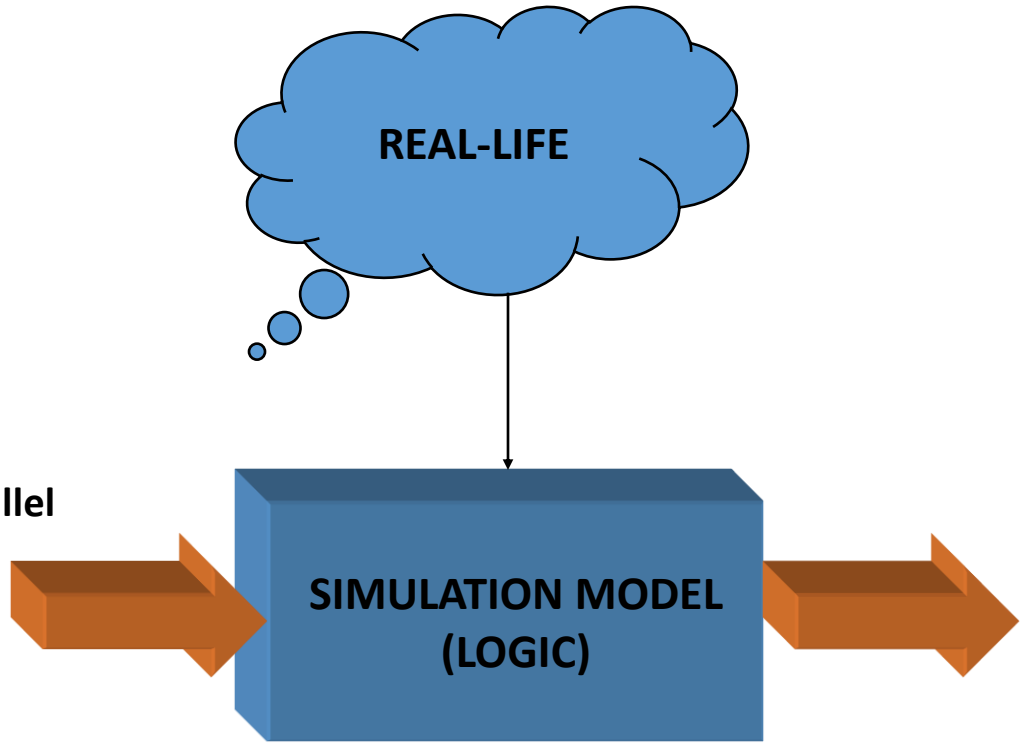
IE 324 SIMULATION

2023 – 2024 SPRING

DISCRETE EVENT SYSTEM SIMULATION BASICS

SIMULATION



Operating Policies
- **Single queue, parallel servers, FIFO, …**

Input Parameters
- **Number of servers, distributions, …**

(X)

System response

- **Waiting times, system size, utilizations…**

(Y)

Y=f(X)

SIMULATION

- In this course we will focus on Discrete-event stochastic dynamic systems.
  - The system is **stochastic** because at least one of the system state variables is random (o.w. deterministic simulation)
  - The system is **dynamic** because system behavior over time is examined. (o.w. static systems – Monte Carlo simulation)
  - The system is **discrete-event** because the changes in the system state variables are associated with events that occur at discrete time instances only.

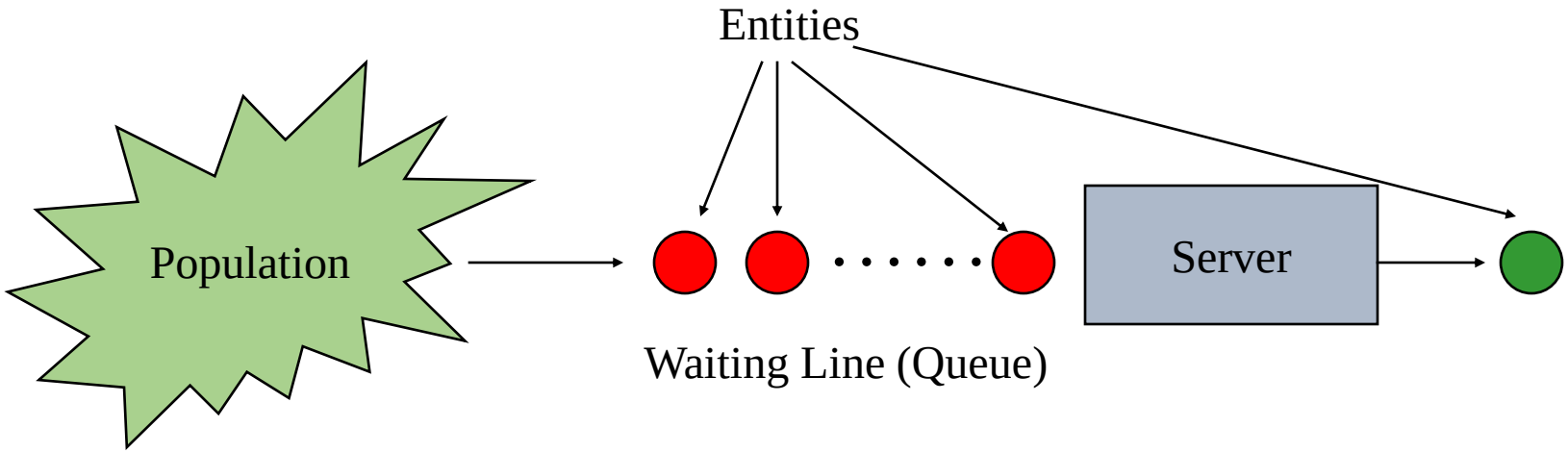
COMPONENTS OF A SIMULATION MODEL

- **STATE:** A collection of variables that contain all information necessary to describe the status of a system at any given point in time.
- **ACTIVITY:** A duration of time of specified length that is known as it begins.
  - Called “Unconditional Wait”
  - It can be:
    - Deterministic
    - Statistical
    - A function of system variables
  - Ex: Inter-arrival times, service times…etc.
- **DELAY:** An indefinite duration of time that depends on activities or system conditions.
  - Called “Conditional Wait”
  - Ex: Waiting time in queue

COMPONENTS OF A SIMULATION MODEL

- **All ACTIVITIES culminate with an EVENT.**
  - Since the duration of the activities are known in advance, an **event notice** is placed for an upcoming event is placed in a **future event list (FEL)** as the activity begins.
  - Delays are not placed in the FEL.
  - Every feasible event has one entry in the FEL.
  - If an event becomes infeasible, it should be removed from the FEL.
    - Ex: Machine breaks down (service completion is not going to happen at the scheduled time)

EXAMPLE: Queueing Systems



Finite vs. Infinite

One line vs. Multiple lines

One server vs. multiple server

CHARACTERISTICS

- Interarrival and Service Times
  - Exponential (M)
  - Deterministic (D)
  - Erlang (E)
  - General (G)
- Queue discipline
  - First Come/In First Served/Out (FCFS/FIFO)
  - Last Come/In First Served/Out (LCFS/LIFO)
  - Earliest Due Date (EDD)
- System Capacity
- Number of Servers

