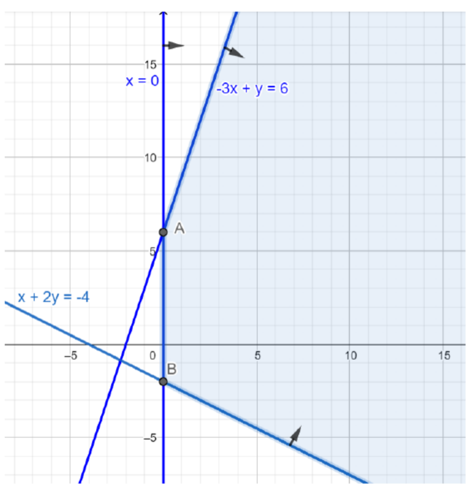
IE 400: Principles of Engineering Management
Study Set 2 Solutions

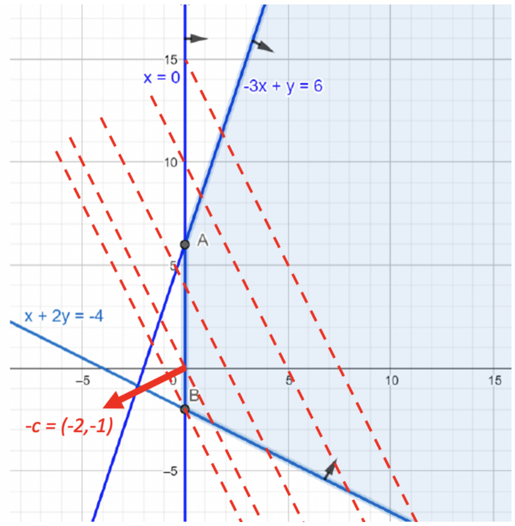
FALL 2022-2023

Question 1.

(a)

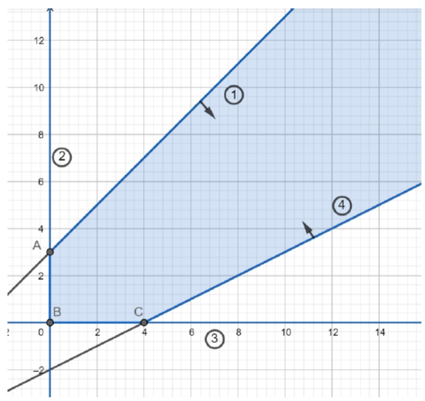


(b) For the given LP, $c = (2, 1)$ and since it is a minimization problem the isocost lines drawn will be perpendicular to the vector $-c = (-2, -1)$ which is the improving direction. Once the lines are drawn, we observe that the lines leave the feasible region at the corner point $B = [0, -2]$

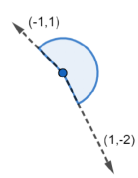


Hence, $B = [0, -2]$ is the unique optimal solution and the optimal value becomes $2 \times 0 + 1 \times (-2) = -2$.

Question 2.

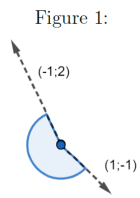


Consider **LP1** (minimization problem):
The feasible region is drawn above. $-c = (-c_1, -c_2)$ should be in the following shaded region for LP1 to be unbounded:

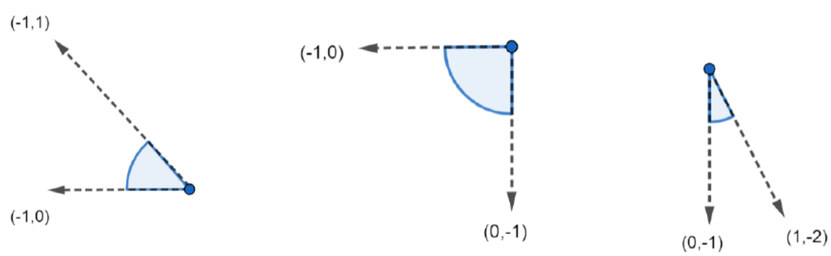


Then c lies in the following shaded region (each vector above is multiplied by -1 for transforming $-c \to c$):

Figure 1:

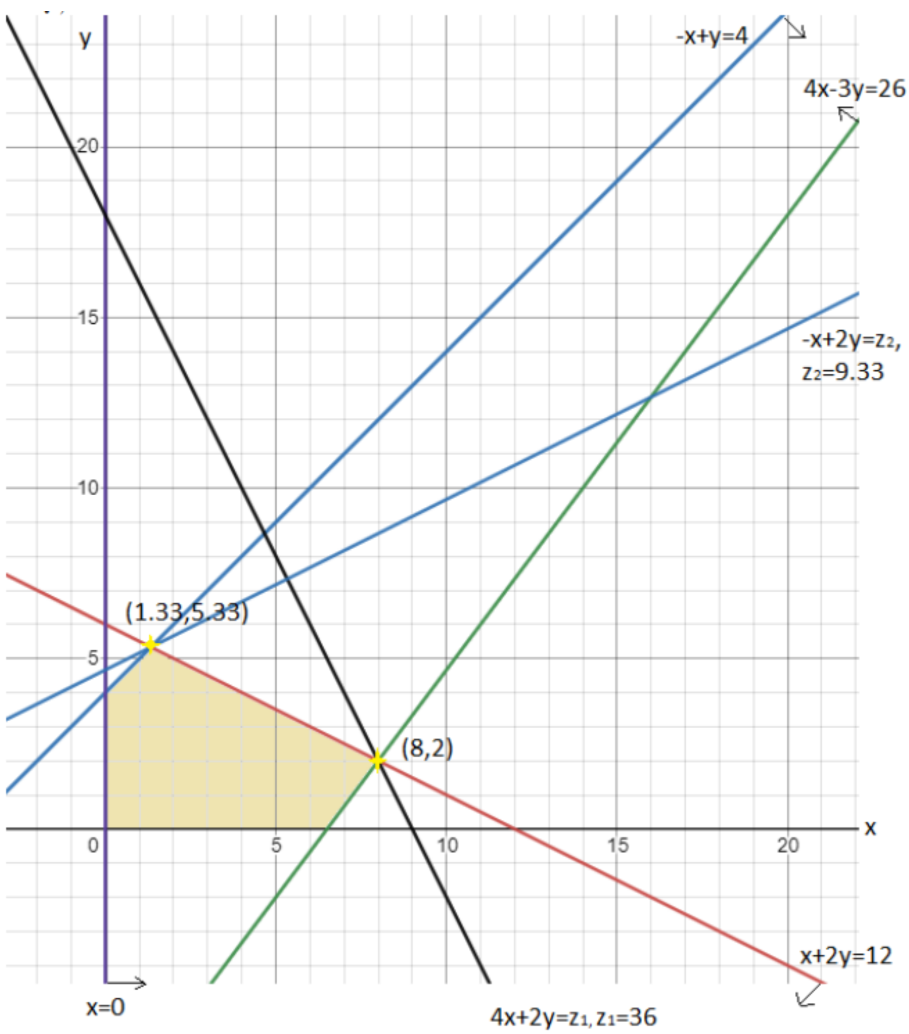


LP1 has unique optimal solution, if $-c$ is in one of the following three regions that make points A,B and C unique optimal solution respectively:

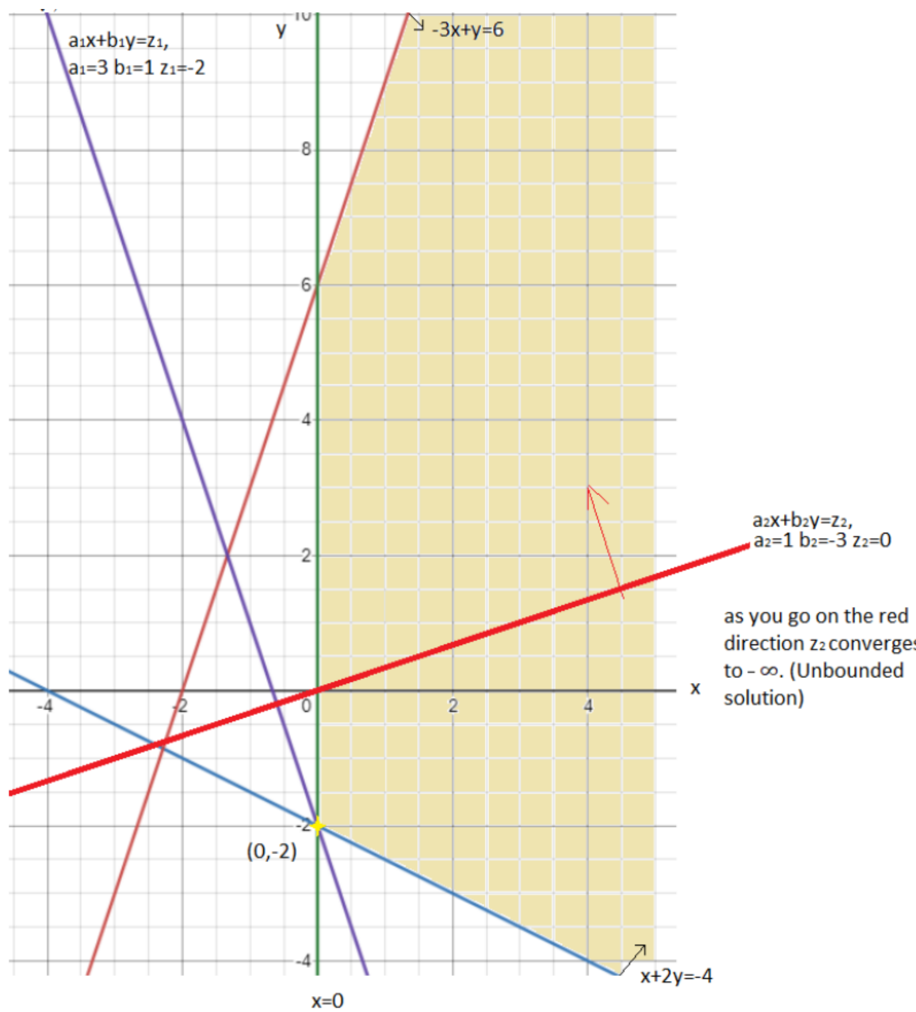


- Take $c = (1, -1/2)$. Then, $-c = (-1, 1/2)$. Then A becomes the unique solution for LP1.
- Take $c = (1, 1)$. Then, $-c = (-1, -1)$. This makes B the unique solution for LP1.
- Take $c = (-1, 3)$. Then, $-c = (1, -3)$. Then C becomes the unique solution for LP1.

Question 3.



Question 4.


a1x+b1y=z1,
a1=3 b1=1 z1=-2
y
-3x+y=6
8
6
4
2
-4
-2
0
2
4
x
-2
(0,-2)
-4
x=0
x+2y=-4
a2x+b2y=z2,
a2=1 b2=-3 z2=0
as you go on the red
direction z2 converges
to – ∞. (Unbounded
solution)

Question 5.

- x_1, x_2 basic,
  x_3, x_4 nonbasic ($x_3 = 0,\ x_4 = 0$)
  Then,

$$\begin{aligned} x_1 - x_2 &= 15 \\ x_1 + x_2 &= 9 \end{aligned}$$

  Solving this system, we obtain $x = (12, -3, 0, 0)$. This is basic but not feasible since $x_2 = -3 < 0$.

- x_1, x_3 basic,
  x_2, x_4 nonbasic ($x_2 = 0,\ x_4 = 0$)
  Then,

$$\begin{aligned} x_1 + 3x_3 &= 15 \\ x_1 &= 9 \end{aligned}$$

  Solving this system, we obtain $x = (9, 0, 2, 0)$. This is a basic feasible solution.

- x_1, x_4 basic,
  x_2, x_3 nonbasic ($x_2 = 0,\ x_3 = 0$)
  Then,

$$\begin{aligned} x_1 + 5x_4 &= 15 \\ x_1 + x_4 &= 9 \end{aligned}$$

  Solving this system, we obtain $x = (15/2, 0, 0, 3/2)$. This is a basic feasible solution.

- x_2, x_3 basic,
  x_1, x_4 nonbasic $(x_1 = 0,\ x_4 = 0)$
  Then,

  $$\begin{aligned} -x_2 + 3x_3 &= 15 \\ x_2 &= 9 \end{aligned}$$

  Solving this system, we obtain $x = (0, 9, 8, 0)$. This is a basic feasible solution.

- x_2, x_4 basic,
  x_1, x_3 nonbasic $(x_1 = 0,\ x_3 = 0)$
  Then,

  $$\begin{aligned} -x_2 + 5x_4 &= 15 \\ x_2 + x_4 &= 9 \end{aligned}$$

  Solving this system, we obtain $x = (0, 5, 0, 4)$. This is a basic feasible solution.

- x_3, x_4 basic,
  x_1, x_2 nonbasic $(x_1 = 0,\ x_2 = 0)$
  Then,

  $$\begin{aligned} 3x_3 + 5x_4 &= 15 \\ x_4 &= 9 \end{aligned}$$

  Solving this system, we obtain $x = (0, 0, -10, 9)$. This is basic but not feasible since $x_3 = -10 < 0$.

Question 6.

First convert the LP into standard form:

$$\begin{aligned} \max\ & 5x_1 + 3x_2 + x_3 \\ \text{s.t. } & x_1 + x_2 + 3x_3 + s_1 = 6 \\ & 5x_1 + 3x_2 + 6x_3 + s_2 = 15 \\ & x_i \geq 0 \ i = 1, 2, 3 \\ & s_1, s_2 \geq 0 \end{aligned}$$

	z	x_1	x_2	x_3	s_1	s_2	RHS
1	1	-5	-3	-1	0	0	0
s_1	0	1	1	3	1	0	6
s_2	0	5	3	6	0	1	15

x_1 enters. Apply MRT: $\min\{6/1, 15/5\}$=3 Hence, s_2 leaves.

	z	x_1	x_2	x_3	s_1	s_2	RHS
1	1	0	0	5	0	1	15
s_1	0	0	2/5	9/5	1	-1/5	3
x_1	0	1	3/5	6/5	0	1/5	3

Since there is no negative row 0 coefficient, the solution is optimal: $x^* = (3, 0, 0)$ and $z^* = 15$.

