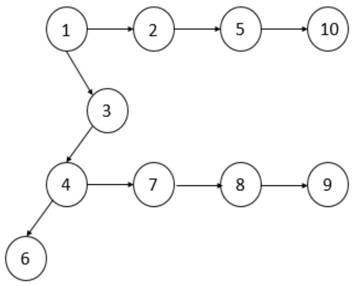
IE 400: Principles of Engineering Management
Study Set 5 Solutions

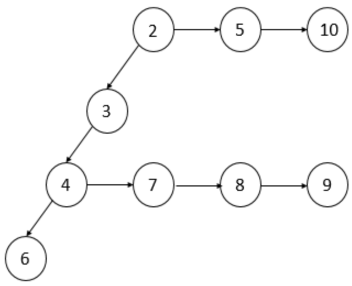
Spring 2022

Question 1.

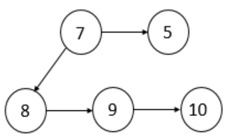
(a) From 1 to 10 (Bob): Shortest path: 1-2-5-10, length=20



From 2 to 10 (Ed): Shortest path: 2-5-10, length=17



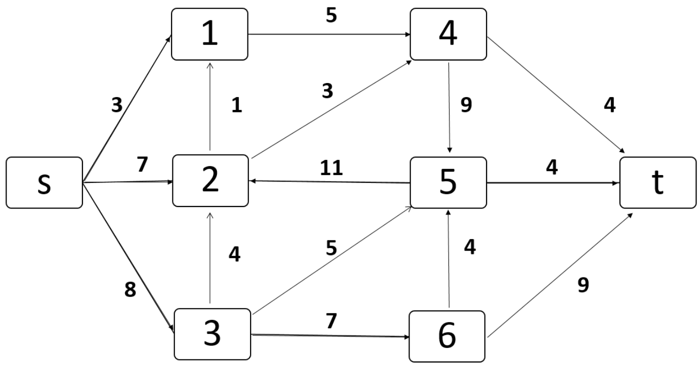
From 7 to 10 (Stu): Shortest path: 7-8-9-10, length=8



(b) Bob should leave 20 minutes earlier than the meeting time, Ed 17 minutes, and Stu 8 minutes.

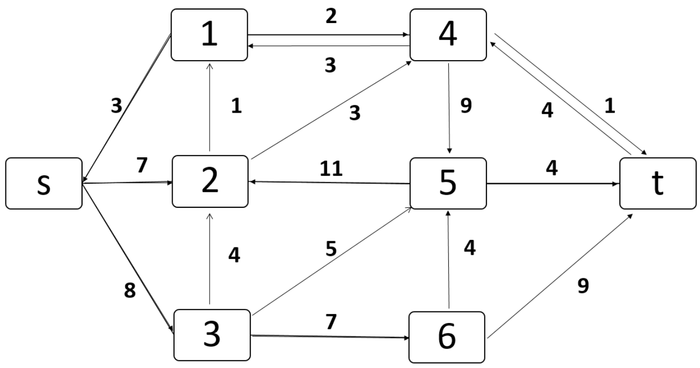
(c) Bob can give a ride to Ed since he is on his shortest path.

Question 2. Consider costs as capacities. Using Ford - Fulkerson algorithm find max-flow.



