

IE 400: Principles of Engineering Management

Study Set 3 Solutions

Fall 2021

Question 1.

(a) Parameters:

f_i = fixed cost of plant i , $i = 1, \dots, 4$

c_{ij} = cost of producing car j at plant i $i = 1, \dots, 4$, $j = 1, 2, 3$

Decision Variables:

$$y_i = \begin{cases} 1 & \text{if plant } i \text{ is used } i = 1, \dots, 4 \\ 0 & \text{o.w} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{if car } j \text{ is produced at plant } i \quad i = 1, \dots, 4, \quad j = 1, 2, 3 \\ 0 & \text{o.w} \end{cases}$$

Model:

$$\begin{aligned} \min \quad & \sum_{i=1}^4 f_i y_i + \sum_{i=1}^4 \sum_{j=1}^3 c_{ij} x_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^3 x_{ij} \leq 1 & i = 1, 2, 3, 4 \end{aligned} \tag{1}$$

$$\sum_{i=1}^4 x_{ij} = 1 \quad j = 1, 2, 3 \tag{2}$$

$$y_3 + y_4 \leq 1 + y_1 \tag{3}$$

$$x_{ij} \leq y_i \quad i = 1, 2, 3, 4 \quad j = 1, 2, 3$$

$$x_{ij} \in \{0, 1\} \quad i = 1, 2, 3, 4 \quad j = 1, 2, 3$$

$$y_j \in \{0, 1\} \quad j = 1, 2, 3$$

Constraint (1): Each plant can produce one type of car.

Constraint (2): The total production of each type of car must be at a single plant; that is, for example, if any Tauruses are made at plant 1, then all Tauruses must be made there.

Constraint (3): If plants 3 and 4 are used, then plant 1 must also be used.

(b)

$$\text{either} \qquad y_1 + y_2 \leq 1 \qquad (*)$$

$$\text{or} \qquad y_3 + y_4 \geq 1 \qquad (**)$$

In order to linearize the constraint, add the following decision variable:

$$w = \begin{cases} 1 & \text{if constraint } (*) \text{ holds} \\ 0 & \text{if constraint } (**) \text{ holds} \end{cases}$$

Then add the following constraints to the model:

$$y_1 + y_2 \leq 1w - (1 - w)M$$

$$y_3 + y_4 \geq 1(1 - w) + Mw$$

$$w \in \{0, 1\}$$

where M is a sufficiently large number.

(For $w = 1$, the constraint $(*)$ holds and $(**)$ becomes redundant. For $w = 0$, the constraint $(**)$ holds and $(*)$ becomes redundant.)

Question 2.

Decision Variables:

$$x_{ij} = \begin{cases} 1 & \text{if astronaut } i \text{ is assigned to post } j \text{ } i = 1, 2, 3, 4, \text{ } j = 1, 2, 3, 4 \\ 0 & \text{o.w} \end{cases}$$

Model:

$$\begin{aligned} \max \quad & \sum_{i=1}^4 \sum_{j=1}^4 p_{ij} x_{ij} \\ \text{s.t.} \quad & \sum_{i=1}^4 x_{ij} = 1 \quad j = 1, 2, 3, 4 \end{aligned} \tag{1}$$

$$\sum_{j=1}^4 x_{ij} = 1 \quad i = 1, 2, 3, 4 \tag{2}$$

$$x_{ij} \in \{0, 1\} \quad i = 1, 2, 3, 4 \text{ } j = 1, 2, 3, 4$$

- Constraint (1) indicates that each post should have a single astronaut.
- Constraint (2) indicates that each astronaut should be assigned to a single post.

Question 3.

