IE 400: Principals of Engineering Management

SET 1- Solutions to In-Class Examples

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Question 1. (**Milk Production**) There are two machines needed to process milk to produce low-fat and regular milk. The processing times are:

	(Pasteurization) Machine 1	**(Homogenization) Machine 2**
Low-fat	3 min/lt	1 min/lt
Regular	2 min/lt	2 min/lt

In a day, machine 1 can be used for 12 hours and machine 2 can be used for 6 hours. The sales price of low-fat milk is 2.5 TL/lt and that of regular milk is 1.5 TL/lt. Find how much low-fat and regular milk should be produced in a day to maximize total revenue.

Decision Variables:

x_1 : amount in liters of low-fat milk produced in a day

x_2 : amount in liters of regular milk produced in a day

Model:

$$\begin{aligned} \max \ & 2.5x_1 + 1.5x_2 \\ \text{s.t.} \ & 3x_1 + 2x_2 \leq 720 \\ & x_1 + 2x_2 \leq 360 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \end{aligned}$$

Question 2. (**Blending Problem 1**)
An oil refinery distills crude oil from SaudiArabia and Venezuela into gasoline, jet fuel and lubricants. The crude differ in chemical composition and yields

- 1 ton of S.A. crude oil $\Longrightarrow$ 0.3 tons of gasoline, 0.4 tons of jet fuel, 0.2 tons of lubricants
- 1 ton of Ven. crude oil $\Longrightarrow$ 0.4 tons of gasoline, 0.2 tons of jet fuel, 0.3 tons of lubricants

Remaining 0.1 tons is lost in refining.
At most 90 tons can be purchased daily from S.A. at a cost of \$600/ton. At most 60 tons can be purchased daily from Ven. at a cost of \$550/ton
The daily demand is 20 tons of gasoline, 15 tons of jet fuel, and 5 tons of lubricants.
How should the demand be met at a **minimum total cost**?

Decision Variable:

x_i : Amount of crude oil purchased from country i,
$i = 1, 2$. (1: S.A., 2: Venezuella)

Model:

$$
\begin{aligned}
\min \quad & 600x_1 + 550x_2 \\
\text{s.t.} \quad & 0.3x_1 + 0.4x_2 \geq 20 \\
& 0.4x_1 + 0.2x_2 \geq 15 \\
& 0.2x_1 + 0.3x_2 \geq 5 \\
& x_1 \leq 90 \\
& x_2 \leq 60 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

Question 3. (**Transportation Problem**)
There are two power plants generating electricity, and three factories that need electricity in their production. power plant 1 has 1250 MWatts capacity and power plant 2 has 2000 MWatts capacity. The demands of factories 1,2 and 3 are 1500 MWatts, 750 MWatts and 750 MWatts respectively. The transmission costs are given as follows:

Table: Transmission costs per MWatt in USD

	F1	**F2**	**F3**
PP1	50	100	60
PP2	30	20	35

How to satisfy the demand of each factory from the two power plants with minimum total cost?

Decision Variable:

x_{ij} : Electricity provided in Mwatts from power plant i to factory j, $i = 1, 2$, $j = 1, 2, 3$

Model:

$$
\begin{aligned}
\min \quad & 50x_{11} + 100x_{12} + 60x_{13} + 30x_{21} + 20x_{22} + 35x_{23} \\
\text{s.t.} \quad & x_{11} + x_{12} + x_{13} \leq 1250 \\
& x_{21} + x_{22} + x_{23} \leq 2000 \\
& x_{11} + x_{21} \geq 1500 \\
& x_{12} + x_{22} \geq 750 \\
& x_{13} + x_{23} \geq 750 \\
& x_{ij} \geq 0 \quad i = 1, 2 \;\; j = 1, 2, 3
\end{aligned}
$$

Question 4. (**General Transportation Problem**)
Say there are m warehouses and n stores.
Let a_i denote the amount of stock at warehouse i for $i = 1, \ldots, m$.
Let d_j denote the demand of store j for $j = 1, \ldots, n$.
Let c_{ij} denote the cost of transporting a unit of product from warehouse i to store j, for $i = 1, \ldots, m$ and $j = 1, \ldots, n$.
How should the demands of stores be met at a minimum total transportation cost while respecting the stock limitations of the warehouses?

