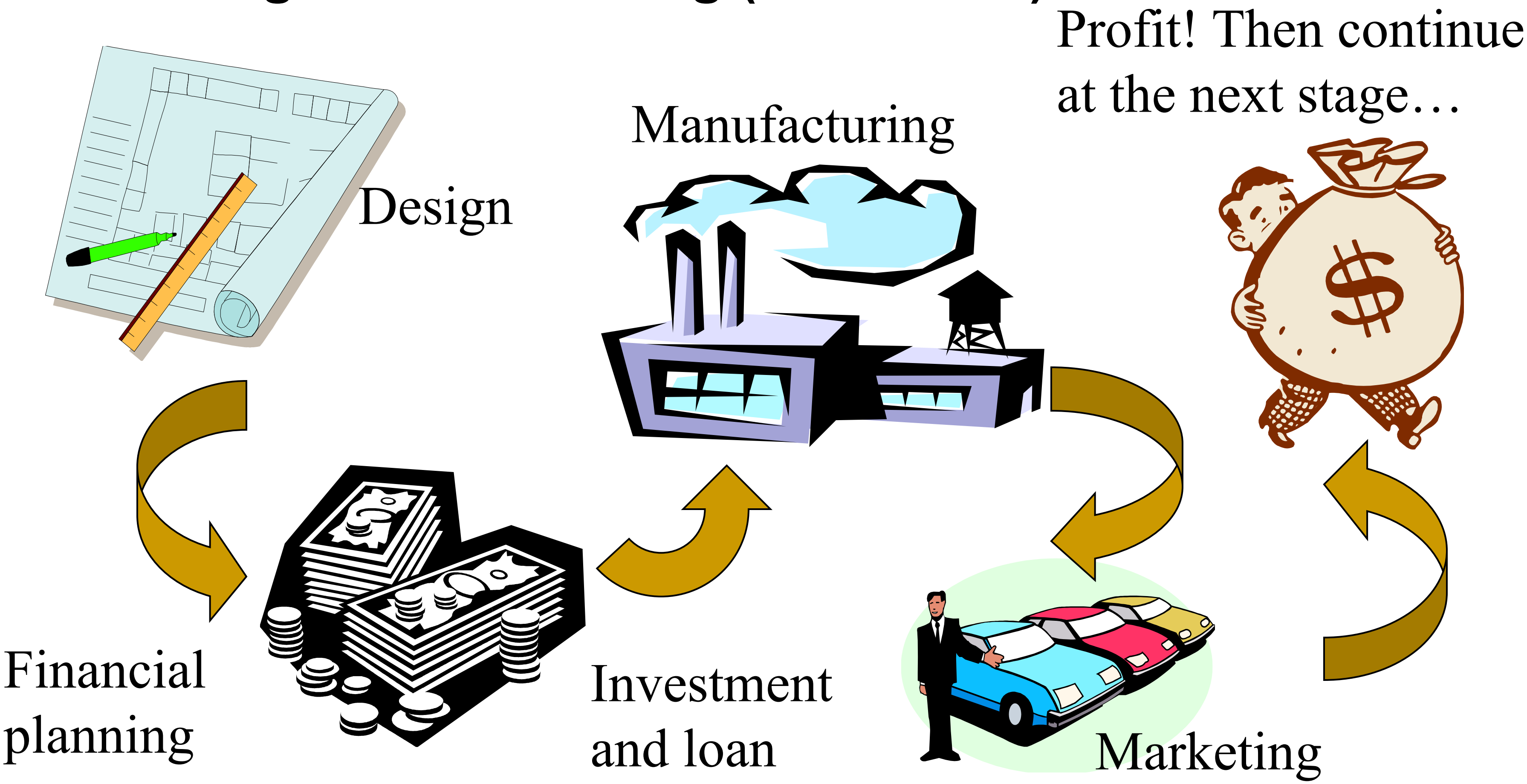
Engineering Economic Decisions

Presentation based on the book

Chan S. Park, *Contemporary Engineering Economics*
Chapter 1, © Pearson Education International Edition

Engineering Economic Decisions (Role of Engineers)

Needed e.g. in the following (connected) areas:



What Makes Engineering Economic Decisions Difficult? Predicting the Future

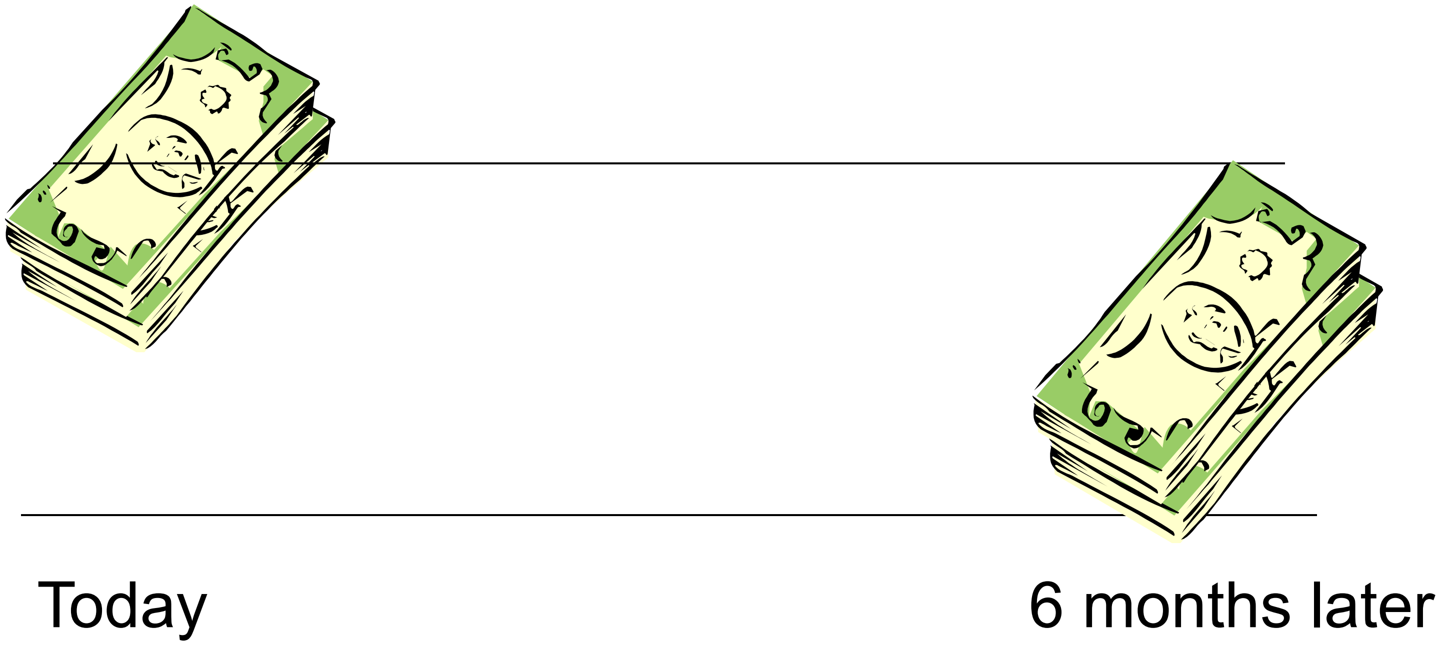
- Estimating the required investments
- Estimating product manufacturing costs
- Forecasting the demand for a brand new product
- Estimating a “good” selling price
- Estimating product life and the profitability of continuing production

The Four Fundamental Principles of Engineering Economics

- 1: An nearby penny is worth a distant dollar
- 2: Only the relative (pair-wise) difference among the considered alternatives counts
- 3: Marginal revenue must exceed marginal cost, in order to carry out a profitable increase of operations
- 4: Additional risk is not taken without an expected additional return of suitable magnitude

Principle 1

An instant dollar is worth more than a distant dollar



Principle 2

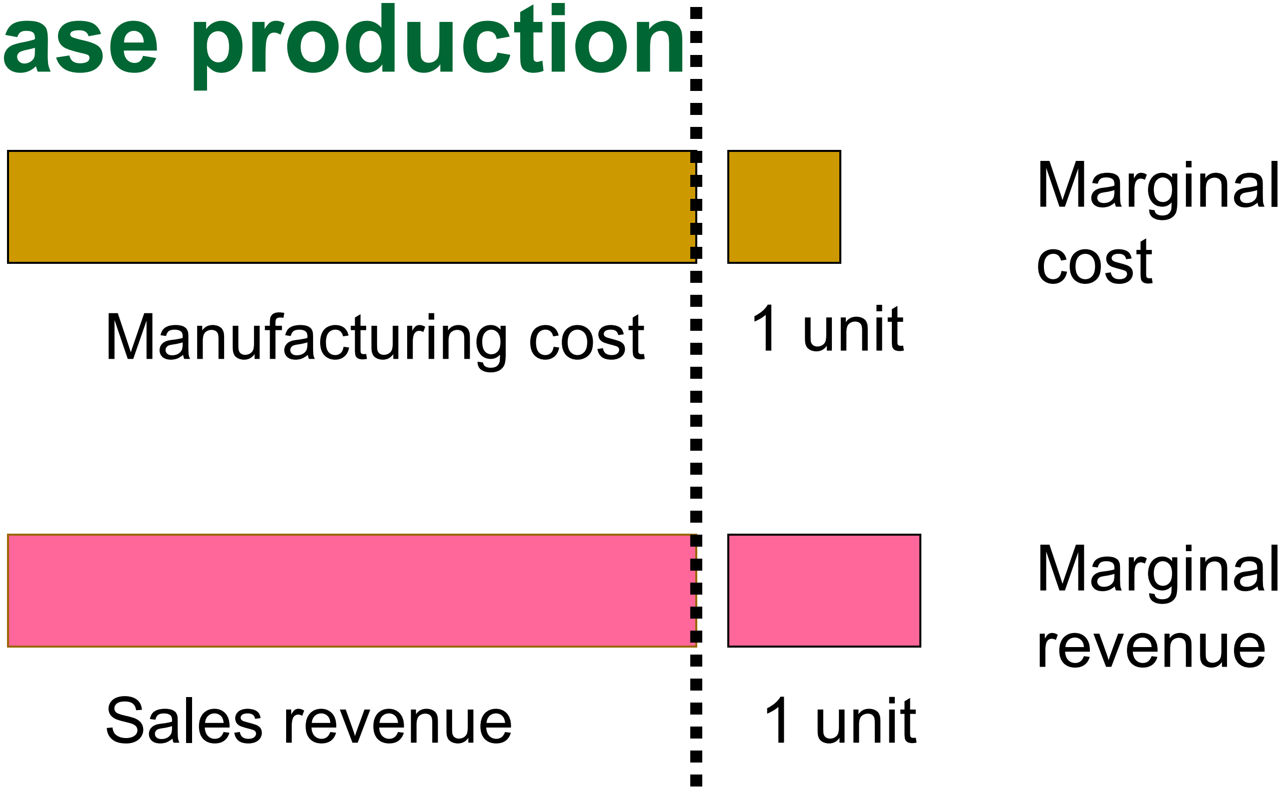
Only the cost (resource) difference among alternatives counts