Parameters: As given in the question

Decision Variables: As given in the question

Model:

$$\begin{aligned} \min \quad & \sum_{i=1}^M x_i \\ \text{s.t.} \quad & \sum_{i=1}^M y_{ik} = d_k & k = 1, \dots, n \\ & \sum_{k=1}^n y_{ik} \cdot l_k \leq l \cdot x_i & i = 1, \dots, M \\ & x_i \in \{0, 1\} & i = 1, \dots, M \\ & y_{ik} \geq 0 & i = 1, \dots, M \quad k = 1, \dots, n \\ & y_{ik} \text{ integer} & i = 1, \dots, M \quad k = 1, \dots, n \end{aligned}$$

Question 4. (a) Parameters:

w_j = weight capacity of compartment j (in tons)

s_j = space capacity of compartment j (in cubic meters)

wt_i = weight of cargo i (in tons)

v_i = weight of cargo i (in cubic meters/ton)

p_i = profit gained from cargo i (TL/ton)

Decision Variables:

x_{ij} = amount of cargo i distributed to compartment j in tons

$$\begin{aligned}
& \max \sum_{i=1}^4 \sum_{j=1}^3 p_i x_{ij} \\
& \text{s.t.} \quad \sum_{j=1}^3 x_{ij} \leq wt_i \quad i = 1, 2, 3, 4 \quad (1) \\
& \quad \sum_{i=1}^4 v_i x_{ij} \leq s_j \quad j = 1, 2, 3 \quad (2) \\
& \quad \sum_{i=1}^4 x_{ij} \leq w_j \quad j = 1, 2, 3 \quad (3) \\
& \quad \frac{\sum_{i=1}^4 x_{i1}}{w_1} = \frac{\sum_{i=1}^4 x_{i2}}{w_2} = \frac{\sum_{i=1}^4 x_{i3}}{w_3} \quad (4) \\
& \quad x_{ij} \geq 0 \quad i = 1, 2, 3, 4 \quad j = 1, 2, 3
\end{aligned}$$

(b) Replace constraint (4) with

$$\begin{aligned}
& \left| \frac{\sum_{i=1}^4 x_{i1}}{w_1} - \frac{\sum_{i=1}^4 x_{i2}}{w_2} \right| \leq 0.1 \\
& \left| \frac{\sum_{i=1}^4 x_{i1}}{w_1} - \frac{\sum_{i=1}^4 x_{i3}}{w_3} \right| \leq 0.1 \\
& \left| \frac{\sum_{i=1}^4 x_{i2}}{w_2} - \frac{\sum_{i=1}^4 x_{i3}}{w_3} \right| \leq 0.1
\end{aligned}$$

You need to linearize these constraints before adding to the model:

$$\begin{aligned}
-0.1 & \leq \frac{\sum_{i=1}^4 x_{i1}}{w_1} - \frac{\sum_{i=1}^4 x_{i2}}{w_2} \leq 0.1 \\
-0.1 & \leq \frac{\sum_{i=1}^4 x_{i1}}{w_1} - \frac{\sum_{i=1}^4 x_{i3}}{w_3} \leq 0.1 \\
-0.1 & \leq \frac{\sum_{i=1}^4 x_{i2}}{w_2} - \frac{\sum_{i=1}^4 x_{i3}}{w_3} \leq 0.1
\end{aligned}$$

(c) (b) is more likely to have a larger optimal value since the feasible region in (b) includes the feasible region in (a).

(d) **Decision Variables:**

x_{ij} = amount of cargo i distributed to compartment j in tons

$$y_{ij} = \begin{cases} 1 & \text{if cargo } i \text{ is shipped in compartment } j \\ 0 & \text{o.w} \end{cases}$$

Model: All constraints in (b) +

$$x_{ij} \leq w_j y_{ij} \quad i = 1, 2, 3, 4 \quad j = 1, 2, 3$$

$$\sum_{j=1}^3 y_{ij} \leq 1 \quad i = 1, 2, 3, 4$$

$$y_{ij} \in \{0, 1\} \quad i = 1, 2, 3, 4 \quad j = 1, 2, 3$$

$$x_{ij} \text{ integer} \quad i = 1, 2, 3, 4 \quad j = 1, 2, 3$$

Question 5. (a) Formulate the model to determine which trucks to use so as to minimize the cost of delivering to all the customers.