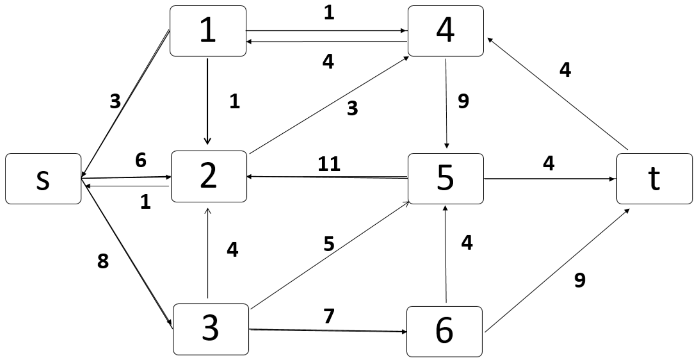
Step 1: Choose path s - 1 - 4 - t

min{3, 5, 4}= 3



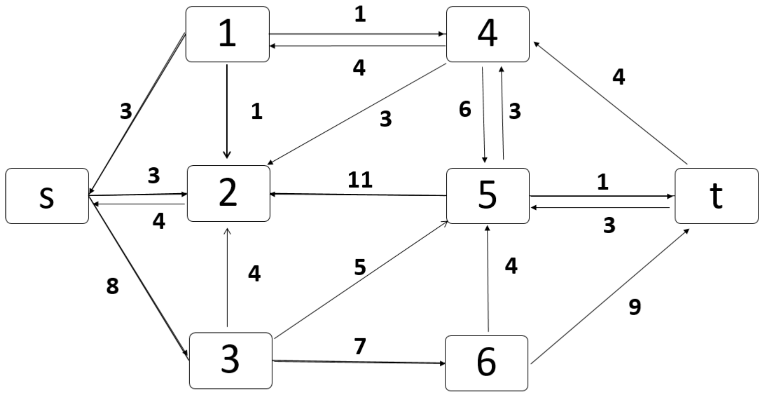
Step 2: Choose path s - 2 - 1 - 4 - t

min{7, 1, 2, 1}= 1



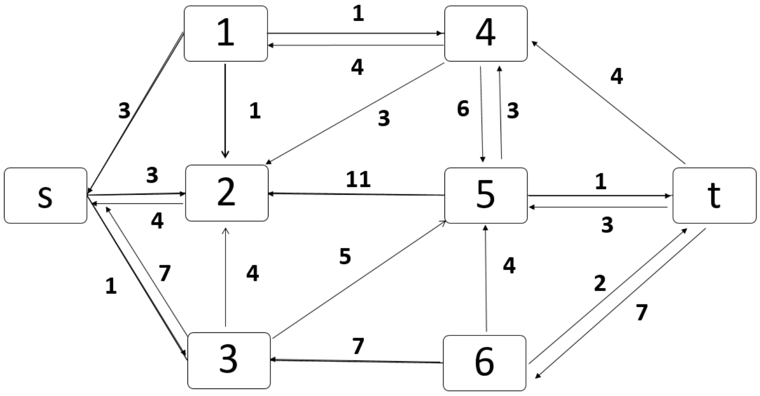
Step 3: Choose path s - 2 - 4 - 5 - t

min{6, 3, 9, 4}= 3



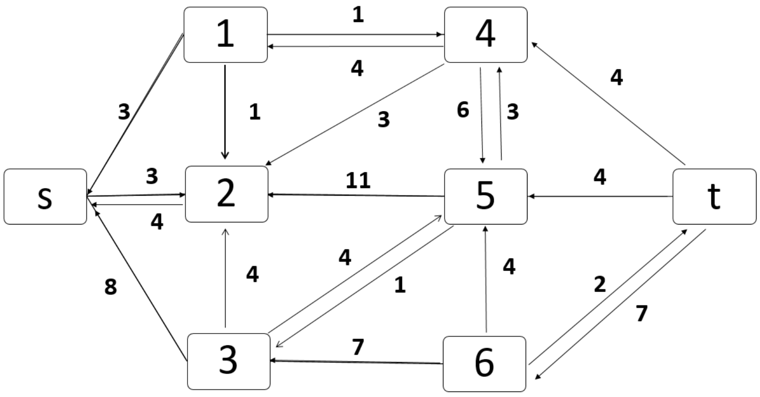
Step 4: Choose path s - 3 - 6 - t

min{8, 7, 9}= 7



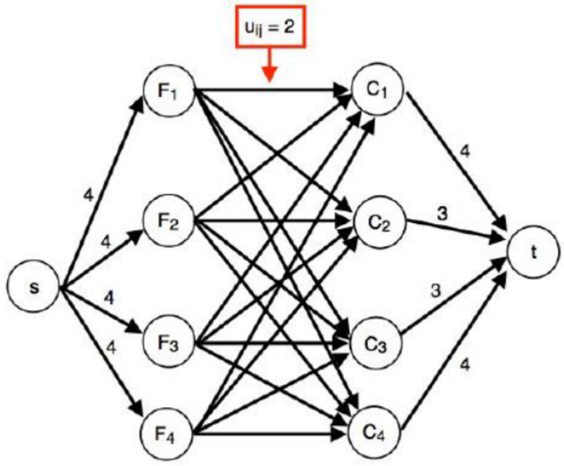
Step 5: Choose path s - 3 - 5 - t

min{1, 5, 1}= 1



There is no other path from s to t.
Since max-flow = min-cut in optimal solution. The minimum cost to block flood's way to city is 15.

