# IE 400: Principles of Engineering Management
# Study Set 3 Solutions

## Fall 2022

**Question 1.**

(a) **Parameters:**

$f_i$= fixed cost of plant $i$, $i = 1, \dots, 4$

$c_{ij}$= cost of producing car j at plant i $i = 1, \dots, 4$, $j = 1, 2, 3$

**Decision Variables:**

$$y_i = \begin{cases} 1 & \text{if plant i is used} \quad i = 1, \dots, 4 \\ 0 & \text{o.w} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{if car j is produced at plant i} \quad i = 1, \dots, 4, \quad j = 1, 2, 3 \\ 0 & \text{o.w} \end{cases}$$

**Model:**

$$\min \sum_{i=1}^{4} f_i y_i + \sum_{i=1}^{4} \sum_{j=1}^{3} c_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_{j=1}^{3} x_{ij} \leq 1 \qquad i = 1, 2, 3, 4 \qquad (1)$$

$$\sum_{i=1}^{4} x_{ij} = 1 \qquad j = 1, 2, 3 \qquad (2)$$

$$y_3 + y_4 \leq 1 + y_1 \qquad (3)$$

$$x_{ij} \leq y_i \qquad i = 1, 2, 3, 4 \quad j = 1, 2, 3$$

$$x_{ij} \in \{0, 1\} \qquad i = 1, 2, 3, 4 \quad j = 1, 2, 3$$

$$y_j \in \{0, 1\} \qquad j = 1, 2, 3$$

Constraint (1): Each plant can produce one type of car.
Constraint (2): The total production of each type of car must be at a single plant; that is, for example, if any Tauruses are made at plant 1, then all Tauruses must be made there.
Constraint (3): If plants 3 and 4 are used, then plant 1 must also be used.

(b)

$$
\begin{array}{lcr}
\text{either} & y_1 + y_2 \leq 1 & (*) \\
\text{or} & y_3 + y_4 \geq 1 & (**)
\end{array}
$$

In order to linearize the constraint, add the following decision variable:

$$
w = \begin{cases} 1 & \text{if constraint (*) holds} \\ 0 & \text{if constraint (**) holds} \end{cases}
$$

Then add the following constraints to the model:

$$
\begin{aligned}
& y_1 + y_2 \leq 1w - (1-w)M \\
& y_3 + y_4 \geq 1(1-w) + Mw \\
& w \in \{0, 1\}
\end{aligned}
$$

where M is a sufficiently large number.

(For $w = 1$, the constraint (*) holds and (**) becomes redundant. For $w = 0$, the constraint (**) holds and (*) becomes redundant.)

**Question 2.**

**Decision Variables:**

$$x_{ij} = \begin{cases} 1 & \text{if astronaut i is assigned to post j } i = 1,2,3,4, \;\; j = 1,2,3,4 \\ 0 & \text{o.w} \end{cases}$$

**Model:**

$$\begin{aligned}
\max \; & \sum_{i=1}^{4} \sum_{j=1}^{4} p_{ij} x_{ij} & & \\
\text{s.t.} \; & \sum_{i=1}^{4} x_{ij} = 1 & j = 1,2,3,4 & \qquad (1) \\
& \sum_{j=1}^{4} x_{ij} = 1 & i = 1,2,3,4 & \qquad (2) \\
& x_{ij} \in \{0,1\} & i = 1,2,3,4 \;\; j = 1,2,3,4 &
\end{aligned}$$

- Constraint (1) indicates that each post should have a single astronaut.
- Constraint (2) indicates that each astronaut should be assigned to a single post.

