IE 400: Principles of Engineering Management
Study Set 1 Solutions

Fall 2021-2022

Question 1.
Decision Variables:

x_{ij} =amount of land allocated by kibbutz j for crop i,
$i = 1, 2, 3$ (1:sugar beets, 2:cotton, 3:sorghum), $j = 1, 2, 3$

Model:

$$
\begin{aligned}
\max\ & 1000(x_{11}+x_{12}+x_{13}) + 750(x_{21}+x_{22}+x_{23}) + 250(x_{31}+x_{32}+x_{33}) \\
\text{s.t. } & x_{11}+x_{12}+x_{13} \le 600 && (1)\\
& x_{21}+x_{22}+x_{23} \le 500 && (2)\\
& x_{31}+x_{32}+x_{33} \le 325 && (3)\\
\\
& 3x_{11}+2x_{21}+x_{31} \le 600 && (4)\\
& 3x_{12}+2x_{22}+x_{32} \le 800 && (5)\\
& 3x_{13}+2x_{23}+x_{33} \le 375 && (6)\\
\\
& x_{11}+x_{21}+x_{31} \le 400 && (7)\\
& x_{12}+x_{22}+x_{32} \le 600 && (8)\\
& x_{13}+x_{23}+x_{33} \le 300 && (9)\\
\\
& (x_{12}+x_{22}+x_{32}) - 2(x_{13}+x_{23}+x_{33}) = 0 && (10)\\
& 3(x_{11}+x_{21}+x_{31}) - 2(x_{12}+x_{22}+x_{32}) = 0 && (11)\\
& 3(x_{11}+x_{21}+x_{31}) - 4(x_{13}+x_{23}+x_{33}) = 0 && (12)\\
\\
& x_{ij} \ge 0 \quad i = 1,2,3 \quad j = 1,2,3
\end{aligned}
$$

(1), (2), (3): Constraints for maximum quota limitations
(4), (5), (6): Constraints for water consumption limitations
(7), (8), (9): Constraints for the usable land

Note: Constraints (10), (11), (12) are obtained by linearizing the following equations that ensures each kibbutz will plant the same proportion of its available irrigable land:

$$\frac{x_{11} + x_{21} + x_{31}}{400} = \frac{x_{12} + x_{22} + x_{32}}{600} = \frac{x_{13} + x_{23} + x_{33}}{300}$$

Question 2.

Decision Variables:

$$\begin{aligned}
x_1 &= \text{Total number of process 1} \\
x_2 &= \text{Total number of process 2} \\
a &= \text{Amount of A that is produced} \\
b_1 &= \text{Amount of B that is sold} \\
b_2 &= \text{Amount of B that is disposed}
\end{aligned}$$

(Note that $b_1 + b_2$ gives the amount of B produced.)
Model:

$$\begin{aligned}
\max\ & 16a + 14b_1 - 2b_2 \\
\text{s.t. } & a = 2x_1 + 3x_2 \\
& b_1 + b_2 = x_1 + 2x_2 \\
& 2x_1 + 3x_2 \leq 60 \\
& x_1 + 2x_2 \leq 40 \\
& b_1 \leq 20 \\
& x_1, x_2, a, b_1, b_2 \geq 0
\end{aligned}$$

Note: One can remove the variables a and b_2 from the model by letting $a = 2x_1 + 3x_2$ and $b_2 = x_1 + 2x_2 - b_1$ (constraints 1 and 2). Then the modified model becomes:

$$
\begin{aligned}
\max\ & 16(2x_1 + 3x_2) + 14b_1 - 2(x_1 + 2x_2 - b_1) \\
\text{s.t.}\ & 2x_1 + 3x_2 \leq 60 \\
& x_1 + 2x_2 \leq 40 \\
& b_1 \leq 20 \\
& b_1 \leq x_1 + 2x_2 \\
& x_1, x_2, b_1 \geq 0
\end{aligned}
$$

Question 3.
Decision Variables:

c_i = Cash on hand at the end of month i, i=1,2,3,4
x_i = Amount of tons purchased at the beginning of month i, i=1,2,3,4
y_i = Amount of tons sold at the end of month i, i=1,2,3,4
I_i = Amount of tons on hand after purchase at month i, i=1,2,3,4

Parameters:

p_i = Price of buying at month i, i=1,2,3,4
s_i = Price of selling at month i, i=1,2,3,4

Model:

$$
\begin{aligned}
\max\ & c_4 & & \\
\text{s.t.}\ & I_i = I_{i-1} + x_i - y_{i-1} & i = 1, \cdots, 4 & \quad \text{(Inventory Balance Equations)} \\
& c_i = c_{i-1} - p_i x_i + s_i y_i & i = 1, \cdots, 4 & \quad \text{(Cash Balance Equations)} \\
& I_i \leq 100 & i = 1, \cdots, 4 & \quad \text{(Capacity Constraint)} \\
& y_i \leq I_i & i = 1, \cdots, 4 & \\
& p_i x_i \leq c_{i-1} & i = 1, \cdots, 4 & \\
& I_0 = 50 & & \\
& c_0 = 1000 & & \\
& y_0 = 0 & & \\
& c_i, x_i, y_i, I_i \geq 0 & i = 1, \cdots, 4 &
\end{aligned}
$$

Question 4.

