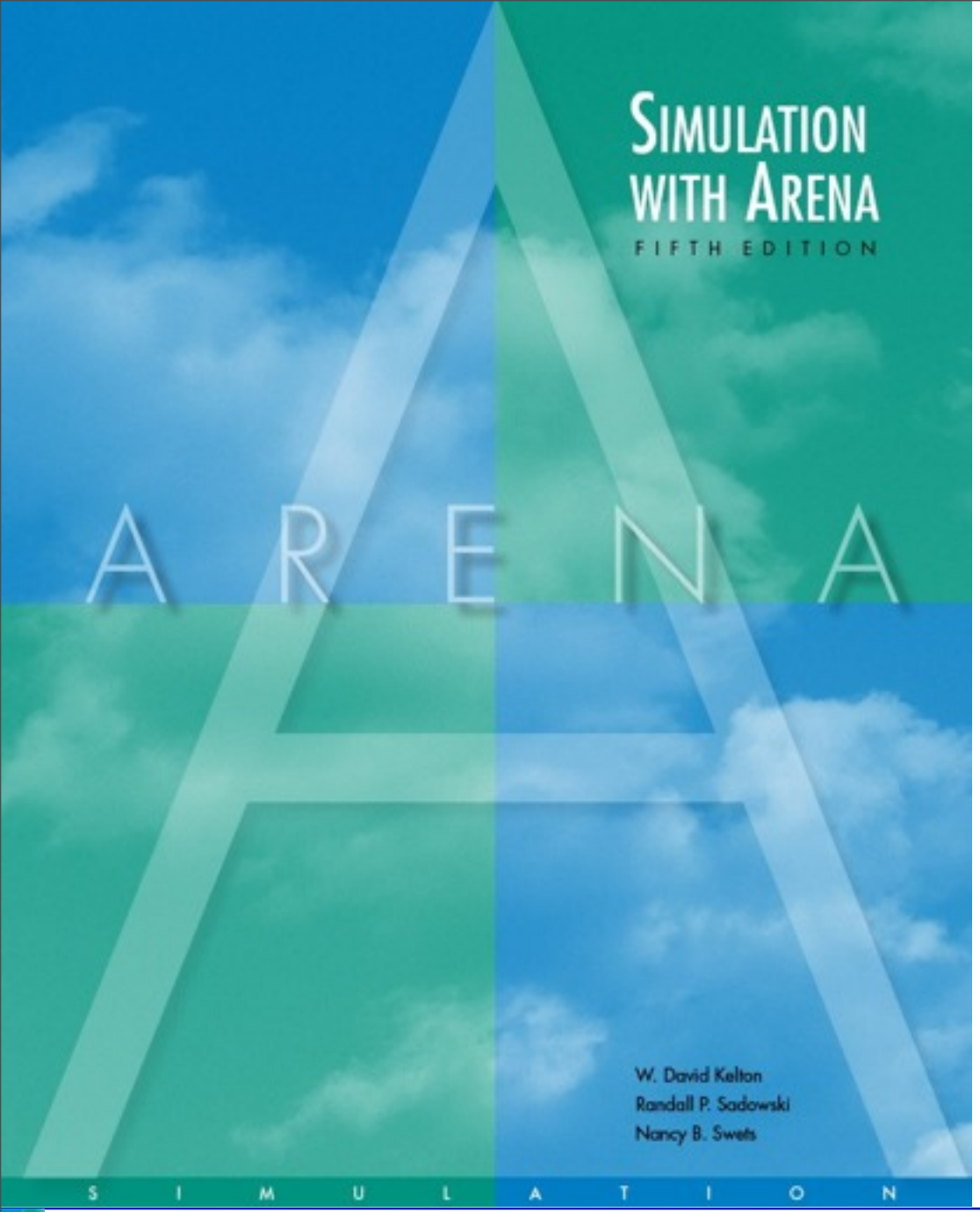


Statistical Analysis of Output from Terminating Simulations

Chapter 6

Last revision September 9, 2009

What We'll Do ...