EXAMPLE

- Consider a bank teller queue with one teller (G/G/1) and FIFO queue discipline
  - State Variables: Number of customers in queue
    Busy/Idle Status of Teller (0,1)
  - Activities: Customer Inter Arrival
    Triggers
  - Events: Customer Arrival Event
  - Delay: Waiting time in queue

Customer Service
Triggers
Customer Departure Event

EXAMPLE

EXAMPLE

- Customer 1 arrives at $t = t_1$
  - If interarrivals are statistical with random variable X_i^A for customer i then the next Customer Arrival Event is scheduled at $t = t_1 + X_2^A$
- Customer 1 enters service at $t = t_2$
  - If service times are statistical with random variable X_i^S for customer i, then the next customer departure event is scheduled at $t = t_2 + X_1^S$

COMPONENTS OF A SIMULATION MODEL

- We advance the system time t (called “Clock”) one by one to each event in a chronological order as we make a simulation run.
  - This is called “Event Driven Simulation”
- WHY?
  - System state remains unchanged between events. (Nothing happens until the next event!)
- There is also an additional type of event called the “Ending Event” that represents the time to stop the simulation run. We add this event to the FEL when simulation run starts (E, T_E).
- Note that, for now interarrival and service times are generated randomly for us.

EXAMPLE (Cont'd)

- What if there were two servers? (G/G/2)
  - We will have two "Customer Service" activities each triggering a separate "Customer Departure Event" in the FEL.

COMPUTATION OF STATISTICS

- As a result of a simulation run, we need to obtain statistical information about a performance measure of interest.
  - Average Waiting Time
  - Maximum Waiting Time
  - Average Number of Entities in the System
  - Maximum Number of Entities in the System
  - Server Utilization
  - Average System Time
  - Maximum System Time

- We need to keep track of some more variables to capture the performance measure we wish to investigate.

COMPUTATION OF STATISTICS

- P: Total number of customers served so far (# left the system)
- N: Total number of customers passed through the queue

- ΣWQ : Sum of queue times observed so far
- WQ^* : Maximum queue time observed so far

} Updated whenever a customer finishes waiting in queue

- ΣTS : Sum of system times observed so far
- TS^*: Max system time observed so far

} Updated whenever a customer leaves the system

COMPUTATION OF STATISTICS

- $Q(t)$: Function of number of customers waiting in queue (state var.)

  $Q(t) \in \{0,1,2, \dots\}$
- $B(t)$: Server busy function (state var.)

  $B(t) \in \{0,1\}$
- $L(t) = Q(t) + B(t)$

COMPUTATION OF STATISTICS

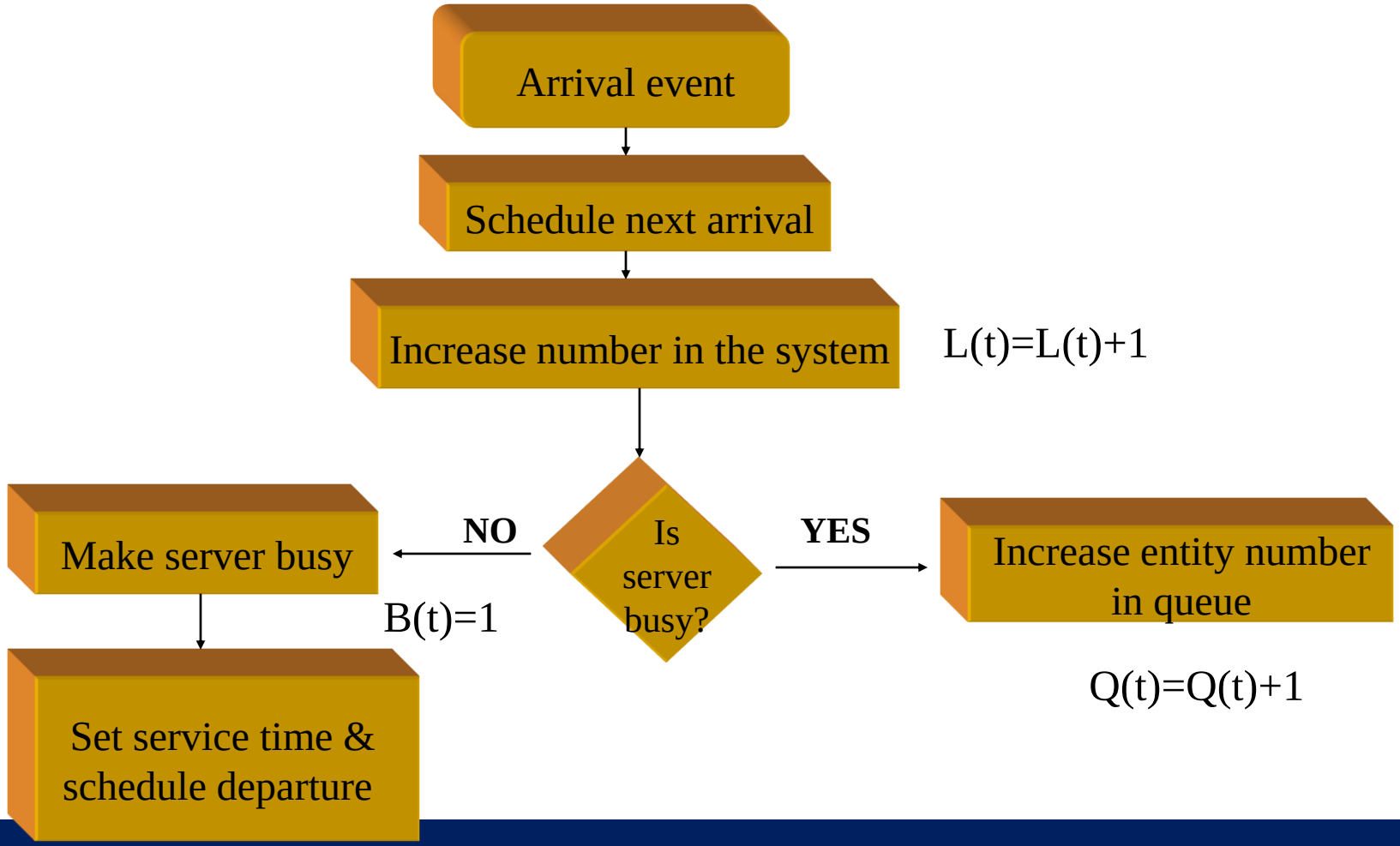
- Using these variables, we can find:
  - Average Waiting Time (In Queue): $\frac{\Sigma WQ}{N}$ (Tally)
  - Average System (Total) Time: $\frac{\Sigma TS}{P}$ (Tally)
  - Average Number of Customers In Queue: $\frac{\int Q(t)}{T_E}$ (Time-persistent)
  - Server Utilization: $\frac{\int B(t)}{T_E}$ (Time-persistent)

- Note that additional variables are needed if further statistical outputs are needed:
  - Maximum idle time, maximum queue length…