**Question 3.**

**Parameters:** As given in the question

**Decision Variables:** As given in the question

**Model:**

$$
\begin{aligned}
\min \ & \sum_{i=1}^{M} x_i \\
\text{s.t.} \ & \sum_{i=1}^{M} y_{ik} = d_k && k = 1, \cdots, n \\
& \sum_{k=1}^{n} y_{ik} \cdot l_k \leq l \cdot x_i && i = 1, \cdots, M \\
& x_i \in \{0, 1\} && i = 1, \cdots, M \\
& y_{ik} \geq 0 && i = 1, \cdots, M && k = 1, \cdots, n \\
& y_{ik} \text{ integer} && i = 1, \cdots, M && k = 1, \cdots, n
\end{aligned}
$$

**Question 4.** (a) **Parameters:**

$w_j$= weight capacity of compartment $j$ (in tons)

$s_j$= space capacity of compartment $j$ (in cubic meters)

$wt_i$= weight of cargo $i$ (in tons)

$v_i$= weight of cargo $i$ (in cubic meters/ton)

$p_i$= profit gained from cargo $i$ (TL/ton)

**Decision Variables:**

$x_{ij}$= amount of cargo $i$ distributed to compartment $j$ in tons

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{4}\sum_{j=1}^{3} p_i x_{ij} \\
\text{s.t.} \quad & \sum_{j=1}^{3} x_{ij} \le wt_i && i = 1,2,3,4 && (1)\\
& \sum_{i=1}^{4} v_i x_{ij} \le s_j && j = 1,2,3 && (2)\\
& \sum_{i=1}^{4} x_{ij} \le w_j && j = 1,2,3 && (3)\\
& \frac{\sum_{i=1}^{4} x_{i1}}{w_1} = \frac{\sum_{i=1}^{4} x_{i2}}{w_2} = \frac{\sum_{i=1}^{4} x_{i3}}{w_3} && && (4)\\
& x_{ij} \ge 0 && i = 1,2,3,4 \quad j = 1,2,3
\end{aligned}
$$

(b) Replace constraint (4) with

$$
\left| \frac{\sum_{i=1}^{4} x_{i1}}{w_1} - \frac{\sum_{i=1}^{4} x_{i2}}{w_2} \right| \le 0.1
$$

$$
\left| \frac{\sum_{i=1}^{4} x_{i1}}{w_1} - \frac{\sum_{i=1}^{4} x_{i3}}{w_3} \right| \le 0.1
$$

$$
\left| \frac{\sum_{i=1}^{4} x_{i2}}{w_2} - \frac{\sum_{i=1}^{4} x_{i3}}{w_3} \right| \le 0.1
$$

You need to linearize these constraints before adding to the model:

$$
\begin{aligned}
-0.1 \le \frac{\sum_{i=1}^{4} x_{i1}}{w_1} - \frac{\sum_{i=1}^{4} x_{i2}}{w_2} \le 0.1 \\
-0.1 \le \frac{\sum_{i=1}^{4} x_{i1}}{w_1} - \frac{\sum_{i=1}^{4} x_{i3}}{w_3} \le 0.1 \\
-0.1 \le \frac{\sum_{i=1}^{4} x_{i2}}{w_2} - \frac{\sum_{i=1}^{4} x_{i3}}{w_3} \le 0.1
\end{aligned}
$$

(c) (b) is more likely to have a larger optimal value since the feasible region in (b) includes the feasible region in (a).

(d) **Decision Variables:**

$x_{ij}$= amount of cargo $i$ distributed to compartment $j$ in tons

$$y_{ij} = \begin{cases} 1 & \text{if cargo i is shipped in compartment j} \\ 0 & \text{o.w} \end{cases}$$

**Model:** All constraints in (b) +

$$\begin{aligned} & x_{ij} \leq w_j y_{ij} && i = 1,2,3,4 \quad j = 1,2,3 \\ & \sum_{j=1}^{3} y_{ij} \leq 1 && i = 1,2,3,4 \\ & y_{ij} \in \{0,1\} && i = 1,2,3,4 \quad j = 1,2,3 \\ & x_{ij} \text{ integer} && i = 1,2,3,4 \quad j = 1,2,3 \end{aligned}$$

**Question 5.** (a) Formulate the model to determine which trucks to use so as to minimize the cost of delivering to all the customers.