- Time frame of simulations
- Strategy for data collection and analysis
- Confidence intervals
- Comparing two scenarios
- Comparing many scenarios via Arena Process Analyzer (PAN)
- Searching for an optimal scenario with OptQuest

Motivation

- **Random input leads to random output (RIRO)**
- **Run a simulation (once) — what does it mean?**
  - Was this run "typical" or not?
  - Variability from run to run (of the same model)?
- **Need statistical analysis of output data**
  - From a single model configuration
  - Compare two or more different configurations
  - Search for an optimal configuration
- **Statistical analysis of output is often ignored**
  - This is a big mistake – no idea of precision of results
  - Not hard or time-consuming to do this – it just takes a little planning and thought, then some (cheap) computer time

Time Frame of Simulations

- ***Terminating*: Specific starting, stopping conditions**
  - Run length will be well-defined (and finite)
- ***Steady-state*: Long-run (technically forever)**
  - Theoretically, initial conditions don't matter
    - But practically, they usually do
  - Not clear how to terminate a run
- **This is really a question of intent of study**
- **Has major impact on how output analysis is done**
- **Sometimes it's not clear which is appropriate**
- **Here: Terminating (steady-state in Section 7.2)**

Strategy for Data Collection and Analysis

- **For terminating case, make IID replications**
  - *Run* > *Setup* > *Replication Parameters*: Number of Replications field
  - Check both boxes for Initialize Between Replications
- **Separate results for each replication – Category by Replication report**
  - Model 5-3, but for 10 replications (= Model 6-1)

Replication	Total Cost ($)	Percent Rejected
1	22,385.64	12.2759
2	20,612.12	11.6059
3	23,837.38	10.4558
4	21,915.24	11.9110
5	22,462.34	13.5546
6	20,573.78	10.9804
7	20,935.88	10.1093
8	22,078.91	9.4256
9	20,056.75	9.4972
10	21,325.23	11.3388

Note cross-replication variability

Strategy for Data Collection and Analysis (cont'd.)

- **Category Overview report has some statistical-analysis results of output across replications**
- **How many replications?**
  - Trial and error (now)
  - Approximate number for acceptable precision (below)
  - Sequential sampling (Chapter 12)
- **Turn off animation altogether for max speed**
  - *Run > Run Control > Batch Run (No Animation)*

Confidence Intervals for Terminating Systems

- **Using formulas in Chapter 2, viewing cross-replication summary outputs as basic data:**

	Total Cost ($)	Percent Rejected
Sample Mean	21,618.33	11.12
Sample Standard Deviation	1,136.24	1.30
95% Confidence Interval Half Width	812.82	0.93
Minimum Summary Output Value	20,056.75	9.43
Maximum Summary Output Value	23,837.38	13.55

- **Possibly most useful part: 95% confidence interval on expected values**
- **This information (except standard deviation) is in Category Overview report**
  - If > 1 replication, Arena uses cross-repl. data as above
  - Other confidence levels, graphics – Output Analyzer

Half Width, Number of Replications

- **Prefer smaller confidence intervals — *precision***
- **Notation:** n = no. replications

  $\bar{X}$ = sample mean

  s = sample standard deviation

  $t_{n-1,1-\alpha/2}$ = critical value from t tables
- **Confidence interval:** $\bar{X} \pm t_{n-1,1-\alpha/2} \frac{s}{\sqrt{n}}$
- **Half-width =** $t_{n-1,1-\alpha/2} \frac{s}{\sqrt{n}}$
- **Can't contro**
- **Must increase *n* — how much?**

Want this to be "small," say ≤ h where h is prespecified

Half Width, Number of Replications (cont'd.)

- **Set half-width = *h*, solve for** $n = t^2_{n-1,1-\alpha/2} \frac{s^2}{h^2}$
- **Not really solved for *n* (*t*, *s* depend on *n*)**
- **Approximation:**
  - Replace *t* by *z*, corresponding normal critical value
  - Pretend that current *s* will hold for larger samples
  - Get $n \cong z^2_{1-\alpha/2} \frac{s^2}{h^2}$
- **Easier approximation:**

s = sample standard deviation from "initial" sample of n_0 replications

$$n \cong n_0 \frac{h_0^2}{h^2}$$

h_0 = half width from "initial" number n_0 of replications

n grows quadratically as h decreases

Half Width, Number of Replications (cont'd.)