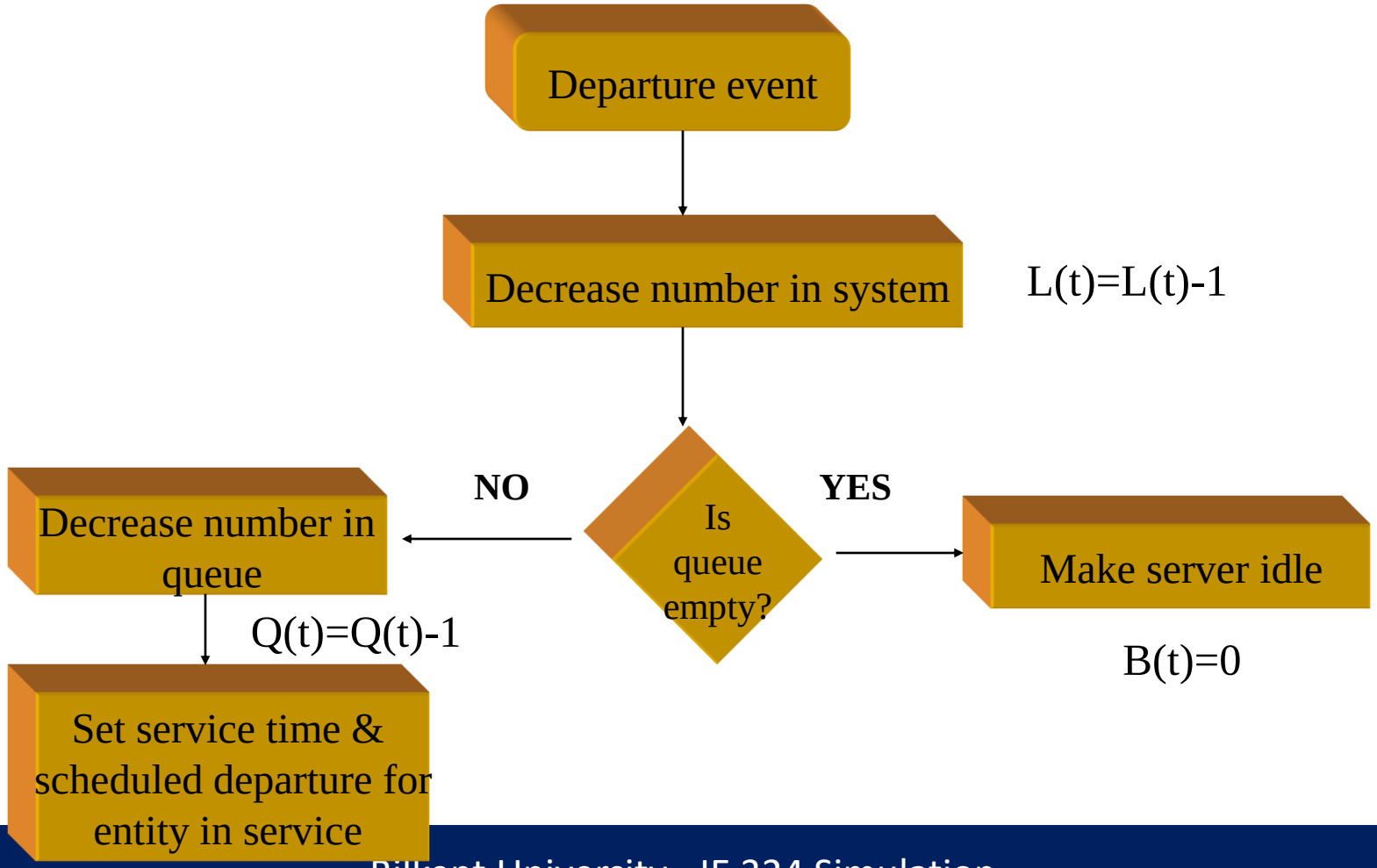
TOWARDS MAKING A SIMUATION RUN

- Now that we know how to advance simulation time, keep track of events and collect statistics, we will now see how to analyze a G/G/1 queue system.

Arrival Event Flowchart (Logical Model)



Departure Event Flowchart (Logical Model)



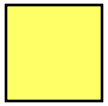
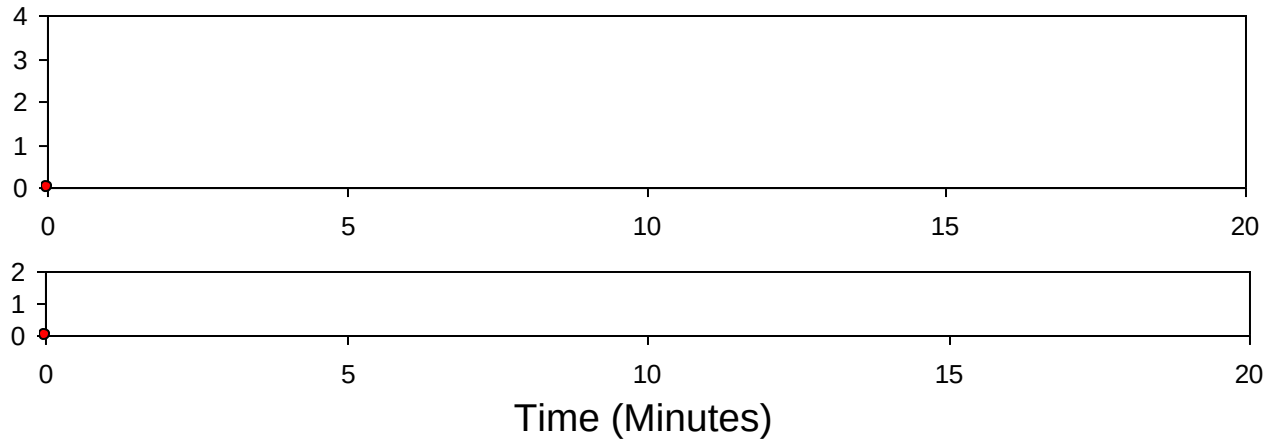
Simulation by Hand

System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S ()	Event calendar
N P	Σ W Σ W TS		Area under		Area under
$Q(t)$ graph $B(t)$ graph	4 3 2 1 0 2 1				15 20 15 20 Time (Minutes)
Interarrival times	1.73, 1.35, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...				
Service times	2.90, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...				