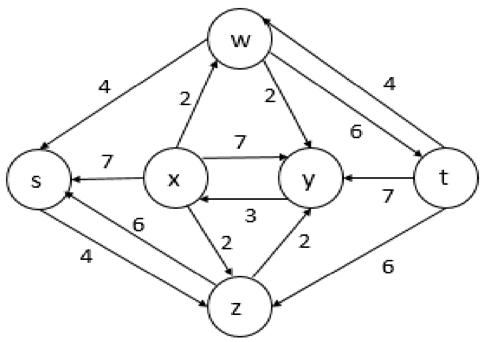
Question 3. The problem of transporting the maximum possible number of people to the picnic can be formulated as a maximum-flow problem on the following network.



Question 4.

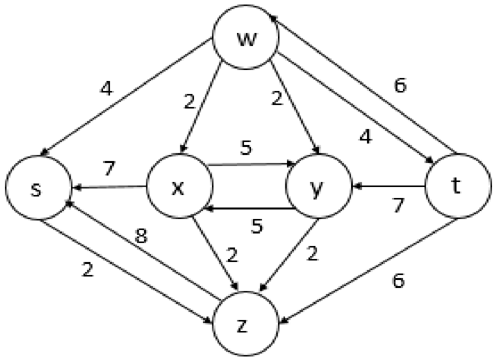
(a) The value of this flow is 17. (The total amount of flow passing from s to t)

(b) No, it is not a maximum flow. We can find a maximum flow by looking at the residual network. In this case, from the given flow, we can get a max flow with only a single augmenting path, as shown below.
First residual network for the given flow network:



Augment flow along path s-z-y-x-w-t. Capacity of this path is 2, so send 2 units of flow on this path.

New residual network:

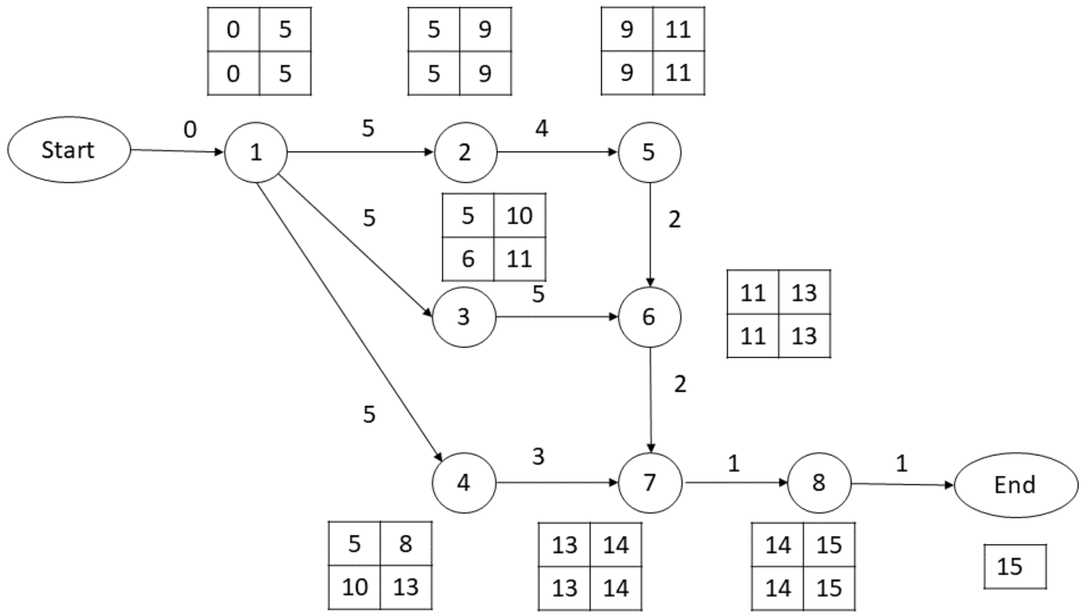


So, the maximum flow has value 19.

(c) We can find a minimum cut using the residual network for the final maximum flow. Starting at s, find all vertices that are reachable from s in the residual network. this forms one set in the minimum cut. The other vertices form the other set part of the cut. So, a minimum cut in this case are the two sets $S^* = \{s, z\}$ and $T^* = \{x, w, y, t\}$.

Question 5.

