# IE 400: Principles of Engineering Management
# Study Set 2

FALL 2022-2023

## 1 Graphical Solution

**Question 1.** For the given LP below:

$$\begin{aligned} \min\ & 2x + y \\ \text{s.t. } & x + 2y \geq -4 \\ & -3x + y \leq 6 \\ & x \geq 0 \\ & y \text{ free} \end{aligned}$$

(a) Draw the feasible region.

(b) What is the optimum solution and optimum solution value?

**Question 2.** Consider the following linear programming problem:

$$\begin{aligned} \min\ & c_1 x + c_2 y \\ \text{s.t. } & x - 2y \leq 4 \\ & -x + y \leq 3 \\ & x \geq 0 \\ & y \geq 0 \end{aligned} \qquad \text{(LP1)}$$

Find coefficients c1 and c2 such that LP1 will have a unique optimal solution.

**Question 3.** There are 2 decision makers (DMs) with the following objectives:

$$\begin{aligned} &\max\ -x + 2y && \text{(DM 1)} \\ &\max\ 4x + 2y && \text{(DM 2)} \end{aligned}$$

and the feasible region is the following:

$$\begin{aligned} \text{s.t. } & x + 2y \leq 12 \\ & -x + y \leq 4 \\ & 4x - 3y \leq 26 \\ & x \geq 0 \\ & y \text{ urs} \end{aligned}$$

(a) Draw the feasible region.

(b) Find the optimal solution and optimal solution value for each DM.

**Question 4.** For the given LP below:

$$\begin{aligned} \min\ & ax + by \\ \text{s.t. } & x + 2y \geq -4 \\ & -3x + y \leq 6 \\ & x \geq 0 \\ & y \text{ urs} \end{aligned}$$

(a) Draw the feasible region.

(b) Give an example for a and b such that LP will have a unique optimal solution. For the chosen a and b, what is the optimal solution and optimal solution value?

## 2 Simplex Method

**Question 5.** Identify the basic solutions and basic feasible solutions in the following linear system.

$$\begin{aligned} & x_1 - x_2 + 3x_3 + 5x_4 = 15 \\ & x_1 + x_2 + x_4 = 9 \\ & x_i \geq 0 \quad i = 1, \cdots, 4 \end{aligned}$$

Try to solve the same question with $x_3$ as urs.

**Question 6.** Use the simplex algorithm to find two optimal solutions to the following LP:

$$\begin{aligned} \max \ & 5x_1 + 3x_2 + x_3 \\ \text{s.t.} \ & x_1 + x_2 + 3x_3 \leq 6 \\ & 5x_1 + 3x_2 + 6x_3 \leq 15 \\ & x_i \geq 0 \ i = 1, 2, 3 \end{aligned}$$

Try to solve the same question with $x_3$ as urs.

**Question 7.** Consider the following problem:

$$\begin{aligned} \max \ & 2x_1 + x_2 + 6x_3 - 4x_4 \\ \text{s.t.} \ & x_1 + 2x_2 + 4x_3 - x_4 \leq 6 \\ & 2x_1 + 3x_2 - x_3 + x_4 \leq 12 \\ & x_1 + x_3 + x_4 \leq 6 \\ & x_i \geq 0 \ i = 1, 2, 3, 4 \end{aligned}$$

Convert this LP to standard form. Construct the simplex tableau for the basis: $x_1$, $x_2$ and $x_4$. Indicate whether the tableau is optimal, if not state the entering and the leaving variables. You do not need to perform further iterations.

**Question 8.** Consider the following LP:

$$\begin{aligned} \max\ & 10x_1 + 8x_2 - 3x_3 + 3x_4 \\ \text{s.t. } & 2x_1 + 4x_2 - 0.5x_3 + 0.5x_4 \leq 3 \\ & -2x_1 + 6x_2 - 4.5x_3 + 4.5x_4 \leq 7 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

Convert the LP into standard form. Then construct the tableau where $x_1$ is the basic variable in the first constraint and $x_4$ is the basic variable in the second constraint (order is important). Then, answer the following:

(a) Is this a basic solution? Why?

(b) Is this a basic feasible solution? Why?

(c) Is this an optimal basic feasible solution? Why?