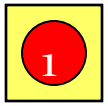
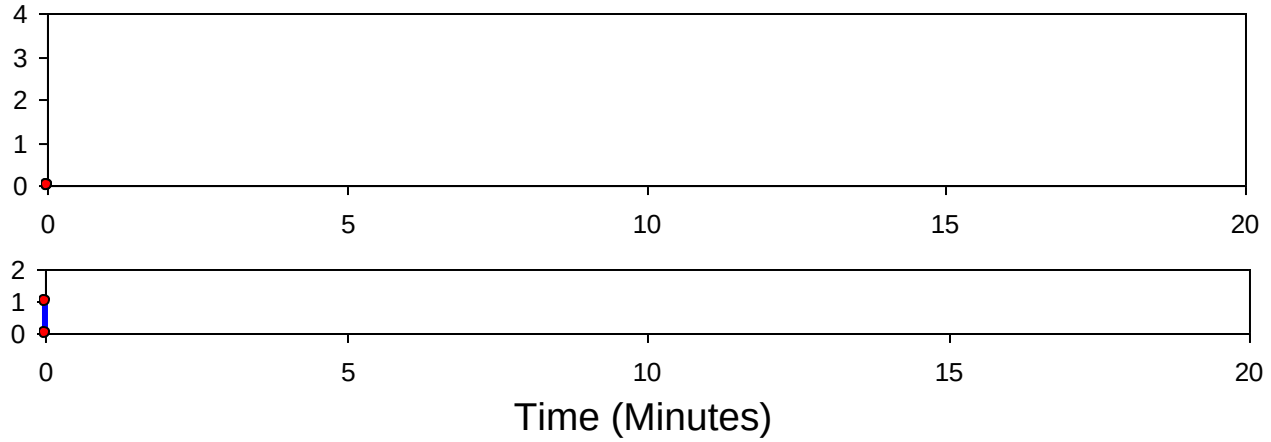
Run simulation for 20 minutes to find
- Average / Max Waiting Time
- Average / Max System Time
- Average Queue Length
- Average Utilization

Assume the first arrival occurs at zero

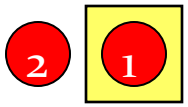
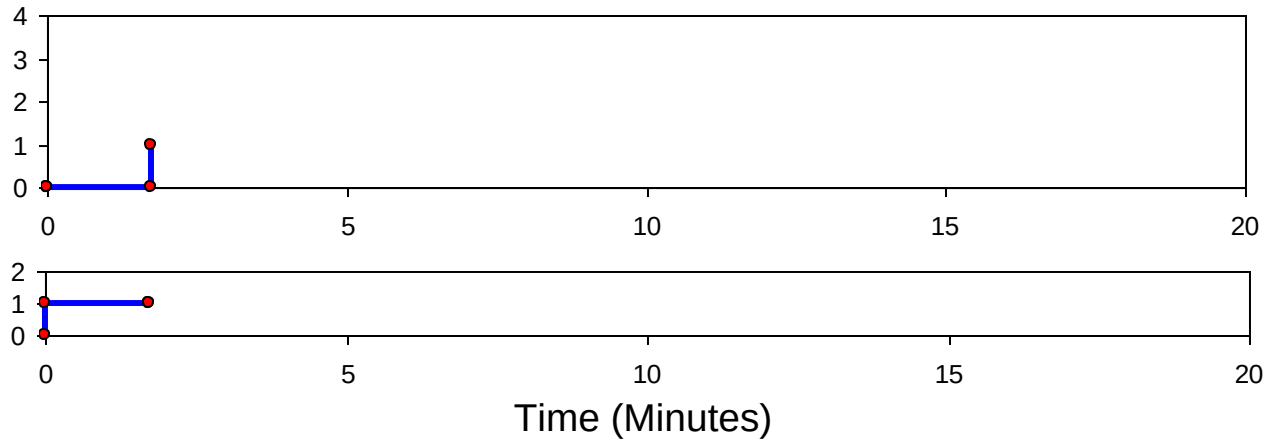
t = 0.00, Initialize

System 	Clock 0.00	$B(t)$ 0	$Q(t)$ 0	Arrival times (Q) S ()	Event calendar [1, 0.00,Arr] [–, 20.00, End]
N: 0 P: 0	Σ WQ: 0.00 Σ TS: 0.00 WQ*: 0.00 TS*: 0.00		Area under $Q(t)$ 0.00		Area under $B(t)$ 0.00
$Q(t)$ graph $B(t)$ graph					
Interarrival times	1.73, 1.35, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...				
Service times	2.90, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...				

t = 0.00, Arrival of Part 1

System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
	0.00	1	0	() 0.00	[2, 1.73,Arr] [1, 2.90,Dep] [–, 20.00, End]
N: 1 P: 0	Σ WQ: 0.00 Σ TS: 0.00 WQ*: 0.00 TS*: 0.00		Area under $Q(t)$ 0.00		Area under $B(t)$ 0.00
$Q(t)$ graph $B(t)$ graph					
Interarrival times	~~1.73~~, 1.35, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...				
Service times	~~2.90~~, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...				

t = 1.73, Arrival of Part 2

System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
	1.73	1	1	(1.73) 0.00	[1, 2.90,Dep] [3, 3.08,Arr] [–, 20.00, End]
N: 1 P: 0	Σ WQ: 0.00 Σ TS: 0.00 WQ*: 0.00 TS*: 0.00		Area under $Q(t)$ 0.00		Area under $B(t)$ 1.73
$Q(t)$ graph $B(t)$ graph					
Interarrival times	~~1.73~~, ~~1.35~~, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...				
Service times	~~2.90~~, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...				

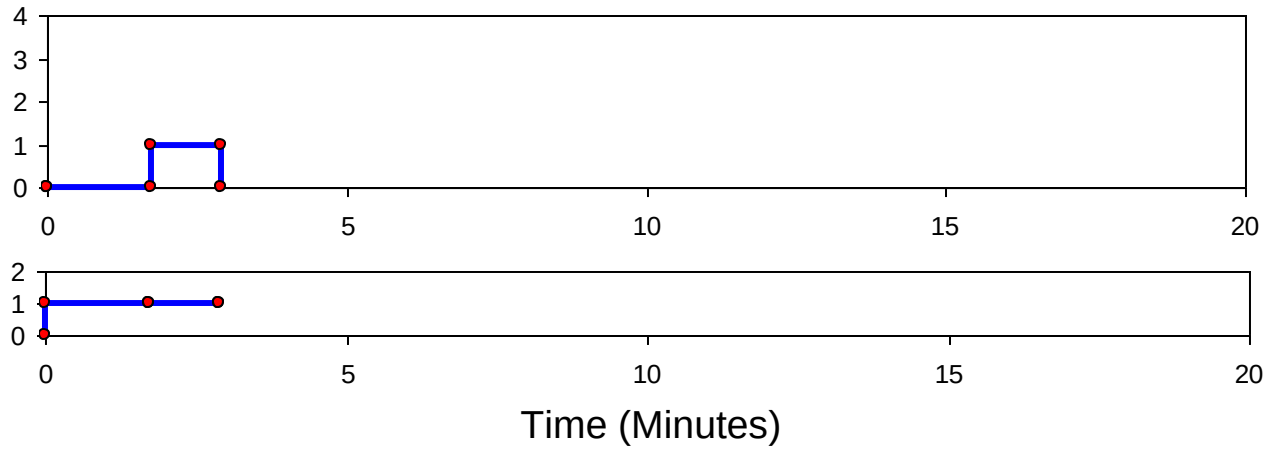
t = 2.90, Departure of Part 1

System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
	2.90	1	0	() 1.73	[3, 3.08,Arr] [2, 4.66,Dep] [–, 20.00, End]