Decision Variable:

x_{ij} : Amount of product transported from warehouse i to store j, $i = 1, \ldots m$ $j = 1, \ldots, n$

Model:

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\text{s.t.} \sum_{j=1}^{n} x_{ij} \leq a_i \qquad i = 1, \ldots m$$

$$\sum_{i=1}^{m} x_{ij} \geq d_j \qquad j = 1, \ldots, n$$

$$x_{ij} \geq 0 \qquad i = 1, \ldots m \; j = 1, \ldots, n$$

Question 5. (Blending Problem 2)
OJ Inc. produces orange juice from 3 different grades of Oranges. Oranges are graded 1(poor), 2(medium), and 3(good) depending on their quality.

Grade	Yield (Juice lt/kg)
1	0.4
2	0.5
3	0.6

The company currently has

- 100 kgs of Grade 1 oranges
- 150 kgs of Grade 2 oranges
- 200 kgs of Grade 3 oranges

Question 5.

OJ Inc produces 3 types of orange juices from these oranges: Superior, premium and regular. The production requires are as follows:

Type	Minimum Average Grade	Profits per Liter	Minimum Daily Production
Superior	2.4	1.5 TL	45 lt
Premium	2.2	1 TL	60 lt
Regular	2	0.75 TL	100 lt

Assume the company may sell more than demand at the same prices. How can the company maximize total profit while satisfying production requirements?

Decision Variable:

x_{ij} : Liters of Orange Juice type i produced using oranges of grade j respectively,

$i = 1, 2, 3$ (1: superior, 2: premium, 3:regular), $j = 1, 2, 3$

Model:

$$\begin{aligned}
\max\ & 1.5(x_{11} + x_{12} + x_{13}) + (x_{21} + x_{22} + x_{23}) + 0.75(x_{31} + x_{32} + x_{33}) \\
\text{s.t.}\ & x_{11} + x_{21} + x_{31} \leq 40 \quad (100 \times 0.4) \\
& x_{12} + x_{22} + x_{32} \leq 75 \quad (150 \times 0.5) \\
& x_{13} + x_{23} + x_{33} \leq 120 \quad (200 \times 0.6) \\
& x_{11} + x_{12} + x_{13} \geq 45 \\
& x_{21} + x_{22} + x_{23} \geq 60 \\
& x_{31} + x_{32} + x_{33} \geq 100 \\
& x_{11} + 2x_{12} + 3x_{13} \geq 2.4(x_{11} + x_{12} + x_{13}) \\
& x_{21} + 2x_{22} + 3x_{23} \geq 2.2(x_{21} + x_{22} + x_{23}) \\
& x_{31} + 2x_{32} + 3x_{33} \geq 2(x_{31} + x_{32} + x_{33}) \\
& x_{ij} \geq 0 \quad i = 1, 2, 3 \quad j = 1, 2, 3
\end{aligned}$$

Question 6. (**Short Term Investment Planning**)
You have 12,500 TL at the beginning of year 2018 and would like to invest your money. Your goal is to maximize your total amount of money at the beginning of year 2021. You have the following options:

Option	Duration (years)	Total Interest Rate	Available at the Beginning of
1	2	26%	2018,2019
2	1	12%	2018,2019,2020
3	3	38%	2018
4	2	27%	2019

Decision Variables:

x_t : Amount of money invested in option 1 in the beginning of year t, $t = 0, 1$ (0: 2018,1: 2019)

y_t : Amount of money invested in option 2 in the beginning of year t, $t = 0, 1, 2$ (0: 2018,1: 2019,2: 2020)

z : Amount of money invested in option 3 in the beginning of year 0.

t : Amount of money invested in option 4 in the beginning of year 1.

Model:

$$
\begin{aligned}
\max\ & 1.26x_1 + 1.12y_2 + 1.38z + 1.27t \\
\text{s.t.}\ & x_0 + y_0 + z = 12500 \\
& x_1 + y_1 + t = 1.12y_0 \\
& y_2 = 1.12y_1 + 1.26x_0 \\
& x_t \geq 0 \qquad\qquad t = 0, 1 \\
& y_t \geq 0 \qquad\qquad t = 0, 1, 2 \\
& z \geq 0 \\
& t \geq 0
\end{aligned}
$$