Decision Variables:

$$x_k = \begin{cases} 1 & \text{if truck } k \text{ is used } k = 1, \dots, 7 \\ 0 & \text{o.w} \end{cases}$$

$$y_{jk} = \begin{cases} 1 & \text{if customer } j \text{ is visited by truck } k \quad j = 1, \dots, 20 \quad k = 1, \dots, 7 \\ 0 & \text{o.w} \end{cases}$$

$$z_{jk} = \text{Amount of good is carried by truck } k \text{ for customer } j \\ j = 1, \dots, 20 \quad k = 1, \dots, 7$$

Model:

$$\begin{aligned}
& \min \sum_{k=1}^7 q_k x_k \\
& \text{s.t.} \quad \sum_{k=1}^7 z_{jk} = d_j & j = 1, \dots, 20 \\
& \quad \sum_{j=1}^{20} z_{jk} \leq C_k x_k & k = 1, \dots, 7 \\
& \quad z_{jk} \leq d_j y_{jk} & j = 1, \dots, 20 \quad k = 1, \dots, 7 \\
& \quad y_{jk} \leq x_k & j = 1, \dots, 20 \quad k = 1, \dots, 7 \\
& \quad x_k \in \{0, 1\} & k = 1, \dots, 7 \\
& \quad y_{jk} \in \{0, 1\} & j = 1, \dots, 20 \quad k = 1, \dots, 7 \\
& \quad z_{jk} \geq 0 & j = 1, \dots, 20 \quad k = 1, \dots, 7
\end{aligned}$$

(b) Add the following requirements to your model:

- A single truck cannot deliver to more than six customers.

$$\sum_{j=1}^{20} y_{jk} \leq 6 \quad k = 1, \dots, 7$$

- Customer 1 and customer 7 cannot be visited by the same truck.

$$y_{1k} + y_{7k} \leq 1 \quad k = 1, \dots, 7$$

- Both customer 2 and customer 12 should be visited by the same truck(s).

$$y_{2,k} = y_{12,k} \quad k = 1, \dots, 7$$

- If a truck visits customer 4 then it has to visit customer 19 or customer 20. (Assume demands of these customers do not exceed truck capacities.)

$$y_{4,k} \leq y_{19,k} + y_{20,k} \quad k = 1, \dots, 7$$

- Either at least two of the customers 5, 6 and 7 or at most three of customers 11, 12, 13 and 14 should be visited by truck 4.

$$\text{either } y_{5,4} + y_{6,4} + y_{7,4} \geq 2 \tag{1}$$

$$\text{or } y_{11,4} + y_{12,4} + y_{13,4} + y_{14,4} \leq 3 \tag{2}$$

In order to linearize the constraint, add the following decision variable:

$$w = \begin{cases} 1 & \text{if constraint 1 holds} \\ 0 & \text{if constraint 2 holds} \end{cases}$$

Then add the following constraints to the model:

$$y_{5,4} + y_{6,4} + y_{7,4} \geq 2w - (1 - w)M$$

$$y_{11,4} + y_{12,4} + y_{13,4} + y_{14,4} \leq 3(1 - w) + Mw$$

$$w \in \{0, 1\}$$

Question 6. Decision Variables:

x_i : Number of workers employed on line i $i = 1, 2$

$$y_i = \begin{cases} 1 & \text{if line } i \text{ is used } i = 1, 2 \\ 0 & \text{o.w} \end{cases}$$

Model:

$$\min 1000y_1 + 2000y_2 + 500x_1 + 900x_2$$

$$\text{s.t. } 20x_1 + 50x_2 \geq 120$$

$$30x_1 + 35x_2 \geq 150$$

$$40x_1 + 45x_2 \geq 200$$

$$x_1 \leq 7y_1$$

$$x_2 \leq 7y_2$$

$$x_1, x_2 \geq 0$$

$$y_1, y_2 \in \{0, 1\}$$