(a) **Decision Variables:**
x_i = Number of hours process i runs, i=1,2,3

$$\begin{aligned}
\max\ & 9(2x_1+2x_3)+10(x_1+3x_2)+24x_3-(5x_1+4x_2+x_3)-2(2x_1+x_2+3x_3) \\
& -3(3x_1+3x_2+2x_3) \\
\text{s.t. } & 2x_1+x_2+3x_3 \leq 200 && (1) \\
& 3x_1+3x_2+2x_3 \leq 300 && (2) \\
& x_1+x_2+x_3 \leq 100 && (3) \\
& x_1, x_2, x_3 \geq 0 && (4)
\end{aligned}$$

(b) Remove constraints (1) and (2) and add the following constraint to the model in part (a)

$$(2x_1+x_2+3x_3)+(3x_1+3x_2+2x_3) \leq 500 \implies 5x_1+4x_2+5x_3 \leq 500$$

(c)

Additional Decision Variable:
y : Amount of Wonka Boxes sold

The following constraint is required:

$$y = \min\left\{2x_1+2x_3, \frac{x_1+3x_2}{2}, x_3\right\}$$

Observe that this is not a linear constraint. It **must** be linearized before it is added to the model. The equality above implies the following:

$$y \leq 2x_1+2x_3, \quad y \leq \frac{x_1+3x_2}{2}, \quad y \leq x_3$$

Then the model becomes:

$$\begin{aligned}
\max\ & 54y-(5x_1+4x_2+x_3)-2(2x_1+x_2+3x_3)-3(3x_1+3x_2+2x_3) \\
\text{s.t. } & (1)-(4) \\
& y \leq 2x_1+2x_3 \\
& 2y \leq x_1+3x_2 \\
& y \leq x_3 \\
& y \geq 0
\end{aligned}$$

Question 5.
Sets:

$i:$ food i=1,2
$j:$ nutrition j=1 (protein),2 (carbohydrate),3 (fat)

Decision Variables:

x_i = amount of food i purchased j, i=1,2

Parameters:

R_j = minimum requirement of nutrition i, j=1,2,3 (120, 60, 25)
p_{ij} = amount of nutrition j in 1 gr of food i, i=1,2 j=1,2,3 ([35, 15, 10],[8, 20, 30])
c_i = price for 1 gr of food i (7, 1.5)

Model:

$$\begin{aligned} \min\ & 7x_1 + 1.5x_2 \\ \text{s.t.}\ & 35x_1 + 8x_2 \geq 120 \\ & 15x_1 + 20x_2 \geq 60 \\ & 10x_1 + 30x_2 \geq 25 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Question 6.
Decision Variables:

x_{ij} = amount of product i produced at period j, i=1,2 j=1,...,12
I_{ij} = amount of product i stored at the end of period j, i=1,2 j=1,...,12

Parameters:

D_{ij} = demand of product i at period j, i=1,2 j=1,...,12

Model:

$$\begin{array}{lll}
\min & \sum_{j=1}^{5}(5x_{1j}+8x_{2j})+\sum_{j=6}^{12}(4.5x_{1j}+7x_{2j})+\sum_{j=1}^{12}(0.2I_{1j}+0.4I_{2j}) & \\
\text{s.t.} & x_{1j}+x_{2j}\leq 120000 & j=1,\cdots,9 \\
& x_{1j}+x_{2j}\leq 150000 & j=10,11,12 \\
& 2I_{1j}+4I_{2j}\leq 150000 & j=1,\cdots,12 \\
& x_{ij}+I_{i(j-1)}=D_{ij}+I_{ij} & i=1,2 \quad j=1,\cdots,12 \\
& I_{i0}=0 & i=1,2 \\
& x_{ij},I_{ij}\geq 0 & i=1,2 \quad j=1,\cdots,12
\end{array}$$

Question 7.
Sets:

i : Candy stype i=1 (Gummy Candy),2 (Hard Candy)
j : ingredient j=1 (sugar),2 (nuts),3 (chocolate)

Decision Variables:

x_{ij} = amount of ingredient j used to produce candy i (in gr) , i=1,2 j=1,2,3
y_i = amount of candy i produced (in gr), i=1,2

Parameters:

$R_{i,j}$ = minimum proportion of ingredient j required in candy i , ([0, 0.08, 0.14],[0, 0.22, 0])
I_j = Stock level of ingredient j (in gr) (15000, 3500, 4500)
c_i = price for 1 gr of candy i (20, 25)

Model:

$$
\begin{aligned}
\max\ & 20y_1 + 25y_2 \\
\text{s.t.}\ & x_{1,1} + x_{2,1} \leq 1500 \\
& x_{1,2} + x_{1,2} \leq 3500 \\
& x_{1,3} + x_{1,3} \leq 4500 \\
& x_{2,2} \geq 0.22y_2 \\
& x_{1,2} \geq 0.08y_1 \\
& x_{1,3} \geq 0.14y_1 \\
& x_{1,1} + x_{1,2} + x_{1,3} = y_1 \\
& x_{2,1} + x_{2,2} + x_{2,3} = y_2 \\
& y_1, y_2, x_{1,1}, x_{1,2}, x_{1,3}, x_{2,1}, x_{2,2}, x_{2,3} \geq 0
\end{aligned}
$$