Option	Monthly Fuel Cost	Monthly Maintenance	Cash paid at signing (cash outlay)	Monthly payment	Salvage Value at end of year 3
Buy	$960	$550	$6,500	$350	$9,000
Lease	$960	$550	$2,400	$550	0

The data shown in the green fields are irrelevant items for decision making, since their financial impact is identical in both cases

Principle 3

Marginal (unit) revenue has to exceed marginal cost, in order to increase production



Principle 4

Additional risk is not taken without a suitable expected additional return

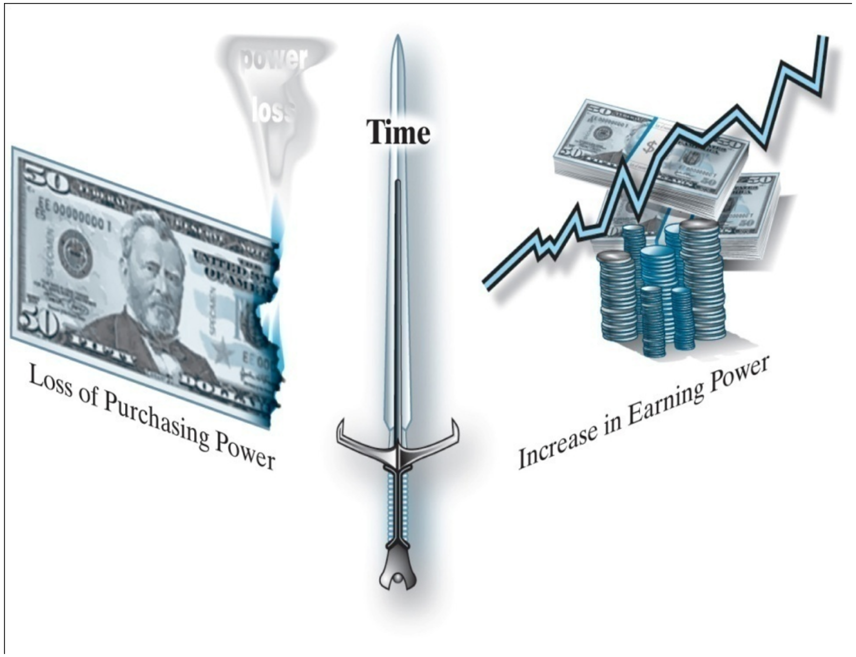
Investment Class	Potential Risk	Expected Return
Savings account (cash)	Lowest	1.5%
Bond (debt)	Moderate	4.8%
Stock (equity)	Highest	11.5%

A simple illustrative example. Note that all investments imply some risk: portfolio management is a key issue in finance

Time Value of Money

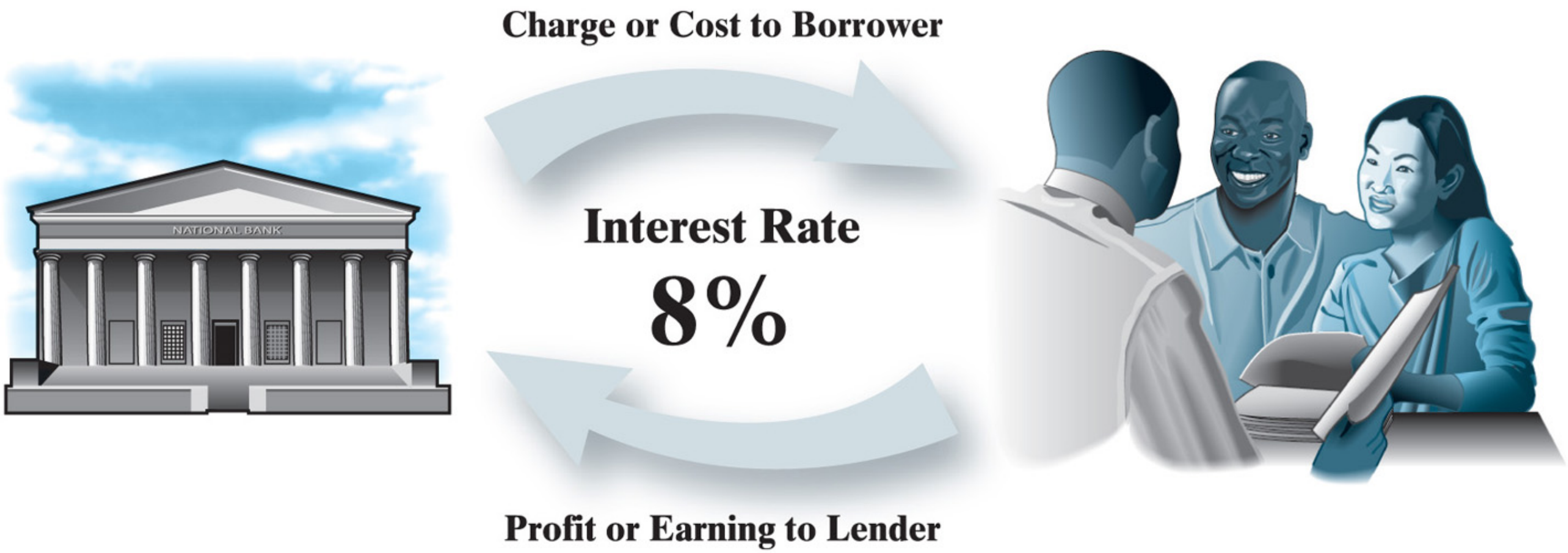
Time Value of Money

- Money has a time value because it can earn more money over time (earning power).
- Money has a time value because its purchasing power changes over time (inflation).
- Time value of money is measured in terms of interest rate.
- Interest is the cost of money—a cost to the borrower and an earning to the lender



This a two-edged sword whereby earning grows, but purchasing power decreases (due to inflation), as time goes by.

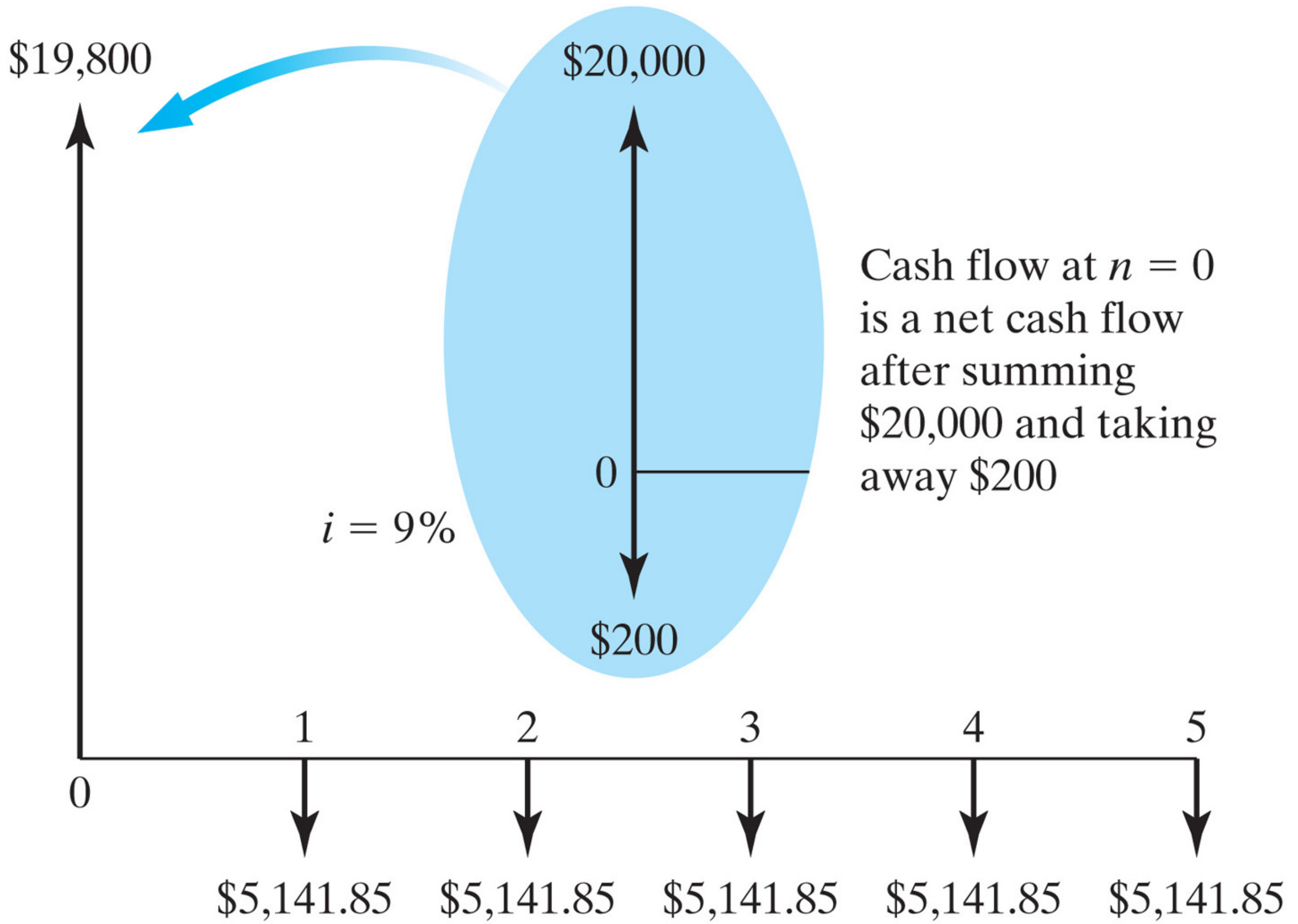
The Interest Rate



Cash Flow Transactions for Two Types of Loan Repayment

End of Year	Receipts	Payments	
		Plan 1	Plan 2
Year 0	$20,000.00	$200.00	$200.00
Year 1		5,141.85	0
Year 2		5,141.85	0
Year 3		5,141.85	0
Year 4		5,141.85	0
Year 5		5,141.85	30,772.48
The amount of loan = $20,000, origination fee = $200, interest rate = 9% APR (annual percentage rate)			

Cash Flow Diagram for Plan 1



End-of-Period Convention

- In practice, cash flows can occur at the beginning or in the middle of an interest period, or indeed, at practically any point in time.
- One of the simplifying assumptions we make in engineering economic analysis is the end-of-period convention.
- **End-of-period convention:**

Unless otherwise mentioned, all cash flow transactions occur at the end of an interest period.