- **Application to Model 6-1**
  - From initial 10 replications, 95% half-width on Total Cost was ± 812.82 (3.8% of $\overline{X}$ = 21,618.33)
    - Let's get this down to ± 250 or less
  - First formula: $n \cong 1.96^2(1136.24^2/250^2) = 79.4$, so 80
  - Second formula: $n \cong 10(812.76^2/250^2) = 105.7$, so 106
  - Modified Model 6-1 into Model 6-2
    - Checked *Run* > *Run Control* > *Batch Run (No Animation)* for speed
    - In *Run* > *Setup* > *Replication Parameters*, changed Number of Replications to 110 (conservative based on above)
  - Got 22175.19 ± 369.54, close to criterion (undershot a bit?)
    - BTW, from 110 replications got 11.745 ± 0.51 on Percent Rejected
    - Use *max* of sample sizes for precisions on multiple outputs

Interpretation of Confidence Intervals

- **Interval with random (data-dependent) endpoints that's supposed to have stated probability of containing, or covering, expected valued**
  - "Target" expected value is a fixed, but unknown, number
  - Expected value = average of infinite number of replications
- **Not an interval that contains, say, 95% of data**
  - That's a *prediction* interval … useful too, but different
- **Usual formulas assume normally-distributed data**
  - Never true in simulation
  - Might be approximately true if output is an average, rather than an extreme
  - Central limit theorem
  - Robustness, coverage, precision – see text (Model 6-3)

Comparing Two Scenarios

- **Usually compare alternative system scenarios, configurations, layouts, sensitivity analysis**
  - For now, just two scenarios ... more later
- **Model 6-4**
  - Model 6-3, except reduce to 110 replications, add file **`Total Cost.dat`** to Statistic module, Output column, **`Total Cost`** row
    - Similarly for percent rejected
  - Saves output statistics to these files for each replication
  - Two scenarios
    - *Base case* – all inputs as original Model 5-3, no extra resources
    - *More-resources case* – Add 3 trunk lines (29), 3 each of New Sales, New Tech 1, New Tech 2, New Tech 3, and New Tech All

      Effect on total cost, percent rejected?

Comparing Two Scenarios (cont'd.)

- **Reasonable but not-quite-right idea**
  - Make confidence intervals on expected outputs from each scenario, see if they overlap; look at Total Cost
  - Base case:

    22175.19 ± 369.54, or [21805.65, 22544.73]
  - More-resources case:

    24542.82 ± 329.11, or [24213.71, 24871.93]
  - But this doesn't allow for a precise, efficient statistical conclusion

No overlap

Compare Means via Output Analyzer

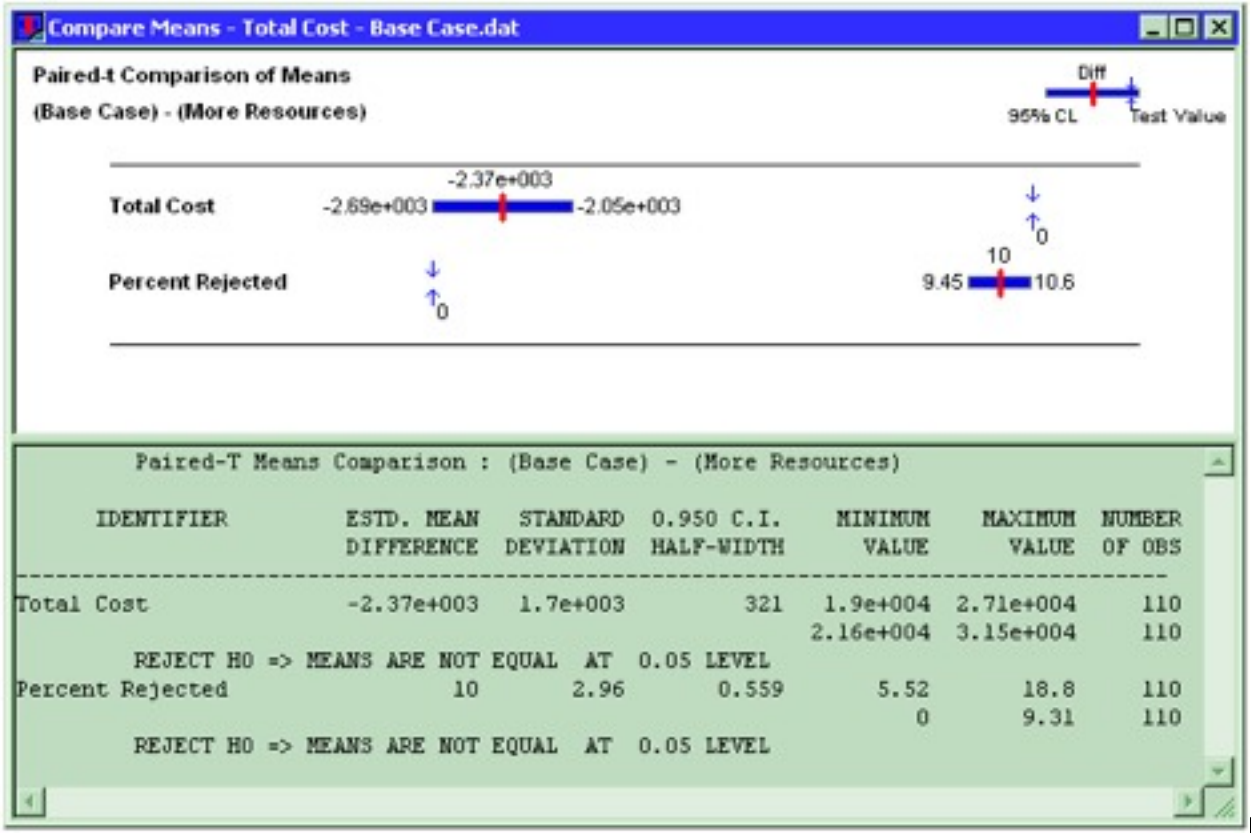
- **Output Analyzer is a separate application that operates on .dat files produced by Arena**
  - Launch separately from Windows, not from Arena
- **To save output values (Expressions) of entries in Statistic data module (Type = Output) – enter filename.dat in Output File column**
  - Did for both Total Cost and Percent Rejected
  - Will overwrite these file names next time
    - Either change names in Arena model, or out in operating system before next run
  - .dat files are binary … can only be read by Output Analyzer