N: 2 P: 1	Σ WQ: 1.17 Σ TS: 2.90 WQ*: 1.17 TS*: 2.90	Area under $Q(t)$ 1.17	Area under $B(t)$ 2.90

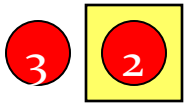
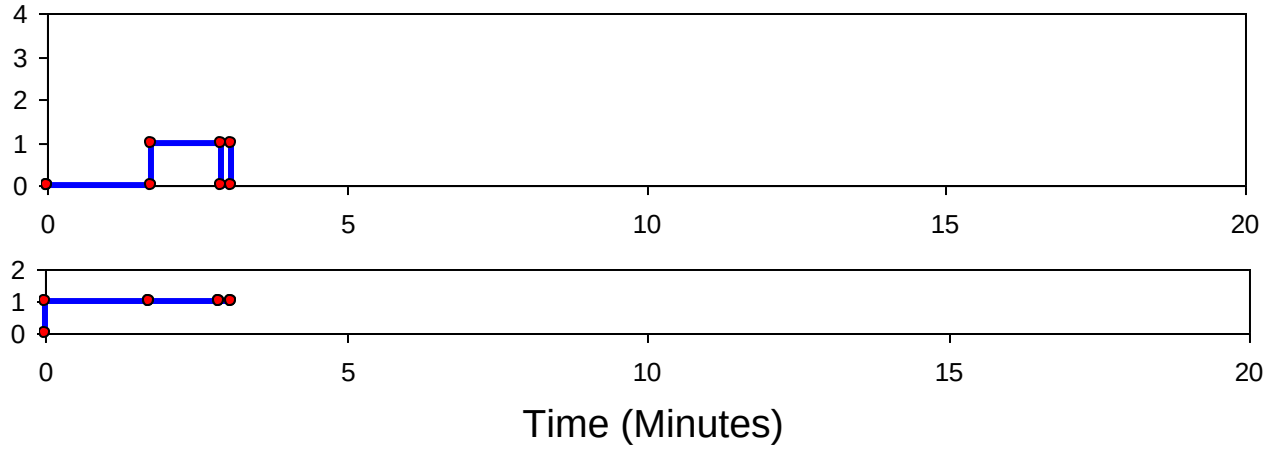
$Q(t)$ graph

$B(t)$ graph

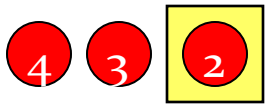
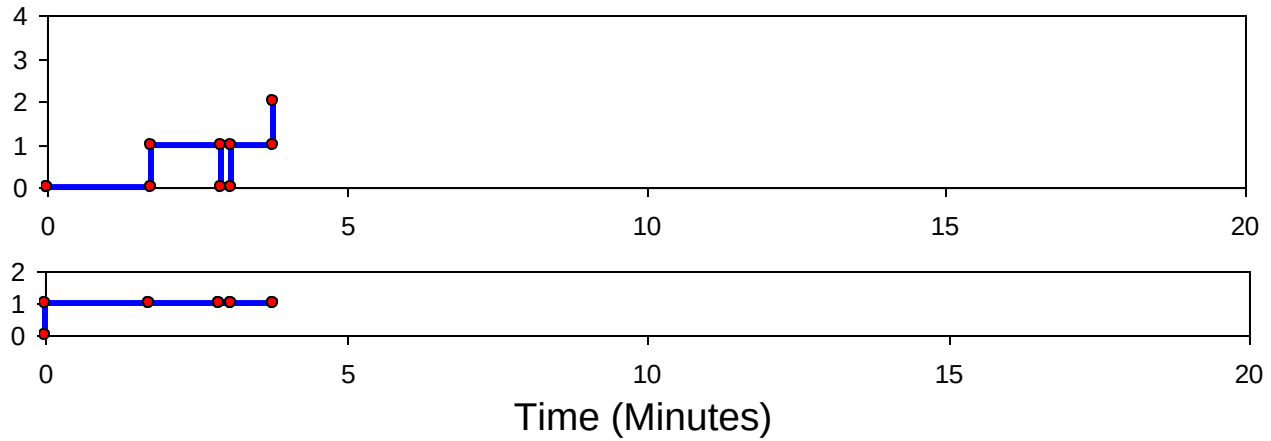


Interarrival times	~~1.73~~, ~~1.35~~, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

t = 3.08, Arrival of Part 3

System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
	3.08	1	1	(3.08) 1.73	[4, 3.79,Arr] [2, 4.66,Dep] [–, 20.00, End]
N: 2 P: 1	Σ WQ: 1.17 Σ TS: 2.90 WQ*: 1.17 TS*: 2.90		Area under $Q(t)$ 1.17		Area under $B(t)$ 3.08
$Q(t)$ graph $B(t)$ graph					
Interarrival times	1.73, 1.35, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...				
Service times	2.90, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...				

t = 3.79, Arrival of Part 4

System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
	3.79	1	2	(3.79, 3.08) 1.73	[5, 4.41,Arr] [2, 4.66,Dep] [–, 20.00, End]
N: 2 P: 1	Σ WQ: 1.17 Σ TS: 2.90 WQ*: 1.17 TS*: 2.90		Area under $Q(t)$ 1.88		Area under $B(t)$ 3.79
$Q(t)$ graph $B(t)$ graph					
Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...				
Service times	~~2.90~~, ~~1.76~~, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...				

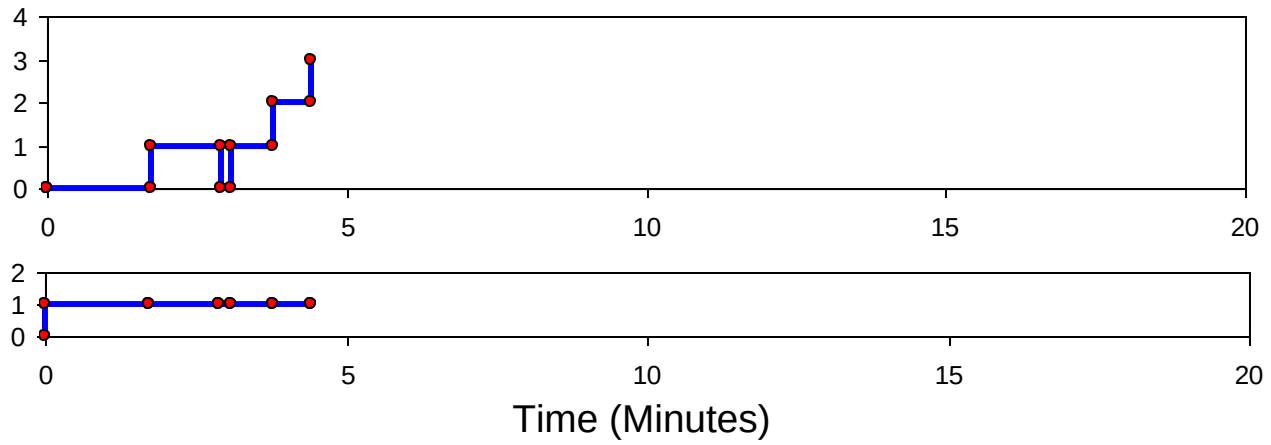
t = 4.41, Arrival of Part 5

System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
5 4 3 **2**	4.41	1	3	(4.41, 3.79, 3.08) 1.73	[2, 4.66, Dep] [6, 18.69, Arr] [–, 20.00, End]