Methods of Calculating Interest

- Simple interest: the practice of charging an interest rate only to an initial sum (principal amount).
- Compound interest: the practice of charging an interest rate to an initial sum and to any previously accumulated interest that has not been withdrawn.

Simple Interest

- *P* = Principal amount
- *i* = Interest rate
- *N* = Number of interest periods
- Example:
  - *P* = $1,000
  - *i* = 10%
  - *N* = 3 years

End of Year	Beginning Balance	Interest earned	Ending Balance
0			$1,000
1	$1,000	$100	$1,100
2	$1,100	$100	$1,200
3	$1,200	$100	$1,300

Simple Interest Formula

$$F = P + (iP)N$$

where

P = Principal amount

i = simple interest rate

N = number of interest periods

F = total amount accumulated at the end of period N

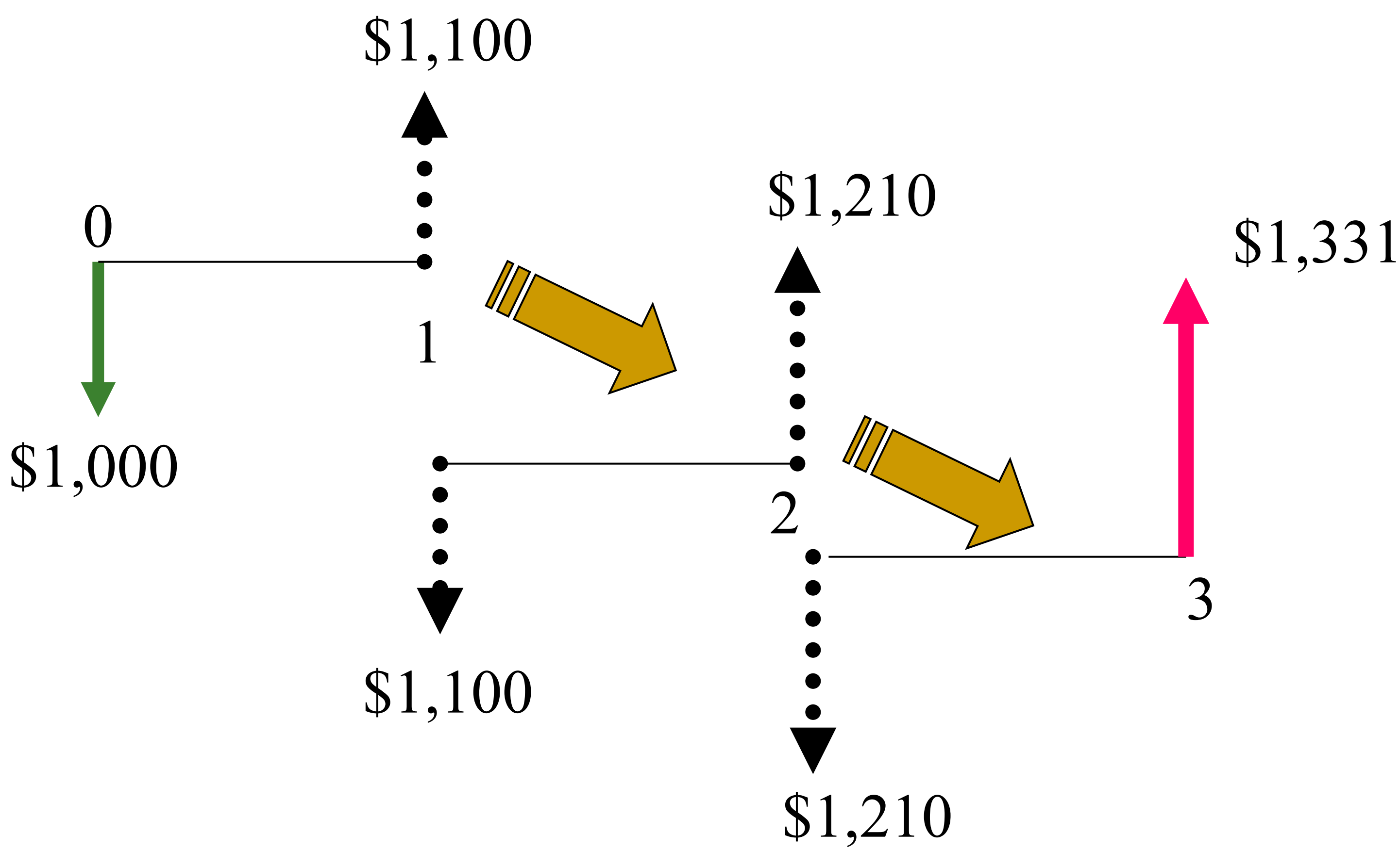
$$\begin{aligned} F &= \$1{,}000 + (0.10)(\$1{,}000)(3) \\ &= \$1{,}300 \end{aligned}$$

Compound Interest

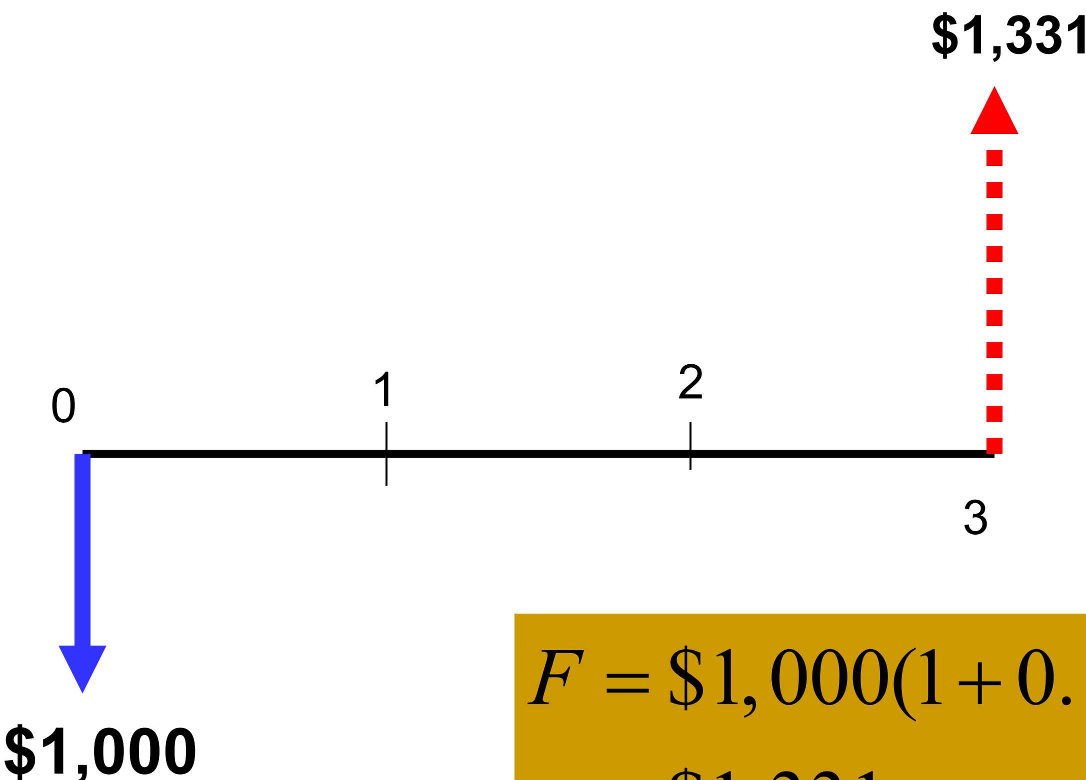
- P = Principal amount
- i = Interest rate
- N = Number of interest periods
- Example:
  - P = $1,000
  - i = 10%
  - N = 3 years

End of Year	Beginning Balance	Interest earned	Ending Balance
0			$1,000
1	$1,000	$100	$1,100
2	$1,100	$110	$1,210
3	$1,210	$121	$1,331