Compare Means via Output Analyzer
(cont'd.)

- **Start Output Analyzer, open a new data group**
  - Basically, a list of .dat files of current interest
  - Can save data group for later use – .dgr file extension
  - Add button to select (Open) .dat files for data group
- ***Analyze* > *Compare Means* menu option**
  - Add data files … "A" and "B" for two scenarios
  - Select "Lumped" for Replications field
  - Title, confidence level, accept Paired-t Test, do not Scale Display since two output performance measures have different units

Compare Means via Output Analyzer
(cont'd.)

- **Results:**



Paired-T Means Comparison : (Base Case) - (More Resources)

IDENTIFIER	ESTD. MEAN DIFFERENCE	STANDARD DEVIATION	0.950 C.I. HALF-WIDTH	MINIMUM VALUE	MAXIMUM VALUE	NUMBER OF OBS
Total Cost	-2.37e+003	1.7e+003	321	1.9e+004	2.71e+004	110
				2.16e+004	3.15e+004	110
REJECT H0 => MEANS ARE NOT EQUAL AT 0.05 LEVEL						
Percent Rejected	10	2.96	0.559	5.52	18.8	110
				0	9.31	110
REJECT H0 => MEANS ARE NOT EQUAL AT 0.05 LEVEL						

- **Confidence intervals on differences both miss 0**
  - Conclude that there *is* a (statistically) significant difference on both output performance measures

Evaluating Many Scenarios with Process Analyzer (PAN)

- **With (many) more than two scenarios to compare, two problems are**
  - Simple mechanics of making many parameter changes, making many runs, keeping track of many output files
  - Statistical methods for drawing reliable, useful conclusions
- **Process Analyzer (PAN) addresses these**
- **PAN operates on program (.p) files – produced when .doe file is run (or just checked)**
- **Start PAN from Arena (*Tools* > *Process Analyzer*) or via Windows**
- **PAN runs on its own, separate from Arena**

PAN Scenarios

- **A *scenario* in PAN is a combination of:**
  - A program (.p) file
  - Set of input *controls* that you choose
    - Chosen from Variables and Resource capacities – think ahead
    - You fill in specific numerical values
  - Set of output *responses* that you choose
    - Chosen from automatic Arena outputs or your own Variables
    - Values initially empty … to be filled in after run(s)
  - To create a new scenario in PAN, double-click where indicated, get Scenario Properties dialog
    - Specify Name, Tool Tip Text, .p file, controls, responses
    - Values of controls initially as in model, but *you can change them in PAN* – this is the real utility of PAN
    - Duplicate (right-click, Duplicate) scenarios, then edit for a new one
  - Think of a scenario as a row

