IE 400: Principles of Engineering Management

In-Class Example Solutions

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1 Work Scheduling (page:13)

The mathematical formulation of the problem is given as follows:

Decision Variables

x_i : number of workers who are NOT working on day i, i = 1,..., 7 (1: Mon, 7: Sun)

Objective is to minimize total number of of workers:

$$
\begin{aligned}
\min \quad & x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 && (1)\\
s.t \quad & x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \geq 8 && (2)\\
& x_1 + x_3 + x_4 + x_5 + x_6 + x_7 \geq 6 && (3)\\
& x_1 + x_2 + x_4 + x_5 + x_6 + x_7 \geq 7 && (4)\\
& x_1 + x_2 + x_3 + x_5 + x_6 + x_7 \geq 6 && (5)\\
& x_1 + x_2 + x_3 + x_4 + x_6 + x_7 \geq 9 && (6)\\
& x_1 + x_2 + x_3 + x_4 + x_5 + x_7 \geq 11 && (7)\\
& x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \geq 9 && (8)\\
& x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0 && (9)\\
& x_i = integer \qquad \forall i = 1, \ldots, 7 && (10)
\end{aligned}
$$

2 The 0-1 Knapsack Problem (page:19)

The mathematical formulation of the problem is given as follows:

Decision Variables

$$
x_i = \begin{cases} 1, & \text{if item i is packed into the suitcase} \\ 0, & \text{otherwise} \end{cases}
$$

Objective is to maximize total utility:

$$
\begin{aligned}
&\max && \sum_{i=1}^{n} c_i x_i && \qquad (11)\\
&s.t && \sum_{i=1}^{n} a_i x_i \leq b && \qquad (12)\\
& && x_i \in \{0,1\} \qquad \forall i = 1,\ldots,n && \qquad (13)
\end{aligned}
$$

3 Workforce Scheduling (page:22)

The mathematical formulation of the problem is given as follows:

Decision Variables

w_t : number of workers working during time period t, t = 0,...,4

f_t : number of workers fired at the beginning of time period t, t = 1,...,4

h_t : number of workers hired at the beginning of time period t, t = 1,...,4

$\imath_t$: number of pairs of shoes in the inventory at the end of time period t, t = 0,...,4

p_t : number of pairs of shoes produced during time period t, t = 1,...,4

Objective is to find a work schedule that meets the total demand at a minimum cost:

$$
\begin{aligned}
&\min && \sum_{t=1}^{4} 5p_t + 30\imath_t + 1500w_t + 2000f_t + 1600h_t && && (14)\\
&s.t && w_0 \geq 3 && && (15)\\
& && \imath_0 = 50 && && (16)\\
& && \imath_{t-1} + p_t = d_t + \imath_t && \forall t = 1,\ldots,4 && (17)\\
& && w_t = w_{t-1} + h_t - f_t && \forall t = 1,\ldots,4 && (18)\\
& && p_t - 40w_t \leq 0 && \forall t = 1,\ldots,4 && (19)\\
& && w_t, f_t, h_t, f_t, \imath_t, p_t \geq 0 && \forall t = 1,\ldots,4 && (20)\\
& && w_t, f_t, h_t, f_t, \imath_t, p_t = integer && \forall t = 1,\ldots,4 && (21)
\end{aligned}
$$

4 Modeling Fixed Costs (page:68)

The mathematical formulation of the problem is given as follows:

Decision Variables

$$y_i = \begin{cases} 1, & \text{if a potential warehouse i is opened i=1,...,m} \\ 0, & \text{otherwise} \end{cases}$$

$x_{ij} =$ Number of units supply from warehouse i to client j, i=1,...,m, j=1,...,n

Objective is order to minimize total fixed costs and transportation costs while meeting the demand and capacity constraints:

$$\min \quad \sum_{i=1}^{m} f_i y_i + \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{22}$$

$$s.t \quad \sum_{i=1}^{m} x_{ij} \geq d_j \qquad \forall j = 1, \ldots, n \tag{23}$$

$$\sum_{j=1}^{n} x_{ij} \leq a_i y_i \qquad \forall i = 1, \ldots, m \tag{24}$$

$$y_i \in \{0, 1\} \qquad \forall i = 1, \ldots, m \tag{25}$$

$$x_{ij} \geq 0 \qquad \forall i = 1, \ldots, m, \forall j = 1, \ldots, n \tag{26}$$