N: 2 P: 1	Σ WQ: 1.17 Σ TS: 2.90 WQ*: 1.17 TS*: 2.90	Area under $Q(t)$ 3.12	Area under $B(t)$ 4.41

$Q(t)$ graph

$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

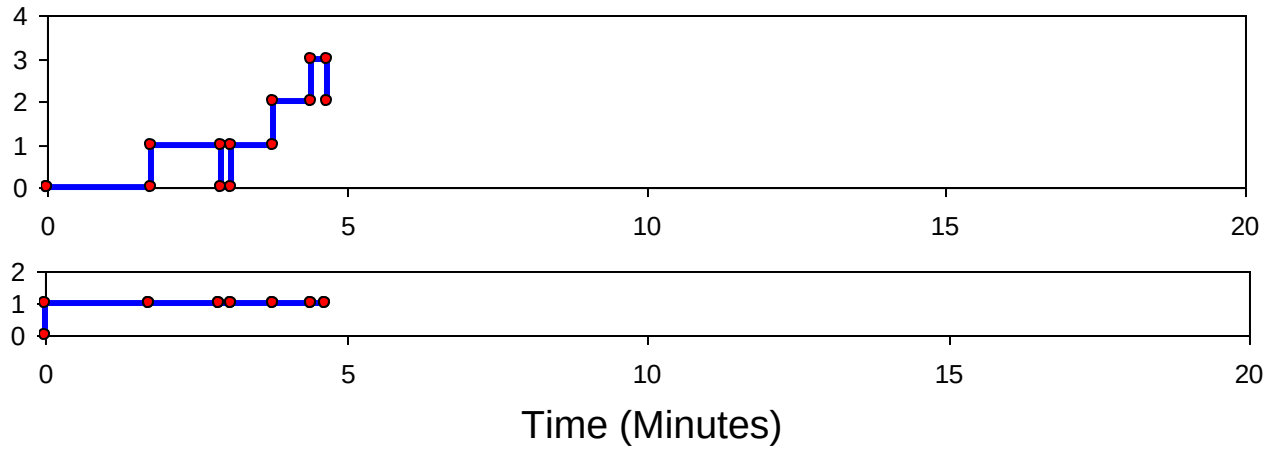
t = 4.66, Departure of Part 2

System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
5 4 3	4.66	1	2	(4.41, 3.79) 3.08	[3, 8.05, Dep] [6, 18.69, Arr] [–, 20.00, End]

N: 3 P: 2	Σ WQ: 2.75 Σ TS: 5.83 WQ*: 1.58 TS*: 2.93	Area under $Q(t)$ 3.87	Area under $B(t)$ 4.66

$Q(t)$ graph

$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

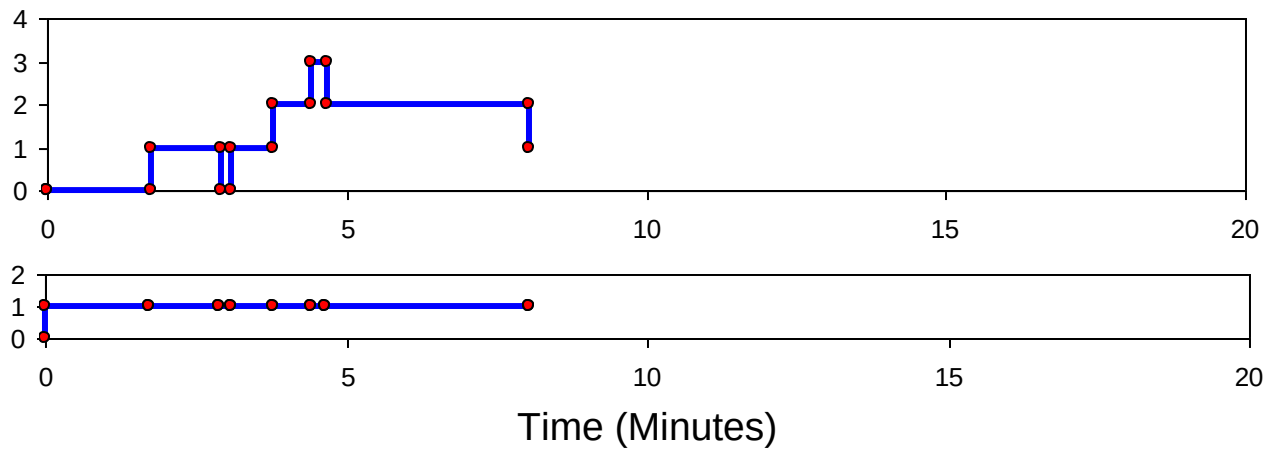
t = 8.05, Departure of Part 3

System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
5 4	8.05	1	1	(4.41) 3.79	[4, 12.57, Dep] [6, 18.69, Arr] [–, 20.00, End]

N: 4 P: 3	Σ WQ: 7.01 Σ TS: 10.80 WQ*: 4.26 TS*: 4.97	Area under $Q(t)$ 10.65	Area under $B(t)$ 8.05

$Q(t)$ graph

$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

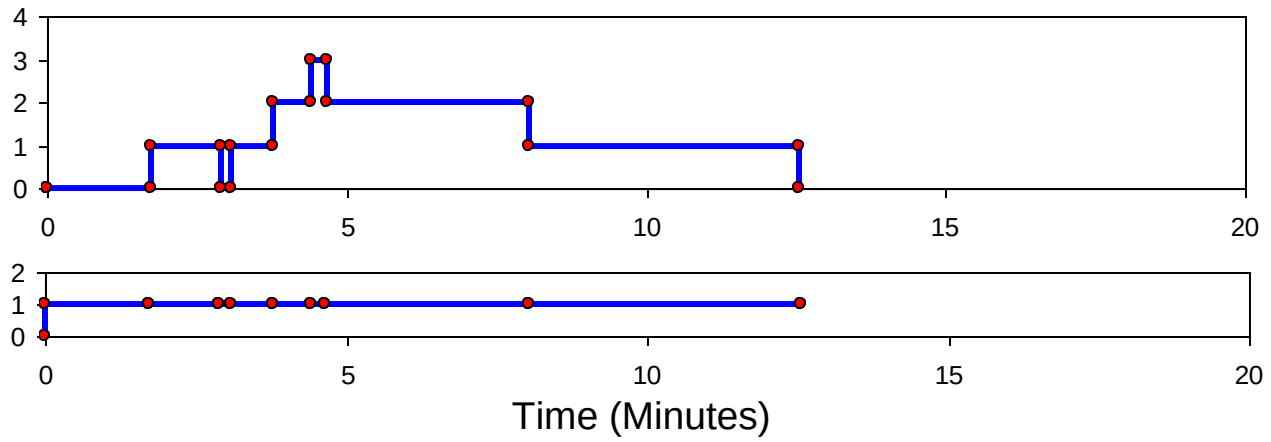
t = 12.57, Departure of Part 4

System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
5	12.57	1	0	() 4.41	[5, 17.03, Dep] [6, 18.69, Arr] [–, 20.00, End]

N: 5 P: 4	Σ WQ: 15.17 Σ TS: 19.58 WQ*: 8.16 TS*: 8.78	Area under $Q(t)$ 15.17	Area under $B(t)$ 12.57

$Q(t)$ graph

$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, 4.36, 2.07, 3.36, 2.37, 5.38, ...