Compounding Process



Cash Flow Diagram



$$F = \$1,000(1+0.10)^3$$
$$= \$1,331$$

Compound Interest Formula

$$n = 0 : P$$

$$n = 1 : F_1 = P(1 + i)$$

$$n = 2 : F_2 = F_1(1 + i) = P(1 + i)^2$$

$$\vdots$$

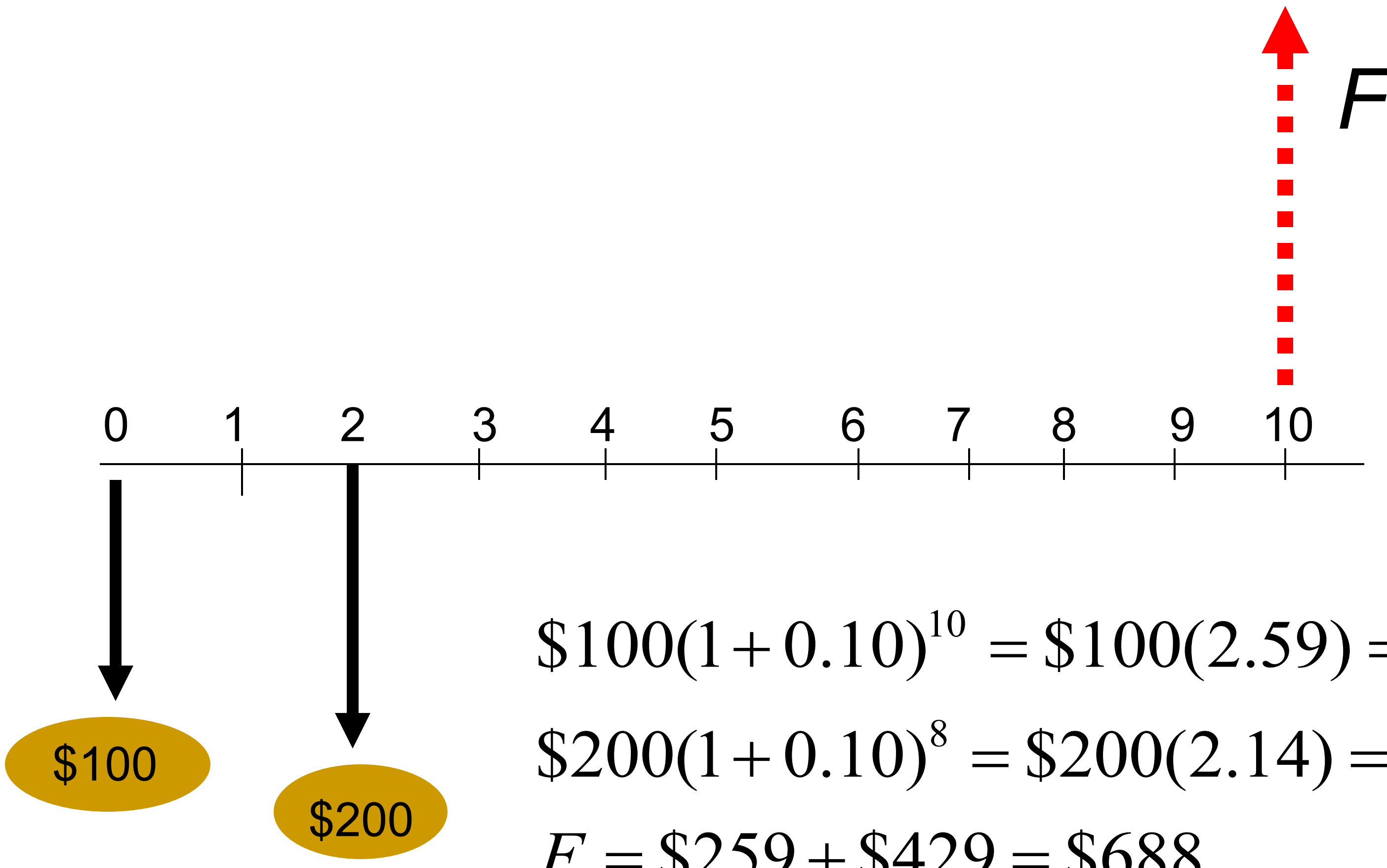
$$n = N : \boxed{F = P(1 + i)^N}$$

Practice Problem

- <u>Problem Statement</u>

  If you deposit \$100 now (n = 0) and \$200 two years from now (n = 2) in a savings account that pays 10% interest, how much would you have at the end of year 10?

Solution



$$\$100(1+0.10)^{10} = \$100(2.59) = \$259$$

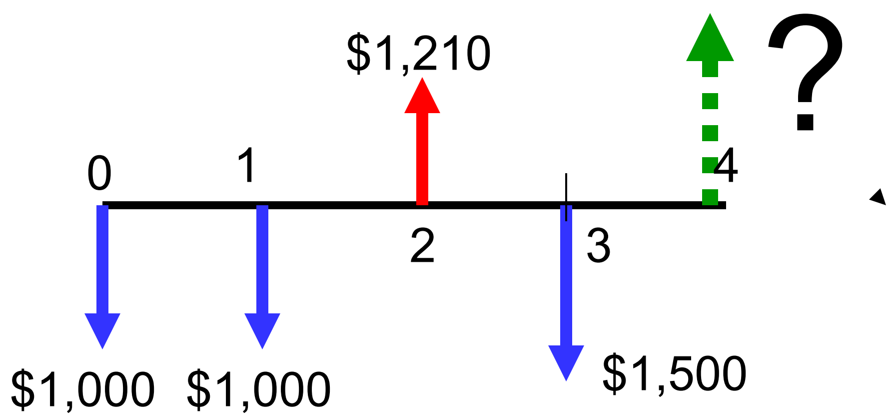
$$\$200(1+0.10)^{8} = \$200(2.14) = \$429$$

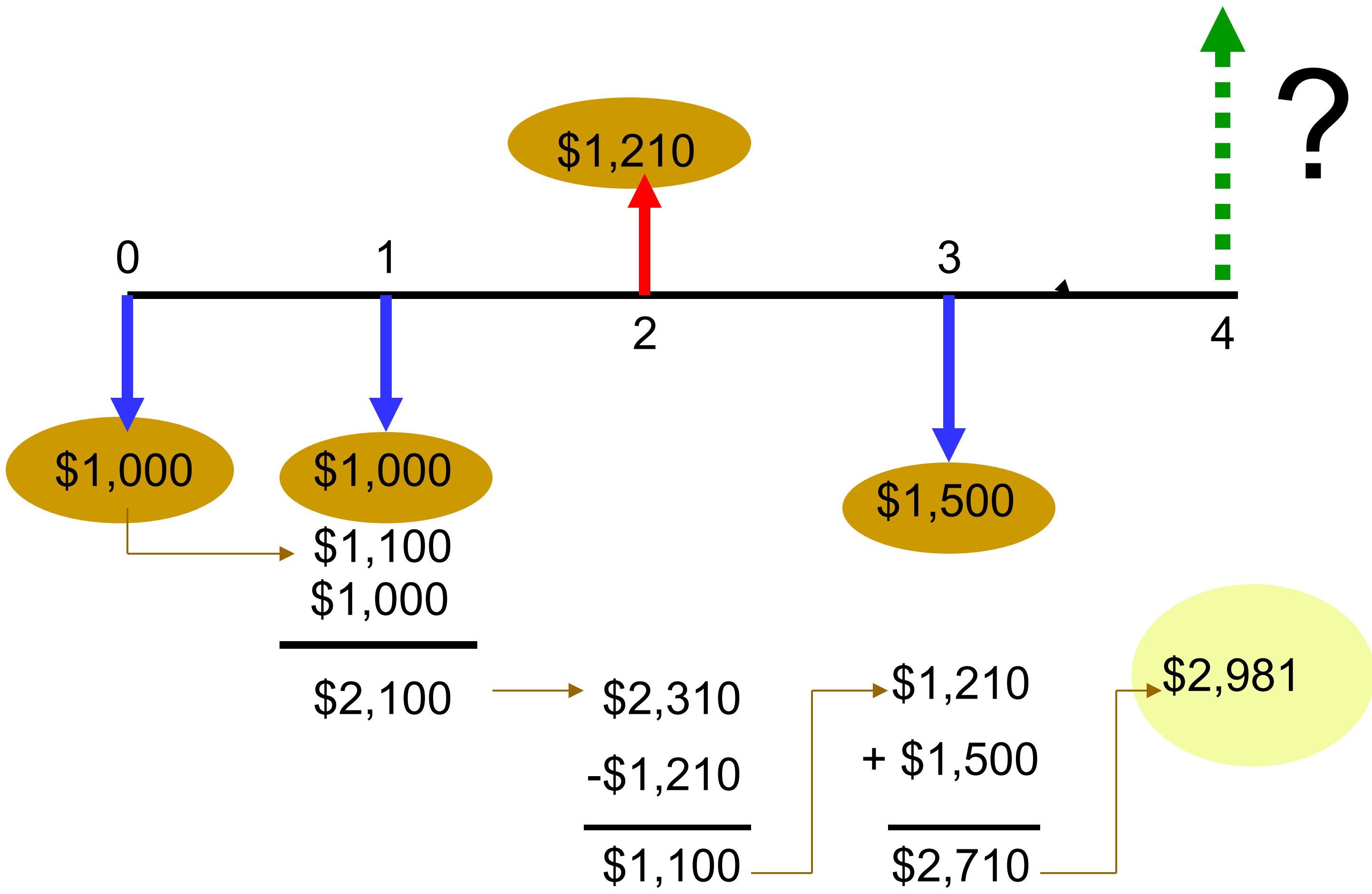
$$F = \$259 + \$429 = \$688$$

Practice problem

- <u>Problem Statement</u>

  Consider the following sequence of deposits and withdrawals over a period of 4 years. If you earn a 10% interest, what would be the balance at the end of 4 years?




$1,210
0
1
3
?
2
4
$1,000
$1,000
$1,500
$1,100
$1,000
$2,100
$2,310
-$1,210
$1,100
$1,210
+ $1,500
$2,710
$2,981

Solution

End of Period	Beginning balance	Deposit made	Withdraw	Ending balance
$n = 0$	0	\$1,000	0	\$1,000
$n = 1$	\$1,000(1 + 0.10) =\$1,100	\$1,000	0	\$2,100
$n = 2$	\$2,100(1 + 0.10) =\$2,310	0	\$1,210	\$1,100
$n = 3$	\$1,100(1 + 0.10) =\$1,210	\$1,500	0	\$2,710
$n = 4$	\$2,710(1 + 0.10) =\$2,981	0	0	\$2,981

Economic Equivalence

- **What** do we mean by "economic equivalence?"
- **Why** do we need to establish an economic equivalence?
- **How** do we measure and compare various cash payments received at different points in time?

What is "Economic Equivalence?"