**Decision Variables:**

$$x_k = \begin{cases} 1 & \text{if truck k is used } k = 1,\ldots,7 \\ 0 & \text{o.w} \end{cases}$$

$$y_{jk} = \begin{cases} 1 & \text{if customer j is visited by truck k } j = 1,\ldots,20 \quad k = 1,\ldots,7 \\ 0 & \text{o.w} \end{cases}$$

$z_{jk}$ =Amount of good is carried by truck k for customer j
$j = 1,\ldots,20 \quad k = 1,\ldots,7$

**Model:**

$$
\begin{aligned}
\min \; & \sum_{k=1}^{7} q_k x_k \\
\text{s.t.} \; & \sum_{k=1}^{7} z_{jk} = d_j && j = 1, \ldots, 20 \\
& \sum_{j=1}^{20} z_{jk} \leq C_k x_k && k = 1, \ldots, 7 \\
& z_{jk} \leq d_j y_{jk} && j = 1, \ldots, 20 \quad k = 1, \ldots, 7 \\
& y_{jk} \leq x_k && j = 1, \ldots, 20 \quad k = 1, \ldots, 7 \\
& x_k \in \{0, 1\} && k = 1, \ldots, 7 \\
& y_{jk} \in \{0, 1\} && j = 1, \ldots, 20 \quad k = 1, \ldots, 7 \\
& z_{jk} \geq 0 && j = 1, \ldots, 20 \quad k = 1, \ldots, 7
\end{aligned}
$$

(b) Add the following requirements to your model:

- A single truck cannot deliver to more than six customers.

$$\sum_{j=1}^{20} y_{jk} \leq 6 \quad k = 1, \ldots, 7$$

- Customer 1 and customer 7 cannot be visited by the same truck.

$$y_{1k} + y_{7k} \leq 1 \quad k = 1, \ldots, 7$$

- Both customer 2 and customer 12 should be visited by the same truck(s).

$$y_{2,k} = y_{12,k} \quad k = 1, \ldots, 7$$

- If a truck visits customer 4 then it has to visit customer 19 or customer 20. (Assume demands of these customers do not exceed truck capacities.)

$$y_{4,k} \leq y_{19,k} + y_{20,k} \quad k = 1, \ldots, 7$$

- Either at least two of the customers 5, 6 and 7 or at most three of customers 11, 12, 13 and 14 should be visited by truck 4.

$$\text{either } y_{5,4} + y_{6,4} + y_{7,4} \geq 2 \tag{1}$$

$$\text{or } y_{11,4} + y_{12,4} + y_{13,4} + y_{14,4} \leq 3 \tag{2}$$

In order to linearize the constraint, add the following decision variable:

$$w = \begin{cases} 1 & \text{if constraint 1 holds} \\ 0 & \text{if constraint 2 holds} \end{cases}$$

Then add the following constraints to the model:

$$\begin{aligned} & y_{5,4} + y_{6,4} + y_{7,4} \geq 2w - (1 - w)M \\ & y_{11,4} + y_{12,4} + y_{13,4} + y_{14,4} \leq 3(1 - w) + Mw \\ & w \in \{0, 1\} \end{aligned}$$

**Question 6. Decision Variables**:

$x_i$ : Number of workers employed on line i $\quad i = 1, 2$

$$y_i = \begin{cases} 1 & \text{if line i is used} \quad i = 1, 2 \\ 0 & \text{o.w} \end{cases}$$

**Model**:

$$\begin{aligned}
\text{min } & 1000y_1 + 2000y_2 + 500x_1 + 900x_2 \\
\text{s.t. } & 20x_1 + 50x_2 \geq 120 \\
& 30x_1 + 35x_2 \geq 150 \\
& 40x_1 + 45x_2 \geq 200 \\
& x_1 \leq 7y_1 \\
& x_2 \leq 7y_2 \\
& x_1, x_2 \geq 0 \\
& y_1, y_2 \in \{0, 1\}
\end{aligned}$$