IE 400: Principles of Engineering Management
Study Set 3

Spring 2022

Question 1. Identify the basic solutions and basic feasible solutions in the following linear system.

$$\begin{aligned} & x_1 - x_2 + 3x_3 + 5x_4 = 15 \\ & x_1 + x_2 + x_4 = 9 \\ & x_i \geq 0 \quad i = 1, \cdots, 4 \end{aligned}$$

Question 2. Use the simplex algorithm to find two optimal solutions to the following LP:

$$\begin{aligned} \max \ & 5x_1 + 3x_2 + x_3 \\ \text{s.t.} \ & x_1 + x_2 + 3x_3 \leq 6 \\ & 5x_1 + 3x_2 + 6x_3 \leq 15 \\ & x_i \geq 0 \ i = 1, 2, 3 \end{aligned}$$

Try to solve the same question with x_3 as urs.

Question 3. Consider the following LP:

$$\begin{aligned} \max \ & 10x_1 + 8x_2 - 3x_3 + 3x_4 \\ \text{s.t.} \ & 2x_1 + 4x_2 - 0.5x_3 + 0.5x_4 \leq 3 \\ & -2x_1 + 6x_2 - 4.5x_3 + 4.5x_4 \leq 7 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

Convert the LP in standard form. Then construct the tableau where x_1 is the basic variable in the first constraint and x_4 is the basic variable in the second constraint (order is important). Then, answer the following

(a) Is this a basic solution? Why?

(b) Is this a basic feasible solution? Why?

(c) Is this an optimal basic feasible solution? Why?

Question 4. Reatix must determine how much investment and debt to undertake during the next year. Each TL invested reduces the net present value (NPV) of the company by 10kr, and each TL of debt increases the NPV by 50kr (due to deductibility of interest payments). Reatix can invest at most 1 million TL during the coming year. Debt can be at most 40% of the investment. Reatix now has 800,000 TL in cash available. All investment must be paid from current cash or borrowed money. Set up an LP that maximizes the NPV of Reatix. Then, solve this LP by **simplex method**.

Question 5. Consider the following simplex tableau of a given **minimization** LP problem.

	z	x_1	x_2	x_3	s_1	s_2	s_3	*RHS*
z	1	0	0	**A**	-10	-1	**B**	32
s_3	0	0	0	**C**	4	-2	1	**D**
x_2	0	0	1	-2	**F**	-1	0	6
x_1	0	**E**	0	-1	1	2	0	4

Give **general** conditions on each of the unknowns **A-F** such that each of the following statements is true. Even if the statement holds independently of the values of a specific variable, you should still mention that variable.

(a) The tableau is final and there exists a unique optimal solution.

(b) The simplex method determines an unbounded solution from this tableau.

(c) The current bfs is degenerate (not necessarily optimal).

(d) The current solution is optimal, there are alternative optimal solutions but no alternative optimal bfs.

Question 6. Use the two-phase method to find the optimal solution to the following LP:

$$\begin{aligned} \max\ & -3x_1 + 4x_2 \\ \text{s.t.}\ & x_1 + x_2 \leq 4 \\ & 2x_1 + 3x_2 \geq 18 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Question 7. Convert the following LP to standard form and solve with the Big-M method.

$$\begin{aligned} \max\ & -x_1 + 3x_2 + x_3 \\ \text{s.t.}\ & -x_1 + x_2 - x_3 \geq -4 \\ & 2x_1 + x_2 + x_3 \geq 10 \\ & x_1 \geq 0 \\ & x_2 \leq 0 \\ & x_3 \text{ free} \end{aligned}$$