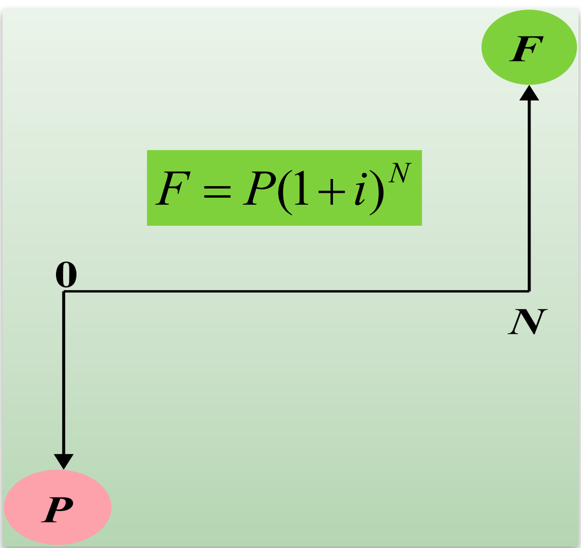
Economic equivalence exists between cash flows that have the <u>same economic effect</u> and could therefore be traded for one another.

Even though the amounts and timing of the cash flows may differ, the appropriate interest rate makes them equal in economic sense.

Equivalence

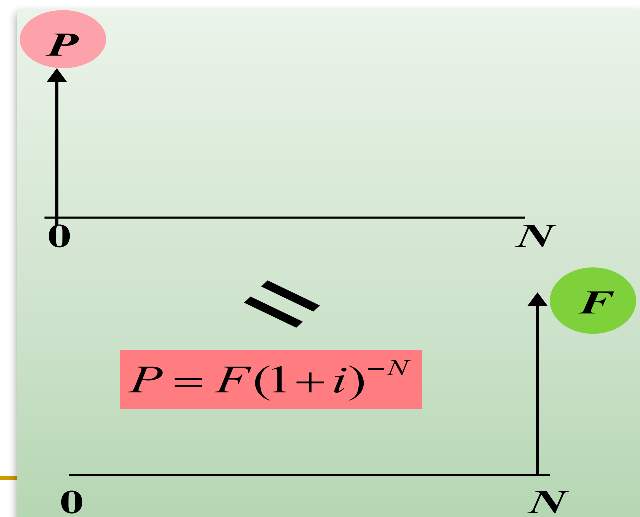
Equivalence from Personal Financing Point of View

If you deposit *P* dollars today for *N* periods at *i*, you will have *F* dollars at the end of period *N*.



Alternate Way of Defining Equivalence

F dollars at the end of period *N* is equal to a single sum *P* dollars now, if your earning power is measured in terms of interest rate *i*.



Example 3.3 Equivalence

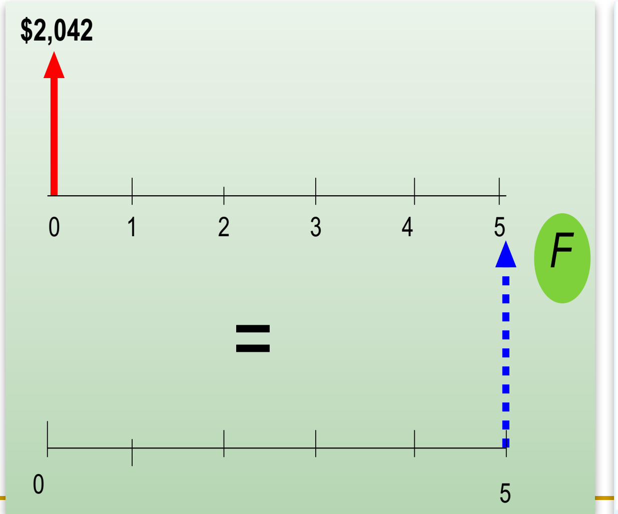
Various dollar amounts that will be economically equivalent to $3,000 in five years, at an interest rate of 8%

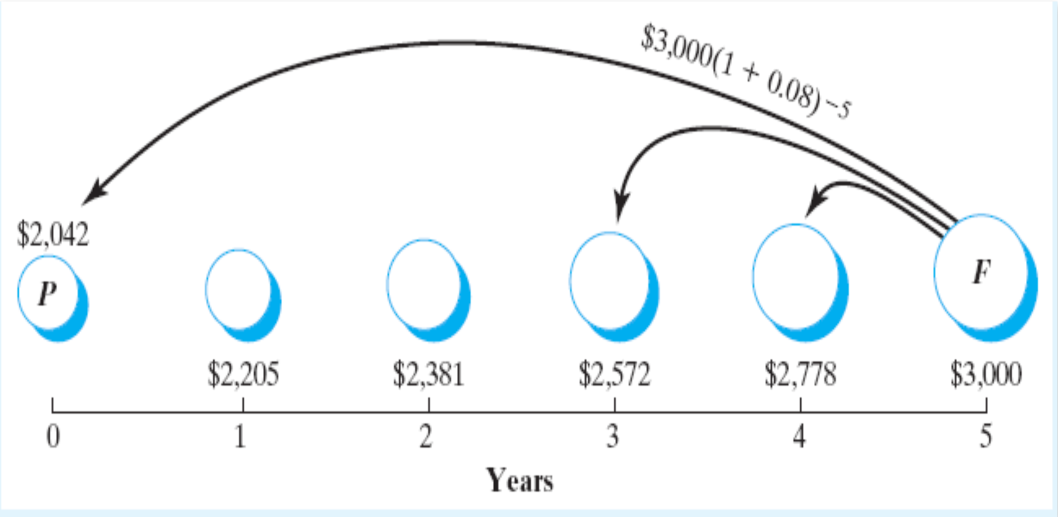
If you deposit $2,042 today in a savings account that pays an 8% interest annually, how much would you have at the end of 5 years?

At an 8% interest, what is the equivalent worth of $2,042 now in 5 years?

$$F = \$2,042(1 + 0.08)^5 = \$3,000$$

$$P = \frac{\$3,000}{(1 + 0.08)^5} = \$2,042$$

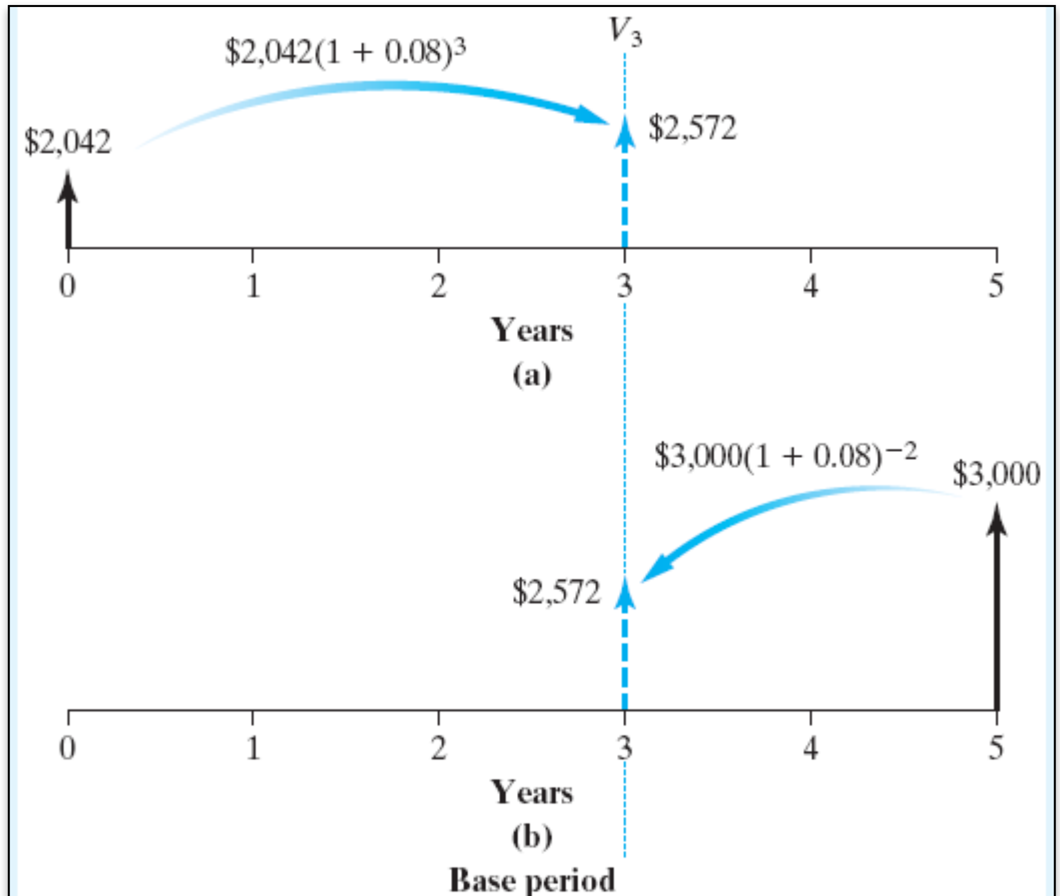




Example 3.4
Equivalent Cash Flows

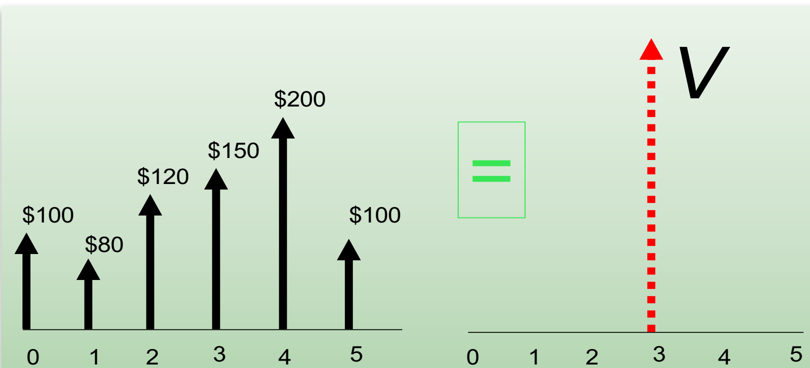
$2,042 today was equivalent to receiving $3,000 in five years, at an interest rate of 8%. Are these two cash flows also equivalent at the end of year 3?

Equivalent cash flows are equivalent at any common point in time, as long as we use the same interest rate (8%, in our example).