Since there exists 0 in row 0, there may be alternative optimal solutions: Let x_2 enter the basis. Apply MRT: $\min\{\frac{3}{2/5}, \frac{3}{3/5}\} = 5$. Then x_1 leaves.

	z	x_1	x_2	x_3	s_1	s_2	RHS
1	1	0	0	5	0	1	15
s_1	0	-2/3	0	1	1	-1/3	1
x_2	0	5/3	1	2	0	1/3	5

$x^* = (0, 5, 0)$ and z^*=15.
(If x_1 enters again, you turn back to the previous tableau!)

Question 7.

Introduce slack variables s_1, s_2 and s_3. Then the LP in the standard form becomes:

$$\begin{aligned}
&\max\ 2x_1 + x_2 + 6x_3 - 4x_4 \\
&\text{s.t. } x_1 + 2x_2 + 4x_3 - x_4 + s_1 = 6 \\
&\quad 2x_1 + 3x_2 - x_3 + x_4 + s_2 = 12 \\
&\quad x_1 + x_3 + x_4 + s_3 = 6 \\
&\quad x_i \geq 0 \ i = 1, 2, 3, 4, 5, 6, 7
\end{aligned}$$

Now construct the simplex tableau for initial basis s_1, s_2 and s_3. Now we will add the required variables to basis one by one. (Be careful! The following are not Simplex iterations to find the optimal solution!)

	z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS
1	1	-2	-1	-6	4	0	0	0	0
s_1	0	1	2	4	-1	1	0	0	6
s_2	0	2	3	-1	1	0	1	0	12
s_3	0	1	0	1	1	0	0	1	6

Let x_1 enter and s_1 leave.

	z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS
1	1	0	3	2	2	2	0	0	12
x_1	0	1	2	4	-1	1	0	0	6
s_2	0	0	-1	-9	3	-2	1	0	0
s_3	0	0	-2	-3	2	-1	0	1	0

Let x_2 enter and s_2 leave.

	z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS
1	1	0	0	-25	11	-4	3	0	12
x_1	0	1	0	-14	5	-3	2	0	6
x_2	0	0	1	9	-3	2	-1	0	0
s_3	0	0	0	15	-4	3	-2	1	0

Let x_4 enter and s_3 leave.

	z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS
1	1	0	0	65/4	0	17/4	-5/2	11/4	12
x_1	0	1	0	19/4	0	3/4	-1/2	5/4	6
x_2	0	0	1	-9/4	0	-1/4	1/2	-3/4	0
x_4	0	0	0	-15/4	1	-3/4	1/2	-1/4	0

The Simplex tableau is constructed for basis x_1, x_2 and x_4. This tableau is not optimal because there is a negative coefficient in row zero. Hence, s_2 enters.

To find the variable that leaves the basis, apply MRT: $\min\{0, 0\} = 0$ Hence, x_2 (or x_4) leaves.

Question 8.

The LP in the standard form:

$$\begin{aligned}
&\max\ 10x_1 + 8x_2 - 3x_3 + 3x_4 \\
&\text{s.t. } 2x_1 + 4x_2 - 0.5x_3 + 0.5x_4 + s_1 = 3 \\
&\quad - 2x_1 + 6x_2 - 4.5x_3 + 4.5x_4 + s_2 = 7 \\
&\quad x_1, x_2, x_3, x_4, s_1, s_2 \geq 0
\end{aligned}$$

	z	x_1	x_2	x_3	x_4	s_1	s_2	RHS
z	1	-10	-8	3	-3	0	0	0
s_1	0	2	4	-0.5	0.5	1	0	3
s_2	0	-2	6	-4.5	4.5	0	1	7

x_1 enters, s_1 leaves.

	z	x_1	x_2	x_3	x_4	s_1	s_2	RHS
z	1	0	12	0.5	-0.5	5	0	15
x_1	0	1	2	-0.25	0.25	0.5	0	3/2
s_2	0	0	10	-5	5	1	1	10

x_4 enters, s_2 leaves.

	z	x_1	x_2	x_3	x_4	s_1	s_2	RHS
z	1	0	13	0	0	5.1	0.1	16
x_1	0	1	1.5	0	0	0.45	-0.05	1
x_4	0	0	2	-1	1	0.2	0.2	2

(a) Yes, it is a basic solution. Any tableu solution in proper form will be basic, since it corresponds to a basis B.

(b) Yes, it is a basic feasible solution. All RHS values are nonnegative.

(c) Yes, it is an optimal BFS. All row zero elements are ≥ 0.