IE375 Spring 2024 Exercise Set 2b

Question 1:

A high-quality grocery wholesaler in Ankara is selling apples and oranges to retailers in Ankara. Oranges are sourced from Antalya and apples are sourced from Isparta. Assume that transportation time is negligible, as well as it is an infinite horizon problem in all the questions posed below. The daily demands and purchase costs for apples and oranges are given below:

Fruit	*Orange*	*Apple*
Demand (tons/day)	1.3	3
Purchase Cost (TL/ton)	3000	1200

Inventory holding costs are 24% of the purchase cost per year. Assume 250 days per year.

A) Currently the wholesaler uses two separate trucks to get the apples and oranges. The shipping cost for oranges from Antalya is 130 TL per trip. The shipping cost for apples from Isparta is 120 TL per trip. The objective is to minimize average total cost.

- How many kilograms of oranges should the wholesaler order each time?
- How many kilograms of apples should the wholesaler order each time?
- What is the total average cost of operating two separate trucks excluding the unit purchasing costs of oranges and apples?

B) The wholesaler is considering using a single large truck that will pick up apples from Isparta and then pick up oranges from Antalya. The shipping cost for this single truck is K=140 TL per trip. The objective is to minimize average total cost.

- (8 points) How many kilograms of oranges and apples should be ordered each time?
- (7 points) What is the total average cost of operating a single truck excluding the unit purchasing costs of oranges and apples?

Question 2:

The inventory of a purchased item is controlled with a continuous review (Q,R) policy. The item demand over 4 weeks of lead time follows a uniform distribution with a mean of 500 units in the interval of (300, 700). Assume that the ordering quantity is fixed and 1000 units.

a) Find the fill rate value if the reorder level is 400 units (Fill rate is the proportion of total demand satisfied directly from the shelf.)

b) Find the probability of having a backorder in an ordering cycle if reorder level is 400 units.

Question 3:

This questions tests your knowledge of using normal tables for calculations. Hence, you are expected to interpolate when necessary. Please, round your numbers with 4 decimal digits, if the number is less than one. **Please, show your computations in detail.**

Consider an inventory system operating under a **(Q, R)** policy. Assume that demand during lead-time is given by a normal distribution with mean 100 and standard deviation 30. There are two equations obtained for the solution two decision variables, namely Q and R. You are asked to implement the iterative algorithm described in class and in the text-book:

- Start with Q_0 = EOQ. Then **compute R_0** using $[1 - F(R)] = hQ/(p\lambda)$.
- Using the second condition given in the formula sheet, **compute Q_1** value.
- Now **compute R_1** value.
- Continue until the difference in Q and R is less than 10^{-3}.

The values of parameters that you need are: h = 1, K = 1280, p = 10, λ = 1000. Note that you need to use normal tables supplied to you.

Question 4:

Consider a retailer which sells N products during a season. The retailer buys items from different suppliers and sells all of them in a single period, i.e. during the same season (as an example, consider a garment store selling for the winter period). The following information is provided for each item i, i=1, ... , N:

c_{io} : cost of overage for item i

c_{iu} : cost of underage for item i

$f_i(d_i)$: pdf of demand during the season for item i

$F_i(d_i)$: cdf of demand during the season for item i

$f_i(d_i) = \lambda_i e^{-d_i\lambda_i}$ for $d_i \geq 0$

$F_i(d_i) = 1 - e^{-d_i\lambda_i}$ for $d_i \geq 0$

$E[d] = 1/\lambda_i$

a) Consider a single item and call it item 1. Show that, if the retailer wants to minimize expected total overage and underage costs, the optimal ordering quantity for item 1 is given by the following equation:

$$Q_1 = -(1/\lambda_1)\ln\{c_{1o}/(c_{1o}+c_{1u})\}$$

b) Now consider the multi-item problem. Consider a situation where there is no salvage value or salvage cost for the items remaining at the end of the season, as well as no cost of goodwill loss for the demand that cannot be satisfied. The only relevant parameters available are given below:

c_i : unit cost of buying item i

r_i : unit revenue obtained by selling item i

Identify the overage and underage costs in terms of the above parameters.

c) The retailer faces a budget problem when all items are ordered from the suppliers for the beginning of the season. Specifically, there is a budget *B* available in the beginning of the season to pay for all items ordered in the beginning of the season. State the meaning of the following inequality:

$$\sum_i -(c_i/\lambda_i)\ln\{c_{io}/(c_{io}+c_{iu})\} \geq B$$

d) Formulate the expected total profit maximization problem subject to this budget constraint, using c_i and r_i rather than underage and overage costs. Write all the terms as explicit as possible. DO NOT SOLVE, but explain how you would solve this problem.