IE 400: Principles of Engineering Management
Study Set 4

Fall 2022

Question 1. Use the branch-and-bound method to find the optimal solution to the following IP:

$$\begin{aligned} \min\ & 9x_1 + 13x_2 + 10x_3 + 8x_4 + 8x_5 \\ \text{s.t. } & 6x_1 + 3x_2 + 2x_3 + 4x_4 + 7x_5 \geq 40 \\ & x_1 \leq 1, x_2 \geq 1, x_3 \geq 2, x_4 \geq 1, x_5 \leq 3 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \\ & x_1, x_2, x_3, x_4, x_5 \text{ integer} \end{aligned}$$

Question 2. Use the branch-and-bound method to find the optimal solution to the following IP:

$$\begin{aligned} \max\ & 4x_1 + 5x_2 \\ \text{s.t. } & 3x_1 + 2x_2 \leq 10 \\ & x_1 + 4x_2 \leq 11 \\ & 3x_1 + 3x_2 \leq 13 \\ & x_1, x_2 \geq 0 \\ & x_1, x_2 \text{ integer} \end{aligned}$$

Question 3. Use the branch-and-bound method to find the optimal solution to the following IP:

$$\begin{aligned} \max\ & 40x_1 + 5x_2 + 60x_3 + 8x_4 \\ \text{s.t. } & 18x_1 + 3x_2 + 20x_3 + 5x_4 \leq 25 \\ & x_1, x_2, x_3, x_4 \in \{0, 1\} \end{aligned}$$

Question 4. Solve the following IP to optimality using the branch and bound method:

$$\begin{aligned} \max\ & 15x_1 + 10x_2 + 9x_3 + 2x_4 \\ \text{s.t. } & 3x_1 + 5x_2 + 3x_3 + 2x_4 \leq 14 \\ & x_i \geq 0 \text{ and integer for } i\text{=1, 2, 3, 4} \end{aligned}$$

Question 5. Suppose we are applying branch and bound algorithm for solving a 0-1 discrete optimization problem and we have reached node 27. Assume that this node corresponds to the following problem:

$$\begin{aligned} \min\ & c_1x_6 + c_2x_7 \\ \text{s.t. } & a_1x_6 + a_2x_7 \geq 15 \\ & x_6, x_7 \in \{0, 1\} \end{aligned}$$

The current incumbent has an objective value of 34. In the following, give conditions on parameters c_1, c_2, a_1 and a_2 such that the statements become true. Assume all parameters are integer valued.

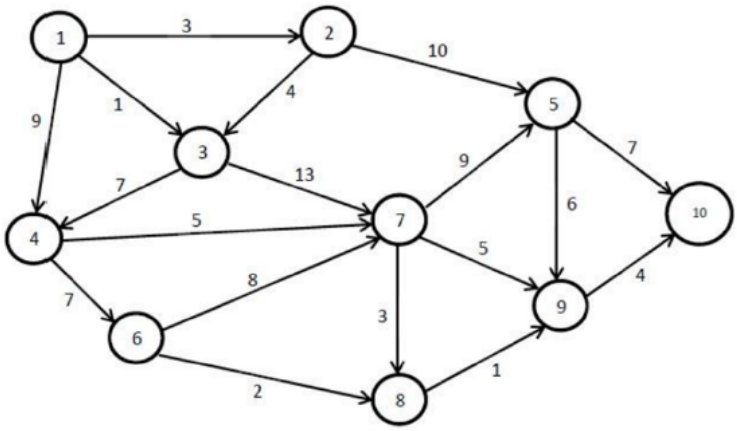
(a) Node 27 is fathomed because of infeasibility.

(b) Node 27 is fathomed because of integrality.

(c) Node 27 is fathomed because of the bound.

(d) Node 27 of the tree is not fathomed.

Question 6. Bob, Ed and Stu are in a car pool. They each live at the points 1, 2, and 7 respectively in the following network. They agree to meet at point 10 every morning at a certain time and proceed from there to their work in a single car. The numbers on the arcs represent the travel times in minutes.

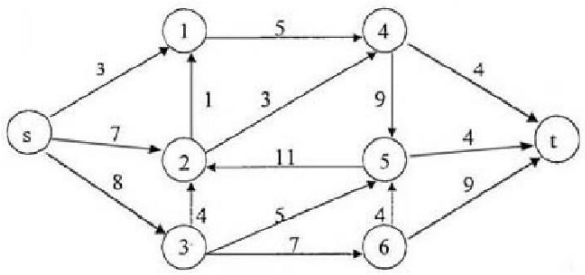


a) What is the fastest route for each man to the meeting point?

b) How early (continuing back from the meeting time) should each man leave?

c) Would anyone be agreeable to give another a ride to point 10?

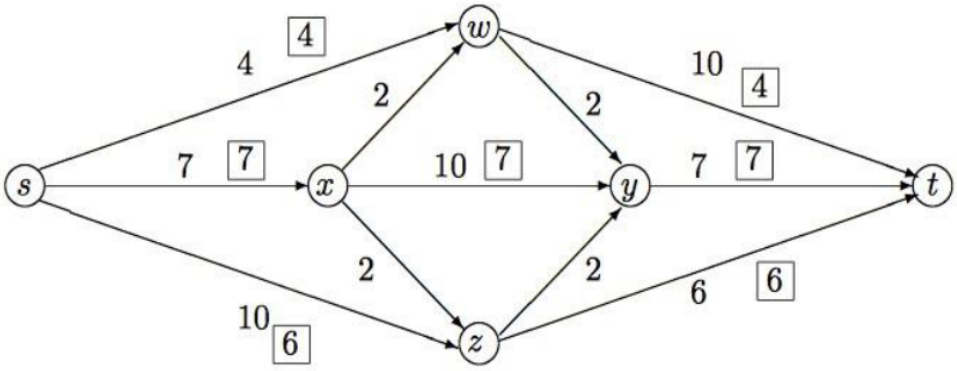
Question 7. Consider the following directed network. At node t, there is a large populated city and nodes 1, 2, 3, 4, 5, 6 are small populated areas. Experts predict that in a month the river at node s will flood and it will reach the city unless we block its way. The arc (i, j) on the network represents the route that a flood can use to reach node j from node i and the numbers on the arcs are the costs required to block that route.



Determine the minimal cost required to prevent the flood from reaching the city, node t. Show all your work and briefly explain your logic.

Question 8. The Hatfields, Montagues, McCoys, and Capulets are going on their annual family picnic. Four cars are available to transport the families to the picnic. The cars can carry the following number of people: car 1, four; car 2, three; car 3, three; and car 4, four. There are four people in each family, and no car can carry more than two people from any one family. Formulate the problem of transporting the maximum possible number of people to the picnic as a maximum-flow problem.

Question 9. The figure below shows a flow network on which an $s - t$ flow is shown. The capacity of each edge appears as a label next to the edge, and the numbers in boxes give the amount of flow sent on each edge. (Edges without boxed numbers have no flow being sent on them.)



(a) What is the value of this flow?

(b) Is this a maximum $s - t$ flow in this graph? If not, find a maximum $s - t$ flow.

(c) Find a minimum $s - t$ cut. (Specify which vertices belong to the sets of the cut.)