PAN Projects and Runs

- **A *project* in PAN is a collection of scenarios**
  - Program files can be the same .p file, or .p files from different model .doe files
  - Controls, responses can be same, or differ across scenarios in a project – usually will be mostly the same
  - Think of a project as a collection of scenario rows – a table
  - Can save as a PAN (.pan extension) file
- **Select scenarios in project to run (maybe all)**
- **PAN runs selected models with specified controls**
- **PAN fills in output-response values in table**
  - Equivalent to setting up, running them all "by hand" but much easier, faster, less error-prone

Model 6-5 for PAN Experiments

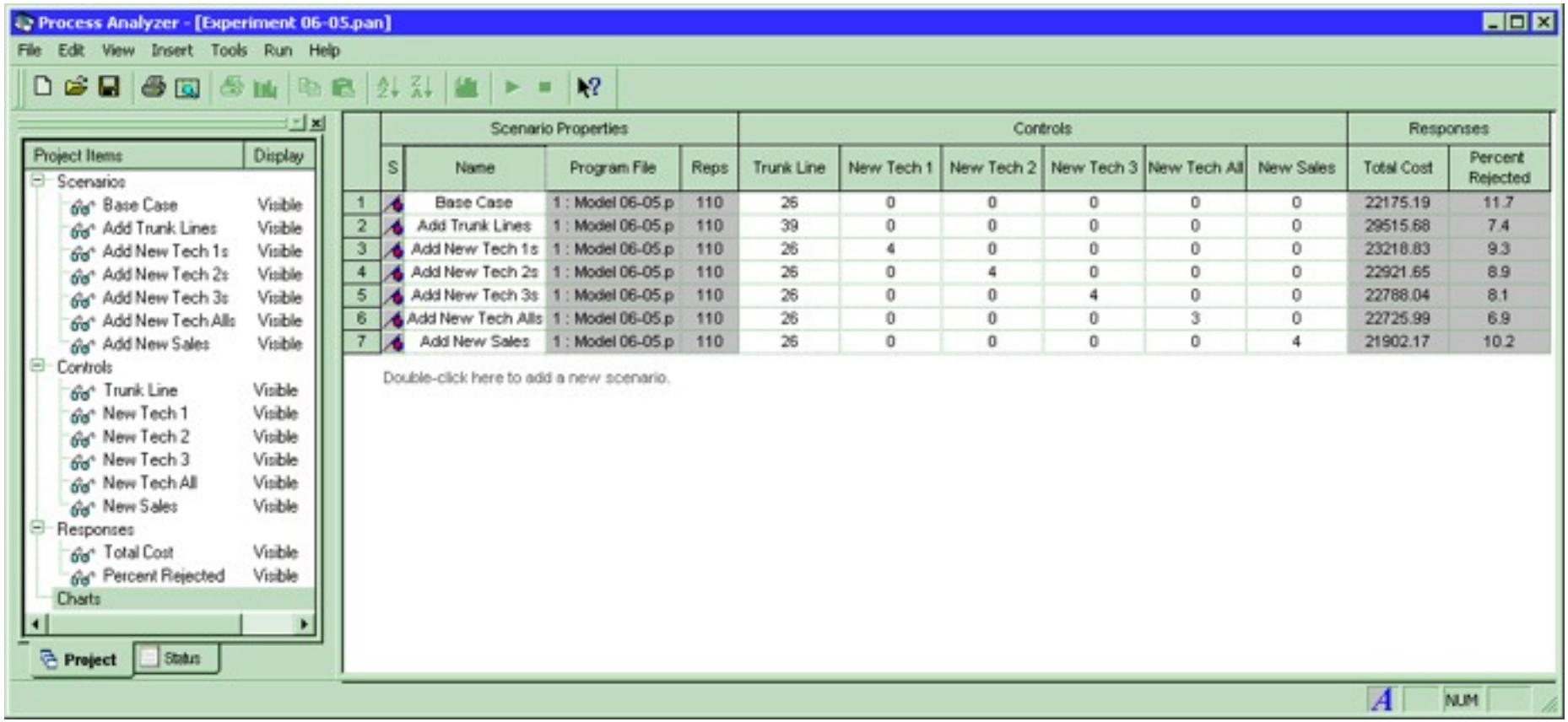
- **Same as Model 6-4 but remove Output File entries in Statistic module**
  - PAN will keep track of outputs itself, so this is faster
  - Stick with 110 replications
- **Start PAN, New project, double-click for scenario**
  - Name = **`Base Case`**
  - Program File = **`Model 06-05.p`** (maybe with path)
- **Six controls – all data type Integer**
  - Resources > capacity of **`Trunk Line`**
  - User Specified > **`New Tech 1`**, **`New Tech 2`**, **`New Tech 3`**, **`New Tech All`**, **`New Sales`**
- **Responses – both from User Specified**
  - **`Total Cost`**, **`Percent Rejected`**

