

IE 400: Principles of Engineering Management

Study Set 4

Fall 2021-2022

Question 1. Use the branch-and-bound method to find the optimal solution to the following IP:

$$\begin{aligned} \min \quad & 9x_1 + 13x_2 + 10x_3 + 8x_4 + 8x_5 \\ \text{s.t.} \quad & 6x_1 + 3x_2 + 2x_3 + 4x_4 + 7x_5 \geq 40 \\ & x_1 \leq 1, x_2 \geq 1, x_3 \geq 2, x_4 \geq 1, x_5 \leq 3 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \\ & x_1, x_2, x_3, x_4, x_5 \text{ integer} \end{aligned}$$

Question 2. Use the branch-and-bound method to find the optimal solution to the following IP:

$$\begin{aligned} \max \quad & 4x_1 + 5x_2 \\ \text{s.t.} \quad & 3x_1 + 2x_2 \leq 10 \\ & x_1 + 4x_2 \leq 11 \\ & 3x_1 + 3x_2 \leq 13 \\ & x_1, x_2 \geq 0 \\ & x_1, x_2 \text{ integer} \end{aligned}$$

Question 3. Use the branch-and-bound method to find the optimal solution to the following IP:

$$\begin{aligned} \max \quad & 40x_1 + 5x_2 + 60x_3 + 8x_4 \\ \text{s.t.} \quad & 18x_1 + 3x_2 + 20x_3 + 5x_4 \leq 25 \\ & x_1, x_2, x_3, x_4 \in \{0, 1\} \end{aligned}$$

Question 4. Solve the following IP to optimality using the branch and bound method:

$$\begin{aligned} \max \quad & 4x_1 + 10x_2 \\ \text{s.t.} \quad & 6x_1 - 2x_2 \leq 21 \\ & -4x_1 + 6x_2 \leq 9 \\ & 4x_1 + 10x_2 \leq 31 \\ & x_1, x_2 \geq 0 \text{ and integer} \end{aligned}$$

Question 5. Solve the following IP to optimality using the branch and bound method:

$$\begin{aligned} \max \quad & 15x_1 + 10x_2 + 9x_3 + 2x_4 \\ \text{s.t.} \quad & 3x_1 + 5x_2 + 3x_3 + 2x_4 \leq 14 \\ & x_i \geq 0 \text{ and integer for } i=1, 2, 3, 4 \end{aligned}$$

Question 6. Suppose we are applying branch and bound algorithm for solving a 0-1 discrete optimization problem and we have reached node 27. Assume that this node corresponds to the following problem:

$$\begin{aligned} \min \quad & c_1x_6 + c_2x_7 \\ \text{s.t.} \quad & a_1x_6 + a_2x_7 \geq 15 \\ & x_6, x_7 \in \{0, 1\} \end{aligned}$$

The current incumbent has an objective value of 34. In the following, give conditions on parameters c_1 , c_2 , a_1 and a_2 such that the statements become true. Assume all parameters are integer valued.

- (a) Node 27 is fathomed because of infeasibility.
- (b) Node 27 is fathomed because of integrality.
- (c) Node 27 is fathomed because of the bound.
- (d) Node 27 of the tree is not fathomed.

Question 7. Suppose branch and bound algorithm is applied to solve the IP and current node is 2. Assume that node 2 corresponds to the following problem:

$$\begin{aligned} \min \quad & 5x_1 + 5x_2 - 13x_3 \\ \text{s.t.} \quad & a_1x_1 + a_2x_2 + a_3x_3 \geq 9 \\ & x_1 \geq 0 \text{ and integer} \\ & x_2, x_3 \in \{0, 1\} \end{aligned}$$

Assume all parameters are integer valued. Consider each statement separately and state whether node 2 fathoms or not. Give reasonings if node 2 fathoms. You are given the constraint:

- (a) $-x_1 + 4x_2 + 5x_3 \geq 9$
- (b) $-4x_1 + x_2 - 5x_3 \geq 9$
- (c) $-6x_1 + x_2 + 8.5x_3 \geq 9$
- (d) $5x_1 + 2x_2 - x_3 \geq 9$