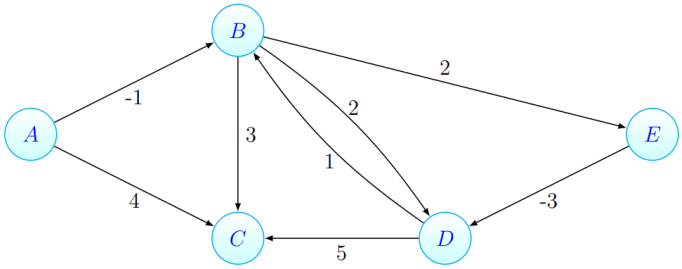
IE 400: Principles of Engineering Management
Study Set 5

FALL 2022-2023

Question 1. Use Bellman Ford algorithm to find the shortest path of the following graph where A is the source node.



Question 2. Given a network, a special source node s and weights on arcs corresponding to heights, find from s to every other node a path on which the highest height encountered is as few as possible.

(a) Does the principle of optimality apply to this problem? Why or why not?

(b) Write down the functional equations corresponding to this variant of the shortest path problem.

Question 3. A company is planning to manufacture a product that consists of three parts (A, B, C). The company anticipates that it will take 5 weeks to design the three parts and to determine the way in which these parts must be assembled to make the final product. Then the company estimates that it will take 4 weeks to make part A, 5 weeks to make part B, and 3 weeks to make part C. The company must test part A after it is completed (2 weeks). The assembly line process will then proceed as follows: assemble parts A and B (2 weeks) and then attach part C (1 week). Then the final product must undergo 1 week of testing. Draw the project network, find the critical path, and free float for each activity. Also, set up an LP that could be used to find the critical path.

Question 4. You and several friends are about to prepare a lasagna dinner. The tasks to be performed, their immediate predecessors and their estimated durations are as follows:

Task	Description	Predecessor	Time (minutes)
A	Buy the mozzarella cheese	-	30
B	Slice the mozzarella	A	5
C	Beat two eggs	-	2
D	Mix eggs and ricotta cheese	C	3
E	Cut up onions and mushrooms	-	7
F	Cook the tomato sauce	E	25
G	Boil water	-	15
H	Boil the lasagna noodles	G	10
I	Drain the lasagna noodles	H	2
J	Assemble all ingredients	B,D,F,I	10
K	Preheat the oven	-	15
L	Bake the lasagna	J,K	30

(a) Construct the project network and find the critical path for preparing this dinner.

(b) Find the earliest start time, earliest finish time, latest start time, latest finish time and the slack for each activity.

(c) Because of a phone call, you were interrupted for 6 minutes when you should have been cutting the onions and mushrooms. By how much will the dinner be delayed? If you use your food processor, which reduces the cutting time from 7 to 2 minutes, will the dinner still be delayed?

(d) Formulate an LP model that would give the latest start time of activities without delaying the dinner.

Question 5. Consider the following list of activities and predecessors.

Activity	Immediate Predecessor	Duration (days)	Crashing cost per day	Max possible reduction in duration
A	-	5	30	2
B	A	8	15	3
C	B	10	20	1
D	B	5	40	2
E	B	4	20	2
F	E	6	30	3
G	C, F	3	40	1

(a) Draw a project network, determine the critical path and project duration (without considering crashing).

(b) Which activities can be delayed without delaying the entire project?

(c) Suppose that by hiring additional workers, the duration of each activity can be reduced. The costs per day of reducing the duration of activities are given in the table above. Write down the LP that minimizes the total cost of completing the project within 20 days.

Question 6. An accounting firm requires the following activities as a first phase of the audit.

Table 1:

Activity	Description	Immediate Predecessor	Duration (days)
A	Determining the terms of engagement	-	3
B	Appraisal of auditability risk and materiality	A	7
C	Identification of types of transactions	A	15
D	Identification of types of possible errors	A	13
E	Systems description	C,D	7
F	Verification of systems description	B	3
G	Evaluation of internal controls	E,F	8
H	Design of audit approach	G	10

(a) Draw the project network. Determine the critical path for the network and slack for each activity.

(b) Set up the LP to find the project's critical path.

(c) If the duration of activity B is increased by 4 days and activity F is increased by 7 days, does the critical path change? If yes, what is the new critical path?

(d) Assume that the project must be completed in 40 days. The duration of each activity can be reduced by incurring the costs shown in Table 2. Formulate an IP that can be used to minimize the cost of meeting the project deadline.

Table 2:

Activity	Cost Per Day of Reducing the Duration of Activity	Maximum Possible Reduction in Duration (Days)
A	10	1
B	21	2
C	15	2
D	23	1
E	12	2
F	14	1
G	13	1
H	20	3

Question 7. Consider the list of activities for an agricultural field:

Activity			Completion Time (Hours)		
	Description	Predecessor	Optimistic	Most Likely	Pessimistic
A	Excavation	-	5	6	7
B	Sprinkler installation	-	4	5	18
C	Planting	A	4	15	20
D	Irrigation	B,C	3	4	5
E	Fertilizer Utilization	A	5	16	18

a) Determine the expected value and the variance of the completion time for each activity and find the critical path.

(b) Assuming that the normal distribution applies, determine the probability that the critical path will take between 18 and 26 days to complete.

Question 8. Solve the following linear programming problem using DP. Define stages, states, and recursive function clearly.

$$\begin{aligned} \max\ & 2x_1 + 5x_2 \\ \text{s.t. } & 2x_1 + x_2 \leq 430 \\ & x_2 \leq 230 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Question 9. An electric power utility forecasts that rt kilowatt-hours (kwh) of generating capacity will be needed during year t (the current year is year 1). At the beginning of each year, the utility must decide by how much generating capacity should be expanded. It costs $c_t(x)$ dollars to change generating capacity by x kwh during year t (x can be both positive and negative). During each year, 10% of the old generating capacity becomes obsolete and unusable (capacity does not become obsolete during its first year of operation). It costs the utility $m_t(i)$ dollars to maintain i units of capacity during year t. At the beginning of year 1, 100,000 kwh of generating capacity are available. Formulate a dynamic programming recursion that will enable the utility to minimize the total cost of meeting power requirements for the next T years.

Question 10. A company needs 15, 30, 10, 30 and 20 workers during each of the next five years. At present, the company has 20 workers. Each worker is paid \$30,000 per year. At the beginning of each year, workers may be hired or fired. It costs \$10,000 to hire a worker and \$20,000 to fire a worker. A newly hired worker can be used to meet the current years worker requirement. During each year, 10% of all workers quit (workers who quit do not incur any firing cost). With dynamic programming, formulate a recursion that can be used to minimize the total cost incurred in meeting the worker requirements of the next five years.

Question 11. Demand d_t during period t(t 1, 2, . . . , T) is known at the beginning of period t. Demand for period t must be met on time from inventory or from period t production. The cost c(x) of producing x units during any period is given by $c(x) = K + cx$, where K is a fixed cost for setting up production during a period, and c is the variable per-unit cost of production. At the end of period t, the inventory level i_t is observed, and a holding cost h_{i_t} is incurred. The goal is to determine a production level x_i for each period t that minimizes the total cost of meeting (on time) the demands for periods 1, 2,..., T. Now, define the following parameters for a five-period dynamic lot-size model.

Week (t)	d(t)	K(t)	c(t)	h(t)
1	210	200	2	1
2	220	250	2	1
3	240	180	2	1
4	200	270	2	1
5	180	200	2	1

We assume that the initial and final inventory level is zero. Determine the optimal production schedule using DP (Wagner-Whitin).