IE 400: Principals of Engineering Management

SET 1- Solutions to In-Class Examples

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Question 7. (**Product Mix Problem**)

Milk chocolate is produced with three ingredients. 1 kg of milk chocolate contains:

- 0.5 kg of milk
- 0.4 kg of cocoa
- 0.1 kg of sweetener

Each of these three ingredients must be processed before they can be used in the production of milk chocolate. The factory has two departments that can process these ingredients:

Ingredient	**Dept 1 (hrs/kg)**	**Dept 2 (hrs/kg)**
Milk	0.4	0.6
Cocoa	0.3	0.2
Sweetener	0.5	0.6

Both departments have 150 hours of available time per week for processing. Formulate an LP problem that maximizes the total amount of milk chocolate produced in a week.

Decision Variables:

x_{ij} : Amount of ingredient i processed by department j, $i = 1, 2, 3$
$j = 1, 2$

z : Total amount of milk chocolate produced in a week

Model:

$$
\begin{aligned}
\max\ & z \\
\text{s.t.}\ & 0.4x_{11} + 0.3x_{21} + 0.5x_{31} \leq 150 \\
& 0.6x_{12} + 0.2x_{22} + 0.6x_{32} \leq 150 \\
& x_{11} + x_{12} \geq 0.5z && (3) \\
& x_{21} + x_{22} \geq 0.4z && (4) \\
& x_{31} + x_{32} \geq 0.1z && (5) \\
& x_{ij} \geq 0 \quad i = 1, 2, 3 \quad j = 1, 2
\end{aligned}
$$

Note that constraints (3)-(4)-(5) follow as:

$$z = \min\left\{\frac{x_{11}+x_{12}}{0.5}, \frac{x_{21}+x_{22}}{0.4}, \frac{x_{31}+x_{32}}{0.1}\right\} \qquad (\star)$$

which implies:

$$\begin{aligned} z &\leq \frac{x_{11}+x_{12}}{0.5} \\ z &\leq \frac{x_{21}+x_{22}}{0.4} \\ z &\leq \frac{x_{31}+x_{32}}{0.1} \end{aligned}$$

(($\star$) cannot be added to the model as a constraint, because it not in linear form.)

Question 8. (**Error Minimization Problem**)
You estimate that there is a linear relationship between the stock price of a beverage company and the number of beverages sold (in thousands) in a day:

Day	**Stock Price**	**Number of Beverages Sold**
1	2	10
2	3	12
3	2	8
4	3	11
5	4	14

Question 8.

You predict the relationship as $s_i = an_i + b$, where s is the stock price and n_i is the number of beverages sold (in thousands) on day i, $i = 1, \ldots, 5$.

For a given choice of a and b, your estimation error on day i is $|s_i - an_i - b|$, $i = 1, \ldots, 5$. Choose the unknown coefficients a and b such that the largest estimation error in any one of the 5 days is as small as possible.

Decision Variables:

z : largest estimation error among days $1, \ldots, 5$

a : coefficient 1

b : coefficient 2

Parameters:

s_i : Stock price at day $i = 1, \ldots, 5$

n_i : number of beverages sold at day $i = 1, \ldots, 5$

Our objective is

$$\min \max_{i=1,\ldots,5} \{|s_i - an_i - b|\} = \min z$$

Model:

$$\begin{aligned} &\min\ z \\ &\text{s.t.}\ \ z \geq s_i - an_i - b && i = 1, \ldots, 5 \\ &\phantom{\text{s.t.}\ \ } z \geq -s_i + an_i + b && i = 1, \ldots, 5 \end{aligned}$$

By plugging in the values of the parameters, we can rewrite the model as:

$$
\begin{aligned}
\min\ & z \\
\text{s.t. } & z \geq 2 - 10a - b \\
& z \geq 3 - 12a - b \\
& z \geq 2 - 8a - b \\
& z \geq 3 - 11a - b \\
& z \geq 4 - 14a - b \\
& z \geq 10a + b - 2 \\
& z \geq 12a + b - 3 \\
& z \geq 8a + b - 2 \\
& z \geq 11a + b - 3 \\
& z \geq 14a + b - 4
\end{aligned}
$$

Question 9. XYZ Company produces three products A, B, and C and has to meet the demand for the next four weeks.

	Weekly Demand			
Product	**1**	**2**	**3**	**4**
A	25	30	15	22
B	15	16	25	18
C	10	17	12	12

Each unit of

- Product A requires 3 hours of labor
- Product B requires 2 hours of labor
- Product C requires 4 hours of labor

Question 9.

During a week, 120 hours of regular-time labor is available at a cost of \$10/hour. In addition 25 hours of overtime labor is available weekly at a cost of \$15/hour. No shortages are allowed and XYZ Company has 18A, 13 B, and 15 C initial stocks for these products. Provide a Linear Programming formulation to determine a production schedule that minimizes the total labor costs for the next four weeks.

Decision Variables:

x_{ij} : Amount of product i produced during week j, $i \in \{A, B, C\}$ and $j = 1, 2, 3, 4$.

I_{ij} : the amount of inventory of product i at the end of week j after meeting demand. $i \in \{A, B, C\}$ and $j = 0, 1, 2, 3, 4$.

O_j : The amount of overtime hours used in week j, $j = 1, 2, 3, 4$.

$$\min\ 10\left(\sum_{j=1}^{4}(3x_{A,1}+2x_{B,1}+4x_{C,1})\right)+5\sum_{j=1}^{4}O_j$$

$$\begin{aligned}
\text{s.t.}\quad & 18+x_{A,1}-25=I_{A,1}\\
& I_{A,1}+x_{A,2}-30=I_{A,2}\\
& I_{A,2}+x_{A,3}-15=I_{A,3}\\
& I_{A,3}+x_{A,4}-22=I_{A,4}\\
\\
& 13+x_{B,1}-15=I_{B,1}\\
& I_{B,1}+x_{B,2}-16=I_{B,2}\\
& I_{B,2}+x_{B,3}-25=I_{B,3}\\
& I_{B,3}+x_{B,4}-18=I_{B,4}
\end{aligned}$$

$$15 + x_{C,1} - 10 = I_{C,1}$$
$$I_{C,1} + x_{C,2} - 17 = I_{C,2}$$
$$I_{C,2} + x_{C,3} - 12 = I_{C,3}$$
$$I_{C,3} + x_{C,4} - 12 = I_{C,4}$$

$$3x_{A,1} + 2x_{B,1} + 4x_{C,1} \leq 120 + O_1$$
$$3x_{A,2} + 2x_{B,2} + 4x_{C,2} \leq 120 + O_2$$
$$3x_{A,3} + 2x_{B,3} + 4x_{C,3} \leq 120 + O_3$$
$$3x_{A,4} + 2x_{B,4} + 4x_{C,4} \leq 120 + O_4$$
$$O_j \leq 25 \quad j = 1, 2, 3, 4$$

$$I_{ij} \geq 0 \quad i \in \{A, B, C\} \quad j = 1, 2, 3, 4$$
$$x_{ij} \geq 0 \quad i \in \{A, B, C\} \quad j = 1, 2, 3, 4$$
$$O_j \geq 0 \quad j = 1, 2, 3, 4$$