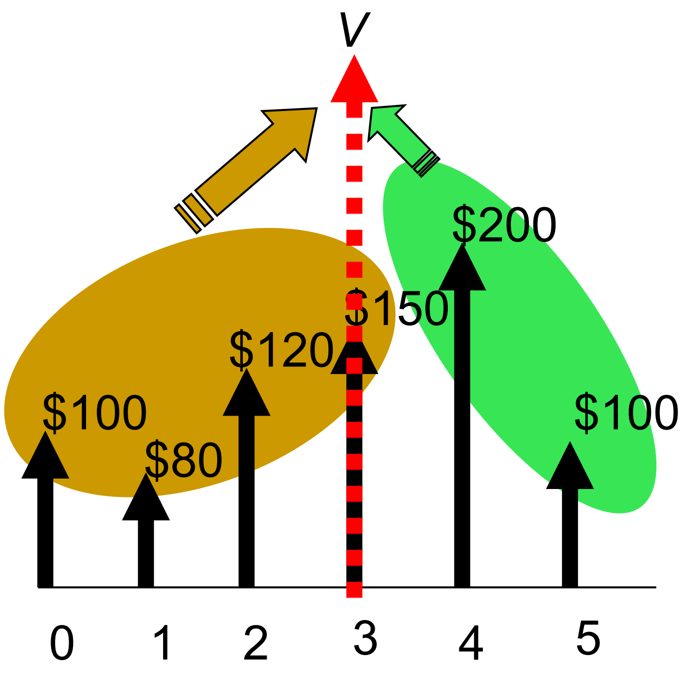


Practice Problem 1

Compute the equivalent value of the cash flow series at $n = 3$, using $i = 10\%$.



Practice Problem 1



V= $100 (1 + 0.10)^3 + $80 (1 + 0.10)^2 + $120 (1 + 0.10) + $150 +

$200 (1 + 0.10)$^{-1}$ + $100 (1 + 0.10)$^{-2}$

= $511.90+$264.46

=$776.36

Practice Problem 2

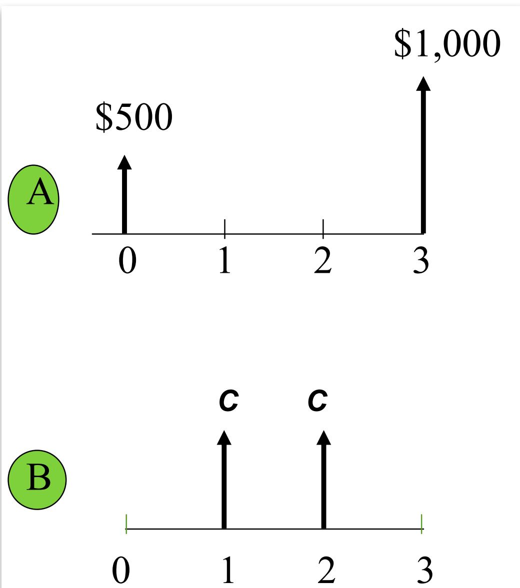
Find C that makes the two cash flow transactions equivalent at $i = 10\%$

Approach:

Step 1: Select a base period to use, say $n = 2$.

Step 2: Find the equivalent lump sum value at $n = 2$ for both A and B.

Step 3: Equate both equivalent values and solve for unknown C.



For A: $V_2 = \$500(1+0.10)^2 + \$1{,}000(1+0.10)^{-1}$

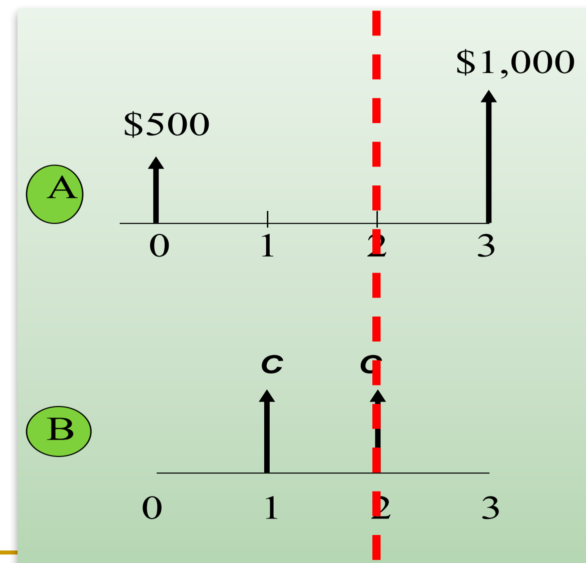
$= \$1{,}514.09$

For B: $V_2 = C(1+0.10) + C$

$= 2.1C$

$2.1C = \$1{,}514.09$

$C = \$721$



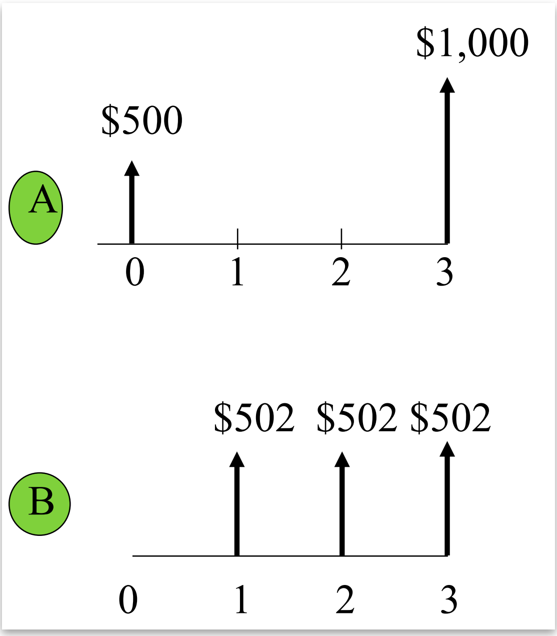
Practice Problem 3

At what interest rate would you be indifferent between the two cash flows?

Approach:

Step 1: Select a base period to compute the equivalent value (say, $n = 3$)

Step 2: Find the equivalent worth of each cash flow series at $n = 3$.



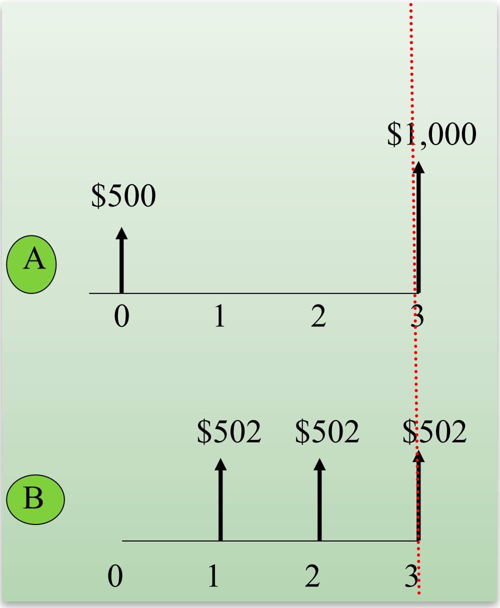
$$\text{Option A}: F_3 = \$500(1+i)^3 + \$1{,}000$$

$$\text{Option B}: F_3 = \$502(1+i)^2 + \$502(1+i) + \$502$$

$i = 8\%$

$$\text{Option A}: F_3 = \$500(1.08)^3 + \$1{,}000 = \$1{,}630$$

$$\text{Option B}: F_3 = \$502(1.08)^2 + \$502(1.08) + \$502 = \$1{,}630$$

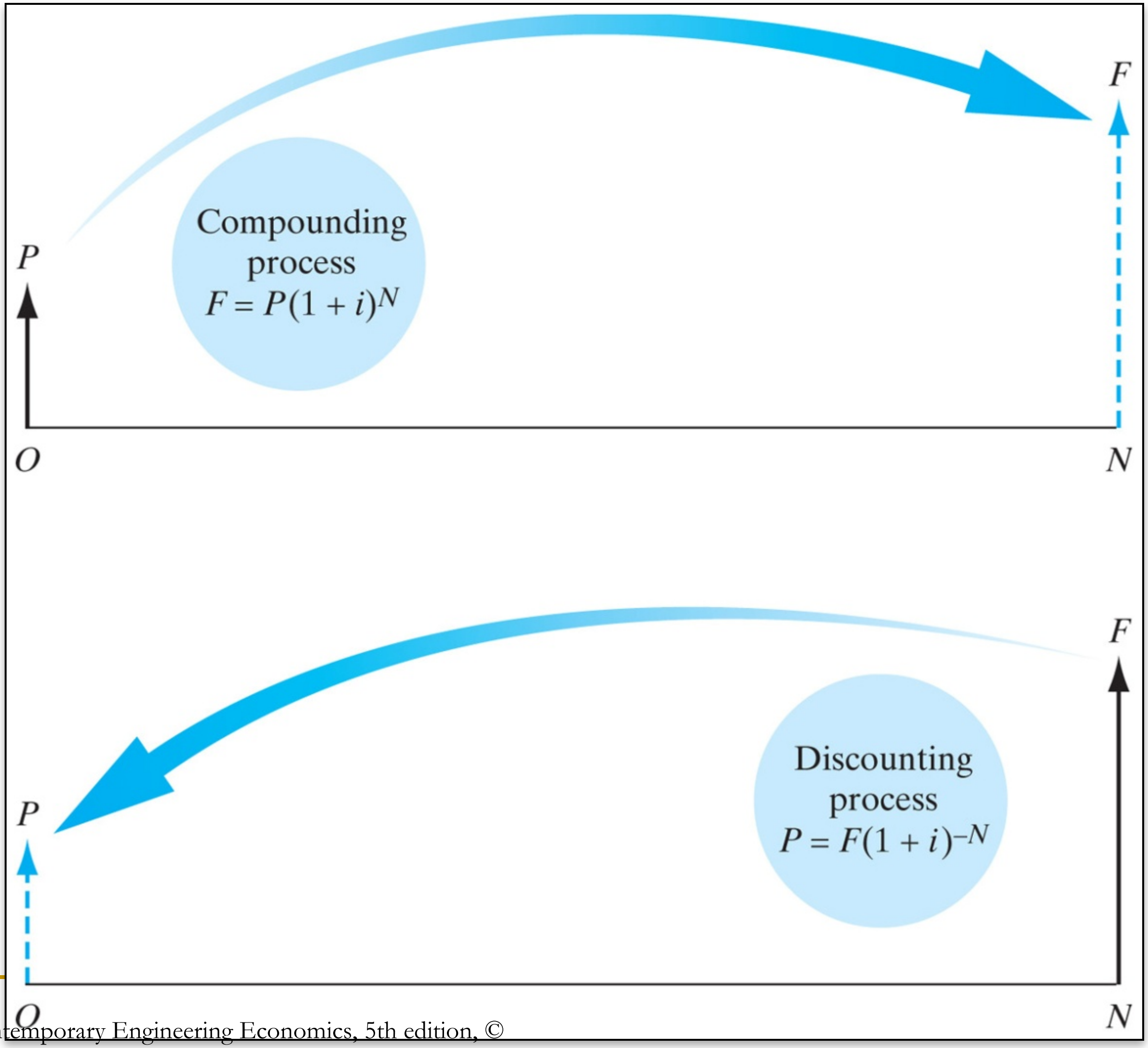


Interest Formulas for Single Cash Flows

Equivalence Relationship Between *P* and *F*

Compounding Process – Finding an equivalent future value of current cash payment

Discounting Process – Finding an equivalent present value of a future cash payment