Activity	Predecessors	Duration
1 (Design)	-	5
2 (make A)	1	4
3 (make B)	1	5
4 (make C)	1	3
5 (test A)	2	2
6 (assembly A&B)	3, 5	2
7 (attach C)	6, 4	1
8 (testing)	7	1


0 5
0 5
5 9
5 9
9 11
9 11
Start
0
1
5
2
4
5
5
5 10
6 11
2
11 13
11 13
3
5
6
2
5
4
3
7
1
8
1
End
5 8
10 13
13 14
13 14
14 15
14 15
15

Critical path: 1 - 2 - 5 - 6 - 7 - 8

Duration: 15

Activity	1	2	3	4	5	6	7	8
Slack	0	0	1	5	0	0	0	0

For the LP define decision variable x_j: start time of activity $j, j = 1, ..., 10, \text{start, end}$

$$
\begin{aligned}
\min \quad & x_{end} \\
\\
\text{s.t.} \quad & x_1 \geq x_{start} \\
& x_2 \geq x_1 + 5 \\
& x_3 \geq x_1 + 5 \\
& x_4 \geq x_1 + 5 \\
& x_5 \geq x_2 + 4 \\
& x_6 \geq x_3 + 5 \\
& x_6 \geq x_5 + 2 \\
& x_7 \geq x_6 + 2 \\
& x_7 \geq x_4 + 3 \\
& x_8 \geq x_7 + 1 \\
& x_{end} \geq x_8 + 1 \\
& x_{start} = 0
\end{aligned}
$$

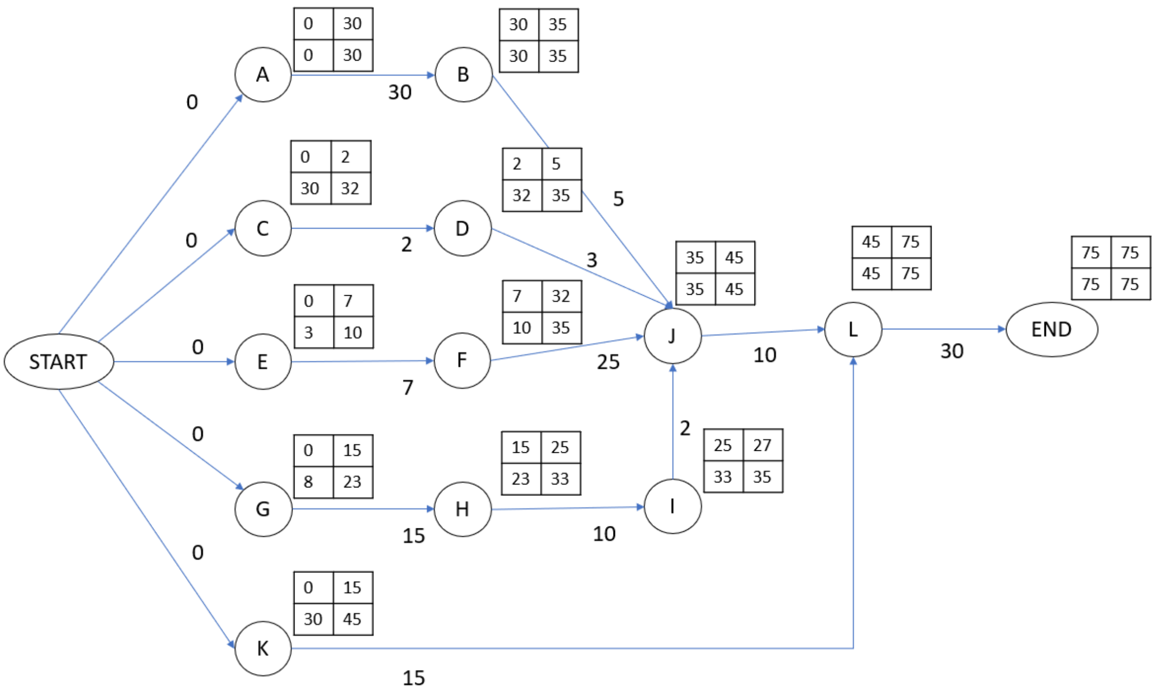
The tight constraints of the optimal solution for this LP, which hold at equality, will give you the critical path.

Question 6.

ES: Earliest Start, EF: Earliest Finish, LS: Latest Start, LF: Latest Finish

ES	EF
LS	LF

(a)-(b)



Activity	Slack
A	0
B	0
C	30
D	30
E	3
F	3
G	8
H	8
I	8
J	0
K	30
L	0

(c) Since the slack for cutting the onions and mushrooms is 3, the dinner will be delayed 6-3=3 minutes.

If the cutting time is 2 minutes, slack is 10 (latest finish time) – 2 (task duration) = 8 minutes. Since the slack time is more than the delay time, it does not delay the dinner.

(d) Let N be the set of activities and A be the set of arc in the project network.