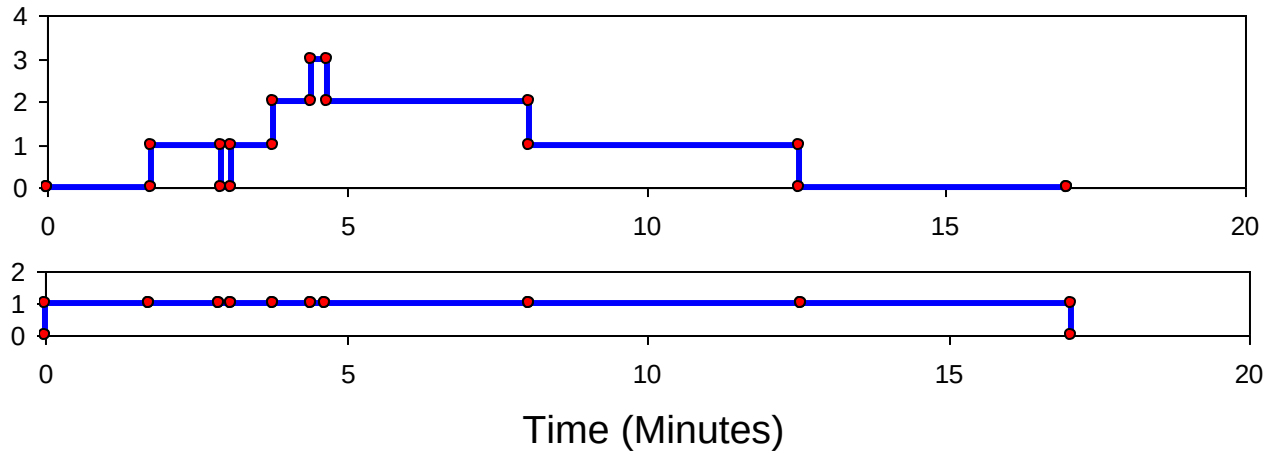
t = 17.03, Departure of Part 5

System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
	17.03	0	0	()	[6, 18.69, Arr] [–, 20.00, End]

N: 5 P: 5	Σ WQ: 15.17 Σ TS: 32.20 WQ*: 8.16 TS*: 12.62	Area under $Q(t)$ 15.17	Area under $B(t)$ 17.03

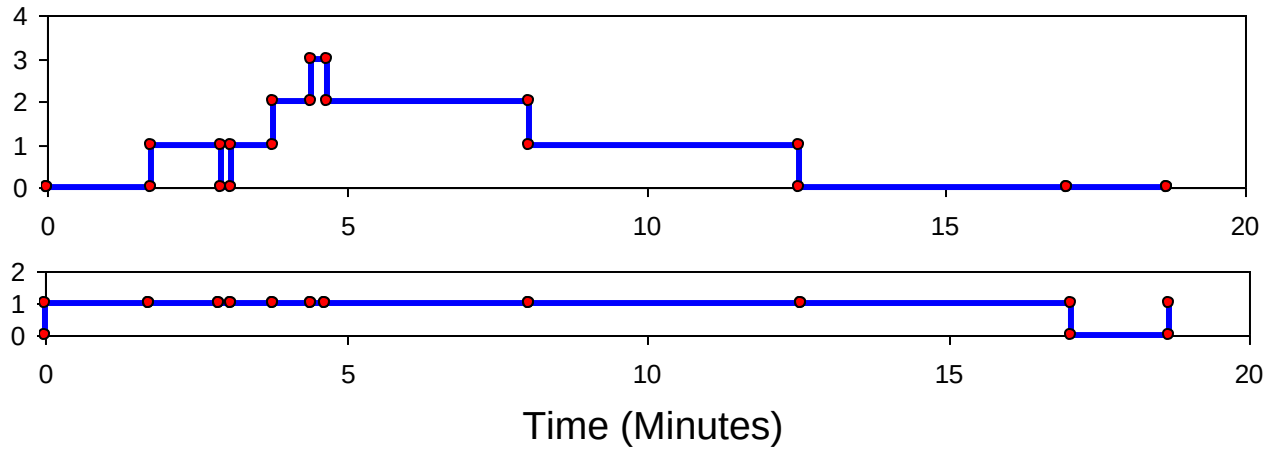
$Q(t)$ graph

$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, 4.36, 2.07, 3.36, 2.37, 5.38, ...

t = 18.69, Arrival of Part 6

System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
6	18.69	1	0	() 18.69	[7, 19.39, Arr] [–, 20.00, End] [6, 23.05, Dep]
N: 6 P: 5	Σ WQ: 15.17 Σ TS: 32.20 WQ*: 8.16 TS*: 12.62		Area under $Q(t)$ 15.17		Area under $B(t)$ 17.03
$Q(t)$ graph $B(t)$ graph					
Interarrival times	1.73, 1.35, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...				
Service times	2.90, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...				

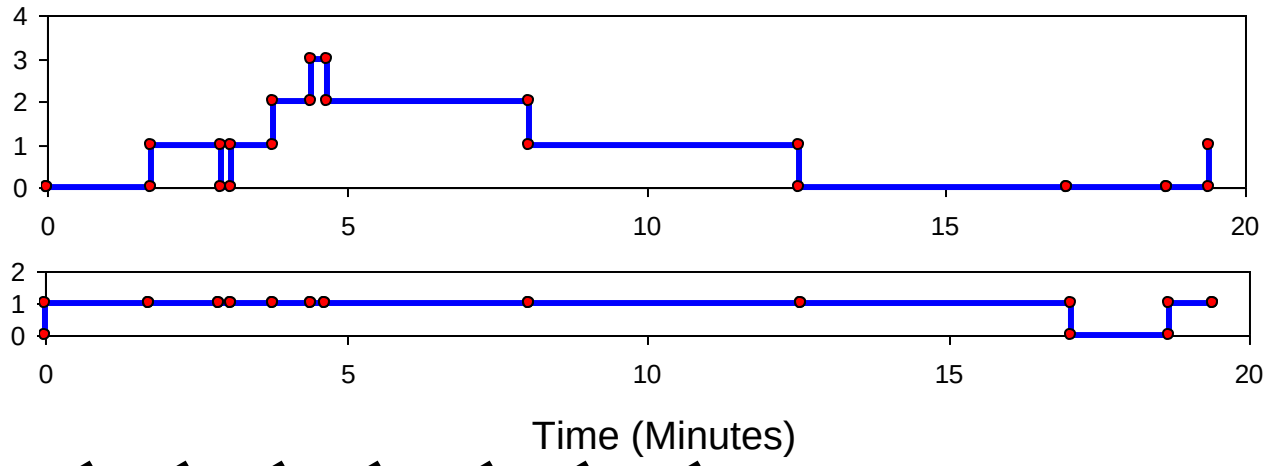
t = 19.39, Arrival of Part 7

System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
7 6	19.39	1	1	(19.39) 18.69	[–, 20.00, End] [6, 23.05, Dep] [8, 34.91, Arr]