Example 3.7 Single Amounts: Find *F*, Given *i*, *N*, and *P*

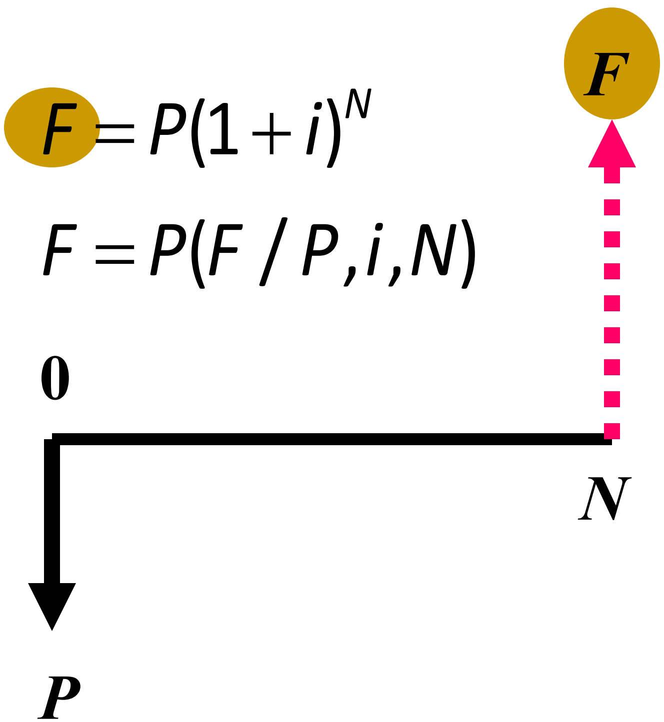
Given: $P = \$2{,}000$, $i = 10\%$, $N = 8$ years

Find: F

$$\begin{aligned} F &= \$2{,}000(1+0.10)^8 \\ &= \$2{,}000(F/P,10\%,8) \\ &= \$4{,}287.18 \end{aligned}$$

$$F = P(1+i)^N$$

$$F = P(F/P,i,N)$$



Single Cash Flow Formula – Present Worth Amount Factor

Example 3.8 Single Amounts: Find *P*, Given *i*, *N*, and *F*

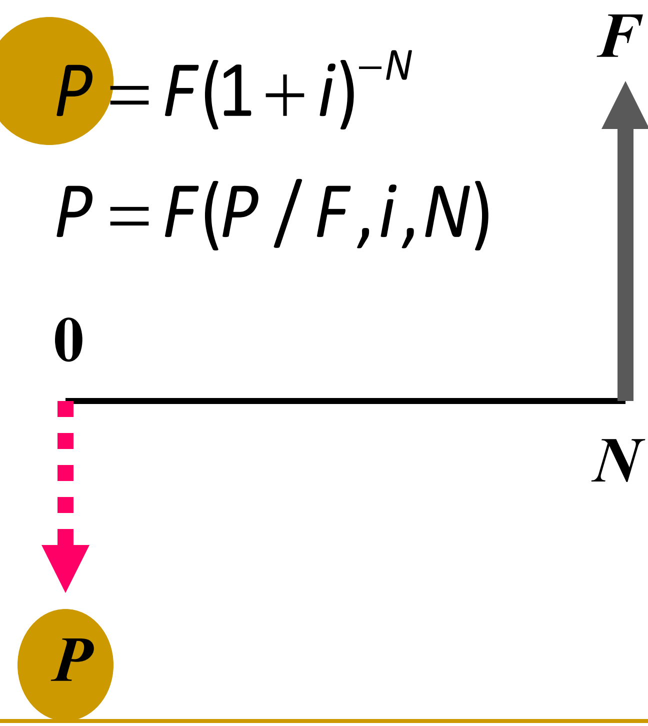
Given: $F = \$1{,}000$, $i = 12\%$, $N = 5$ years