Could also do a designed experiment with PAN, for more efficient study of controls' effects, interactions

Model 6-5 for PAN Experiments (cont'd.)

- **Experimental (non-base-case) scenarios**
  - Suppose you get $1360 more per week for more resources
    - Must spend all $1360 on a single type of resource; could get
      - 13 more trunk lines @ $98 each
      - 4 more of any one of single-product tech-support people @ $320 each
      - 3 more of all-product tech-support people @ $360 each
      - 4 more sales people @ $340 each
  - Create six more PAN scenarios
    - Right-click, Duplicate Scenario(s), edit fields
    - See saved PAN file **`Experiment 06-05.pan`**
  - Execute scenarios
    - Select which to run (click on left, Ctrl-Click, Shift-Click)
    - or *Run* > *Go* or F5

Model 6-5 for PAN Experiments (cont'd.)



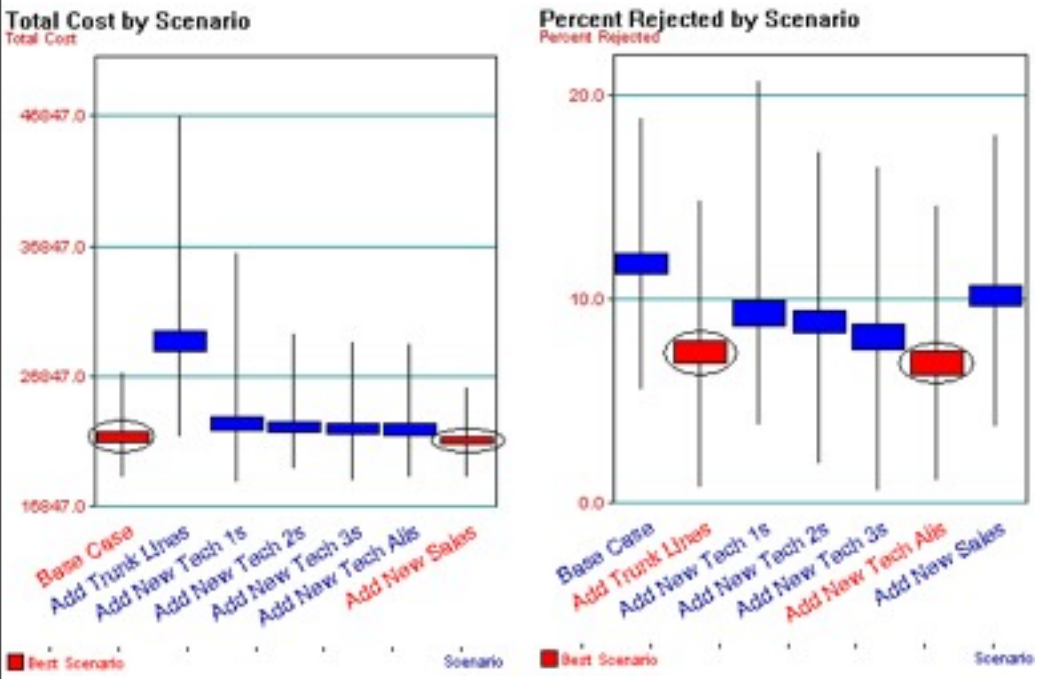
What to make of all this?
Statistical meaningfulness?

Statistical Comparisons with PAN

- **Model 6-5 scenarios were made with 110 replications each**
  - Better than one replication, but what about statistical validity of comparisons, selection of "the best"?
- **Select Total Cost column, *Insert* > *Chart* (or  or right-click on column, then Insert Chart)**
  - Chart Type: Box and Whisker
  - Next, **`Total Cost`**; Next defaults
  - Next, Identify Best Scenarios
    - Smaller is Better, Error Tolerance = 0 (not the default)
    - Show Best Scenarios; Finish

Repeat for **`Percent Rejected`**

Statistical Comparisons with PAN (cont'd.)



