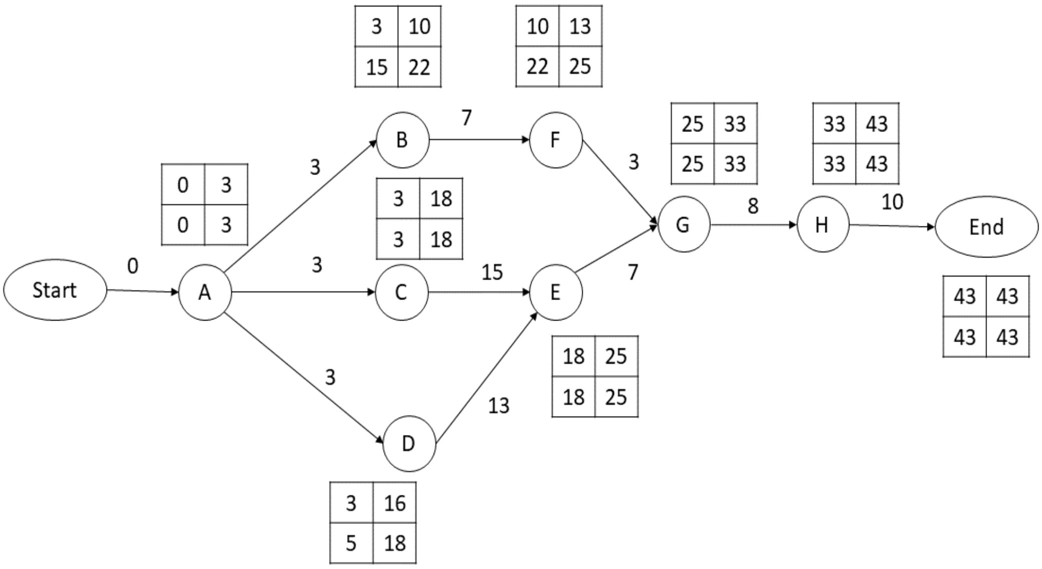
c) **Decision Variables:**
x_j: start time of activity j
y_j: number of days duration of activity j is reduced
Parameters:
c_j: cost of reduction
r_j: max reduction
d_j: duration of activity

Model:

$$
\begin{array}{lll}
\min & \sum_j (c_j y_j) & \\
& & \\
\text{subject to} & y_j \leq r_j & \\
& x_j \geq x_i + d_i \text{ - } y_i & \forall (i,j) \in A \\
& x_{start} = 0 & \\
& x_{end} \leq 20 & \\
& y_j \geq 0 & \forall j
\end{array}
$$

Question 8.

(a)



Activity	A	B	C	D	E	F	G	H
Slack	0	12	0	2	0	12	0	0

Critical path: A - C - E - G - H

(b) N: set of activities

Decision Variables:

x_i: start time of activity i, $i \in N$

Model:

$$
\begin{aligned}
\min \quad & \textstyle\sum_{i \in N} x_i \\
\\
\text{s.t.} \quad & x_{end} \leq 43 \\
& x_{start} = 0 \\
& x_A \geq x_{start} \\
& x_B \geq x_A + 3 \\
& x_C \geq x_A + 3 \\
& x_D \geq x_A + 3 \\
& x_E \geq x_C + 15 \\
& x_E \geq x_D + 13 \\
& x_F \geq x_B + 7 \\
& x_G \geq x_F + 3 \\
& x_G \geq x_F + 7 \\
& x_H \geq x_G + 8 \\
& x_{end} \geq x_H + 10 \\
& x_i \geq 0 \ \forall \ i \in N
\end{aligned}
$$

(c) No, the critical path does not change. The increase in the path A - B - F - G - H is $4 + 7 = 11$, which is smaller than the slack of activities B and F.

(d) **Additional Decision Variable:**

y_i: number of days that the duration of activity i is reduced, $i \in N$

Model:

$$
\begin{aligned}
\min \quad & 10y_A + 21y_B + 15y_C + 23y_D + 12y_E + 14y_F + 13y_6 + 20y_H \\
\\
\text{s.t.} \quad & x_A \geq x_{start} \\
& x_B \geq x_A + 3 - y_A \\
& x_C \geq x_A + 3 - y_A \\
& x_D \geq x_A + 3 - y_A \\
& x_E \geq x_C + 15 - y_C \\
& x_E \geq x_D + 13 - y_D \\
& x_F \geq x_B + 7 - y_B \\
& x_G \geq x_F + 3 - y_F \\
& x_G \geq x_F + 7 - y_E \\
& x_H \geq x_G + 8 - y_G \\
& x_{end} \geq x_H + 10 - y_H \\
& x_{end} \leq 40 \\
& y_A \leq 1 \\
& y_B \leq 2 \\
& y_C \leq 2 \\
& y_D \leq 1 \\
& y_E \leq 2 \\
& y_F \leq 1 \\
& y_G \leq 1 \\
& y_H \leq 3 \\
& y_i \geq 0, \text{integer } \forall i \in N \\
& x_i \geq 0 \ \forall \ i \in N
\end{aligned}
$$