N: 6 P: 5	Σ WQ: 15.17 Σ TS: 32.20 WQ*: 8.16 TS*: 12.62	Area under $Q(t)$ 15.17	Area under $B(t)$ 17.73

$Q(t)$ graph

$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, ~~0.70~~, ~~15.52~~, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, ~~4.36~~, 2.07, 3.36, 2.37, 5.38, ...

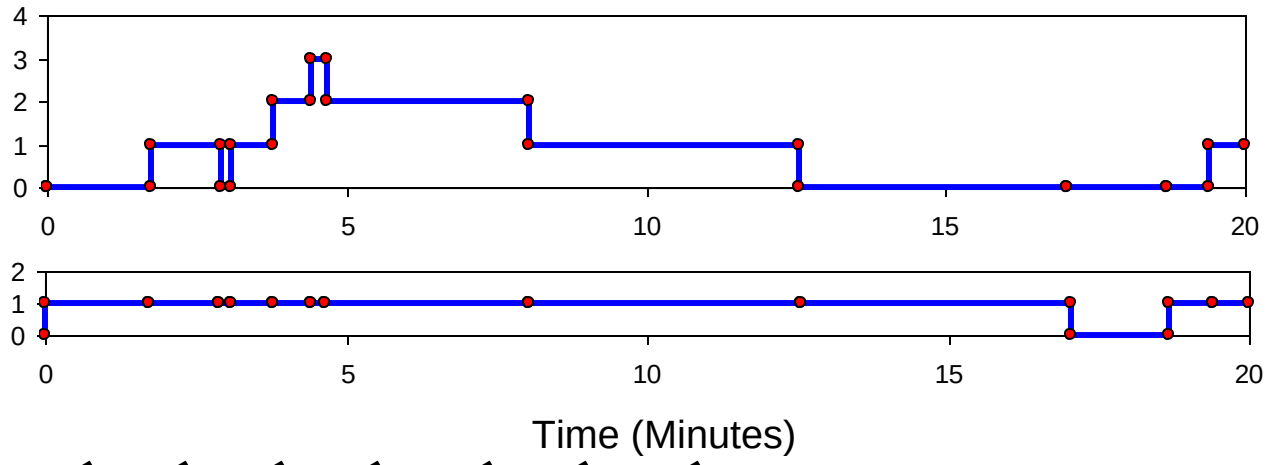
t = 20.00, The End

System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
7 6	20.00	1	1	(19.39) 18.69	[6, 23.05, Dep] [8, 34.91, Arr]

N: 6 P: 5	Σ WQ: 15.17 Σ TS: 32.20 WQ*: 8.16 TS*: 12.62	Area under $Q(t)$ 15.78	Area under $B(t)$ 18.34

$Q(t)$ graph

$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, ~~0.70~~, ~~15.52~~, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, ~~4.36~~, 2.07, 3.36, 2.37, 5.38, ...

Complete Record of Hand Simulation

Just-Finished Event			Variables		Attributes		Statistical Accumulators							Event Calendar
Entity No.	Time t	Event Type	$Q(t)$	$B(t)$	Arrival Times: (In Queue)	In Service	P	N	Σ_{WQ}	WQ^*	Σ_{TS}	TS^*	$\int_Q$	Q^*	$\int_B$	[Entity No., Time, Type]
–	0.00	Init	0	0	( )	–	0	0	0.00	0.00	0.00	0.00	0.00	0	0.00	[1, 0.00, Arr] [–, 20.00, End]
1	0.00	Arr	0	1	( )	0.00	0	1	0.00	0.00	0.00	0.00	0.00	0	0.00	[2, 1.73, Arr] [1, 2.90, Dep] [–, 20.00, End]
2	1.73	Arr	1	1	(1.73)	0.00	0	1	0.00	0.00	0.00	0.00	0.00	1	1.73	[1, 2.90, Dep] [3, 3.08, Arr] [–, 20.00 End]
1	2.90	Dep	0	1	( )	1.73	1	2	1.17	1.17	2.90	2.90	1.17	1	2.90	[3, 3.08, Arr] [2, 4.66, Dep] [–, 20.00, End]
3	3.08	Arr	1	1	(3.08)	1.73	1	2	1.17	1.17	2.90	2.90	1.17	1	3.08	[4, 3.79, Arr] [2, 4.66, Dep] [–, 20.00, End]
4	3.79	Arr	2	1	(3.79, 3.08)	1.73	1	2	1.17	1.17	2.90	2.90	1.88	2	3.79	[5, 4.41, Arr] [2, 4.66, Dep] [–, 20.00, End]
5	4.41	Arr	3	1	(4.41, 3.79, 3.08)	1.73	1	2	1.17	1.17	2.90	2.90	3.12	3	4.41	[2, 4.66, Dep] [6, 18.69, Arr] [–, 20.00, End]
2	4.66	Dep	2	1	(4.41, 3.79)	3.08	2	3	2.75	1.58	5.83	2.93	3.87	3	4.66	[3, 8.05, Dep] [6, 18.69, Arr] [–, 20.00, End]
3	8.05	Dep	1	1	(4.41)	3.79	3	4	7.01	4.26	10.80	4.97	10.65	3	8.05	[4, 12.57, Dep] [6, 18.69, Arr] [–, 20.00, End]
4	12.57	Dep	0	1	( )	4.41	4	5	15.17	8.16	19.58	8.78	15.17	3	12.57	[5, 17.03, Dep] [6, 18.69, Arr] [–, 20.00, End]
5	17.03	Dep	0	0	( )	–	5	5	15.17	8.16	32.20	12.62	15.17	3	17.03	[6, 18.69, Arr] [–, 20.00 End]
6	18.69	Arr	0	1	( )	18.69	5	6	15.17	8.16	32.20	12.62	15.17	3	17.03	[7, 19.39, Arr] [–, 20.00, End] [6, 23.05, Dep]
7	19.39	Arr	1	1	(19.39)	18.69	5	6	15.17	8.16	32.20	12.62	15.17	3	17.73	[–, 20.00, End] [6, 23.05, Dep] [8, 34.91, Arr]
–	20.00	End	1	1	(19.39)	18.69	5	6	15.17	8.16	32.20	12.62	15.78	3	18.34	[6, 23.05, Dep] [8, 34.91, Arr]