- **Vertical boxes: 95% confidence intervals**
- **Red scenarios statistically significantly better than blues**
  - More precisely, red scenarios are 95% sure to contain best one
  - Narrow down red set – more replications, or Error Tolerance > 0
  - More details in text

Numerical values (including c.i. half widths) in chart – right click on chart, Chart Options, Data

So which scenario is "best"? Criteria disagree. Combine them somehow?

Searching for an Optimal Scenario with OptQuest (not included in student version of Arena)

- **Scenarios considered via PAN are just a few of many**
- **Seek input controls minimizing `Total Cost` while keeping `Percent Rejected` ≤ 5**
  - Explore all possibilities – add resources in any combination
  - New rules:

    26 ≤ number of trunk lines ≤ 50

    Total number of new employees of all five types ≤ 15

Searching for an Optimal Scenario with OptQuest – Formulation

- **Formulate as an *optimization* problem:**

  Minimize **`Total Cost`** ← *Objective function is a simulation-model output*

  Subject to

  $26 \le$ **`MR(Trunk Line)`** ≤ 50

  $0 \le$ **`New Sales + New Tech 1 + New Tech 2 + New Tech 3 + New Tech All`** ≤ 15

  `Percent Rejected` ≤ 5 ← *Constraint on another output*

  Constraints on input control (decision) variables

  - Reasonable start – best acceptable scenario so far
    - No PAN scenarios satisfied **`Percent Rejected`** ≤ 5, so start with more-resources case earlier (29 trunk lines, 3 new employees of each of five types)
  - Where to go from here? Explore all of feasible six-dimensional space exhaustively? ***No***.
    - For this problem, choice (decision) variables are discrete, so can enumerate that there are 1,356,600 feasible scenarios – with 110 replications per scenario, would take two months on 2.1GHz PC

Searching for an Optimal Scenario with OptQuest – Operation

- **OptQuest searches intelligently for an optimum**
  - Like PAN, OptQuest ...
    - runs as a separate application … can be launched from Arena
    - "takes over" running of your model
    - asks you to identify input controls, the output (just one) objective
  - Unlike PAN, OptQuest ...
    - allows you to specify constraints on input controls
    - allows you to specify "constraints" on outputs
    - decides itself what input-control-value combinations to try
    - uses internal heuristic algorithms to decide how to change input controls to move toward an optimum configuration
- **There are various stopping criteria for search**
  - Default is no significant improvement for 100 scenarios

Searching for an Optimal Scenario with OptQuest – Example

- **Model 6-6 for OptQuest**
  - Model 6-5, but OptQuest requires finite Replication Length
  - Make sure Model 6-6 model window is active
- **Make sure desired model window is active**
- ***Tools > OptQuest for Arena***
  - New Optimization or Browse for saved one (.opt)
  - Tree on left, expand for Controls and Responses

Searching for an Optimal Scenario with OptQuest – Controls, Responses

- **Controls → Resources → Trunk Line**
  - Integer, Lower Bound = 26, Suggested Value = 29, Upper Bound = 50
- **Controls → User Specified → New Sales**
  - Integer, Lower Bound = 0, Suggested Value = 3, Upper Bound = 15
  - Similarly for others ... open `Optimum Seeking 06-06.opt`
  - Click on "Included" to collect selections at top or bottom
- **Responses → User Specified → Output**
  - Check Percent Rejected, Total Cost

Searching for an Optimal Scenario with OptQuest – Constraints, Objective

- **Constraints**
  - Add button, then each of first five controls, “+”, then “<= 15”
  - Add button, then **`Percent Rejected`**, then “<= 5”
- **Objectives**
  - Add button, **`Total Cost`**, Minimize radio button
- **Options**
  - Stopping rules
  - Tolerance for regarding results as “equal”
  - Replications per simulation
  - Solutions log file location
    - Stores all scenarios tried, results – valuable for second best, etc.

Searching for an Optimal Scenario with OptQuest – Running

- ▶ or *Run* > *Start* or F5
  - Optimization branch on tree to watch progress, scenarios so far, best scenario so far
- **Can't absolutely guarantee a true optimum**
  - But usually finds far better configuration than possible by hand