Parameters:

a_i : Duration of activity i, for all i $\in N$

d : Project duration (75)

Decision Variables:

x_i : Start time of activity i, for all $i \in N$

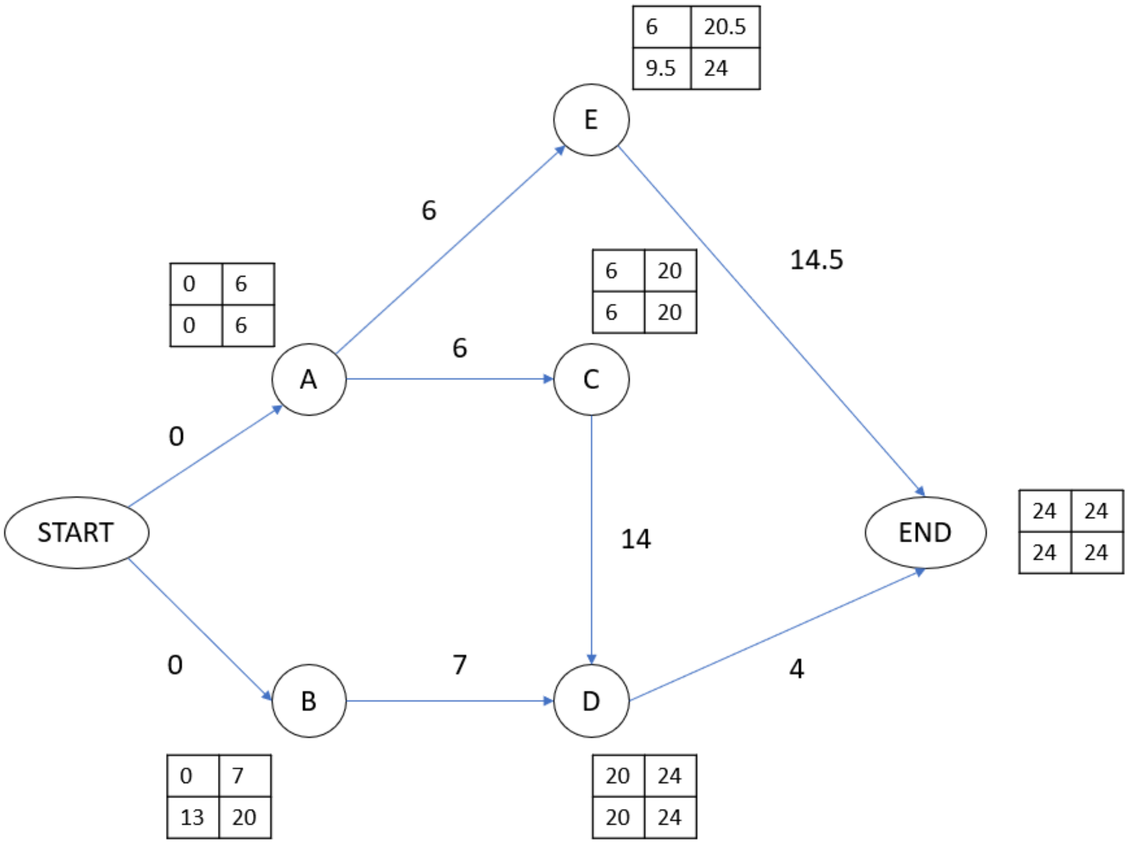
Model:

$$
\begin{aligned}
\max \ & \sum_{i \in N} x_i \\
\text{s.t.} \ & x_j \geq x_i + a_i && (i,j) \in A \\
& x_{end} \leq d \\
& x_i \geq 0 && i \in N \\
& x_{start} = 0
\end{aligned}
$$

Question 7.

(a)

Activity	**Mean**	**Variance**
A	6	4/36
B	7	196/36
C	14	256/36
D	4	4/36
E	14.5	169/36



The critical path is A-C-D.

(b) Note that

$$\mathbb{P}(18 \leq t \leq 26) = \mathbb{P}(t \leq 26) - \mathbb{P}(t \leq 18)$$

We have, $\sigma_p = 2.708$

Note that $Z_{18} = \frac{18-24}{2.708} = -2.2156$

Then, $\mathbb{P}(t \leq 18) = \mathbb{P}(Z \leq -2.2156) = 0.013$

$Z_{26} = \frac{26-24}{2.708} = 0.739$

$\mathbb{P}(t \leq 26) = \mathbb{P}(Z \leq 0.739) = 0.770$

Hence,

$$\mathbb{P}(18 \leq t \leq 26) = \mathbb{P}(t \leq 26) - \mathbb{P}(t \leq 18) = 0.757$$