Find: P

$$P = \$1{,}000(1+0.12)^{-5}$$
$$= \$1{,}000(P/F, 12\%, 5)$$
$$= \$567.43$$

$$P = F(1+i)^{-N}$$

$$P = F(P/F, i, N)$$



Solving for *i*

Example 3.9 Single Amounts: Find *i*, Given *P*, *F*, and *N*

Given: $F = \$20$, $P = \$10$, $N = 5$ years

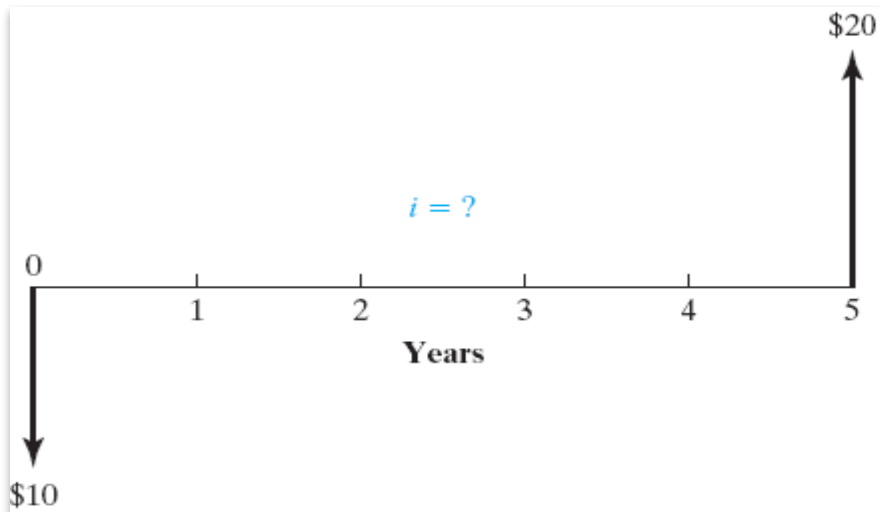
Find: i

$$F = P(1+i)^N$$

$$\$40 = \$20(1+i)^5; \text{ solve for } i$$

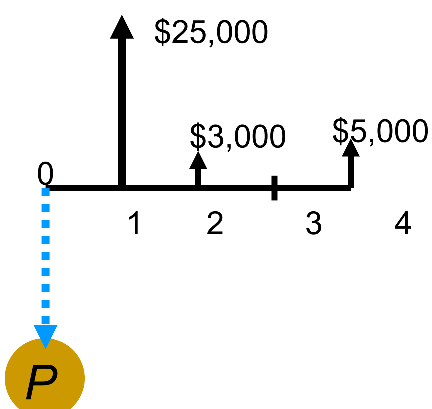
$$i = 2^{1/5} - 1$$

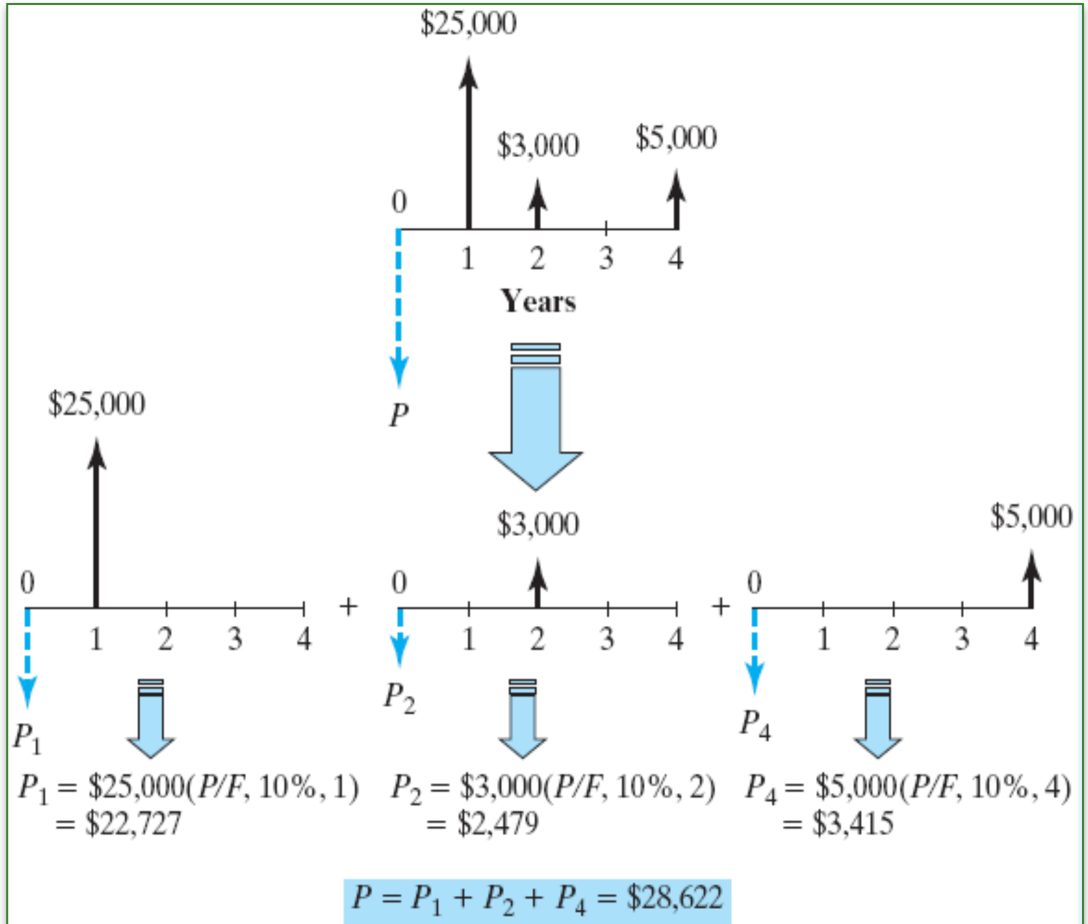
$$= 14.87\%$$



Example 3.11 Uneven Payment Series

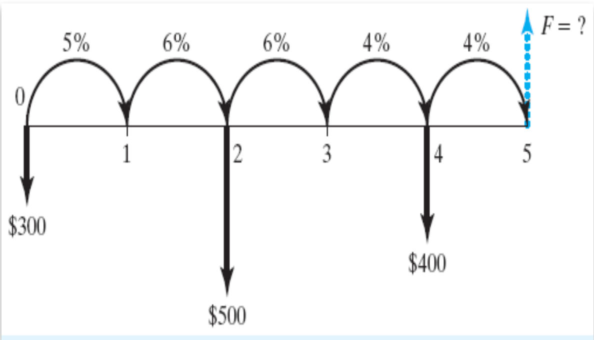
How much do you need to deposit today (P) to withdraw \$25,000 at n =1, \$3,000 at n = 2, and \$5,000 at n =4, if your account earns 10% annual interest?





Example 3.12 Future Value of an Uneven Series with Varying Interest Rates

Given: Deposit series as given over 5 years



Find: Balance at the end of year 5

- Contribution of \$300 at $n = 0$ toward F_5:

$$\underbrace{\underbrace{\overbrace{\$300(F/P, 5\%, 1)}^{\text{Balance at } n=1}(F/P, 6\%, 2)}_{\text{Balance at } n=3}(F/P, 4\%, 2)}_{\text{Balance at } n=5} = \$382.82$$

- Contribution of \$500 at $n = 2$ toward F_5:

$$\underbrace{\overbrace{\$500(F/P, 6\%, 1)}^{\text{Balance at } n=3}(F/P, 4\%, 2)}_{\text{Balance at } n=5} = \$573.25$$

- Contribution of \$400 at $n = 4$ toward F_5:

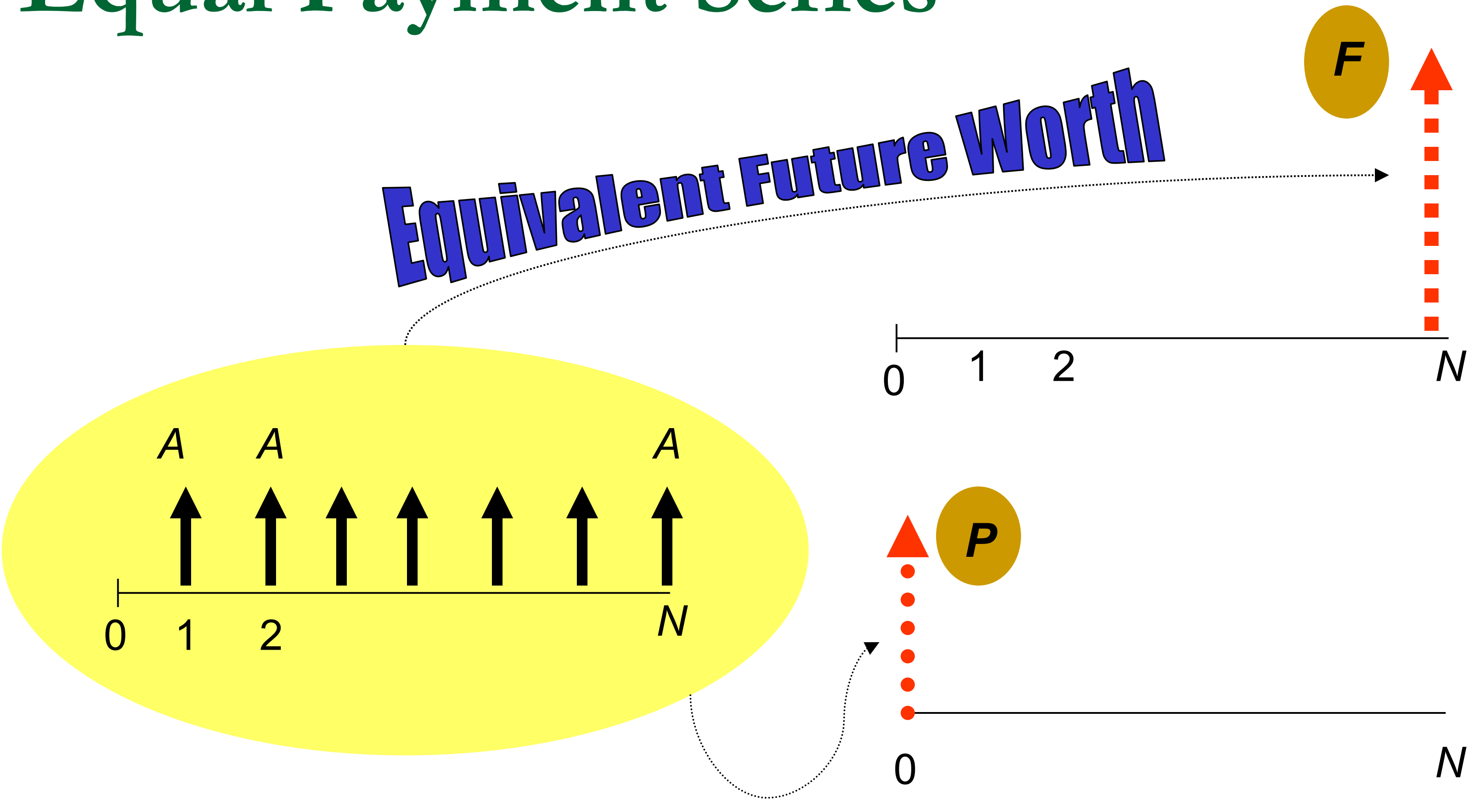
$$\$400(F/P, 4\%, 1) = \$416$$

- Total balance at $n = 5$:

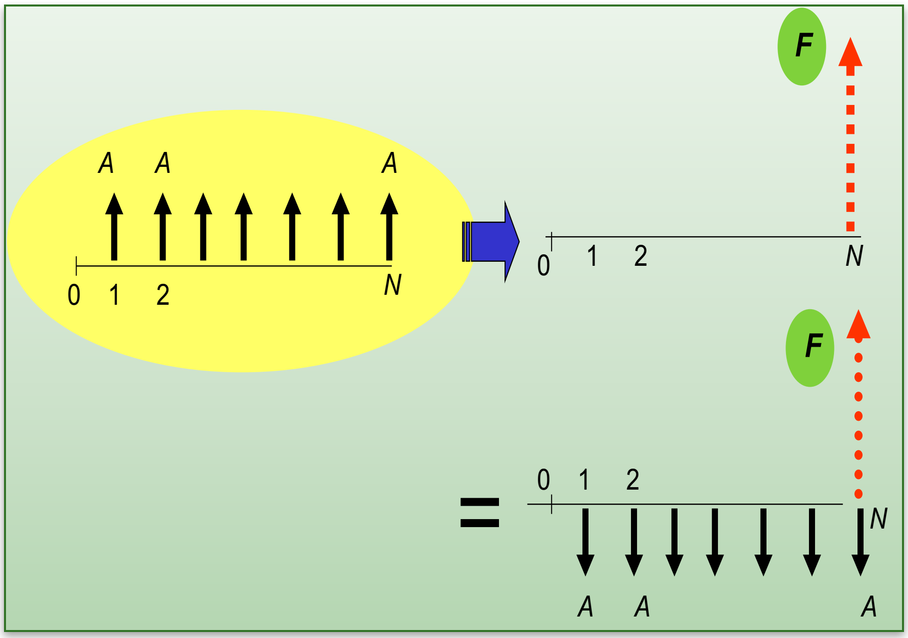
$$F_5 = \$382.82 + \$573.25 + \$416.00 = \$1{,}372.06$$

Interest Formulas – Equal Payment Series

Equal Payment Series

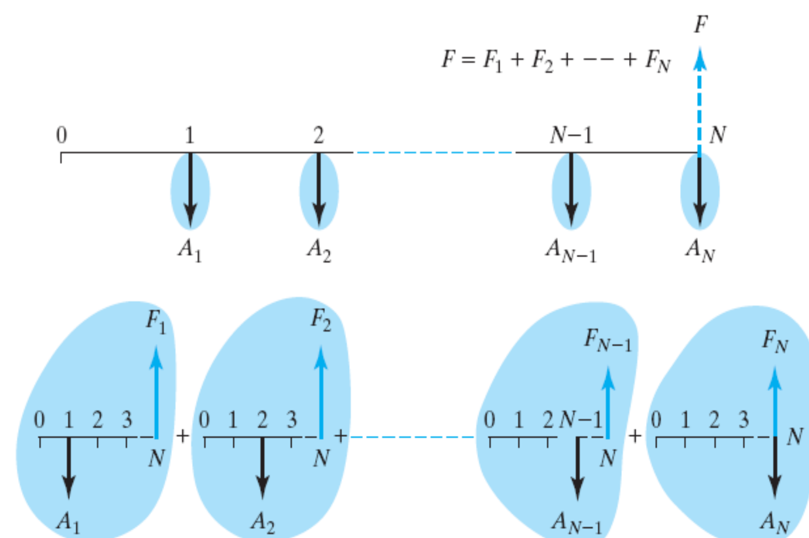


Equal-Payment Series Compound Amount Factor

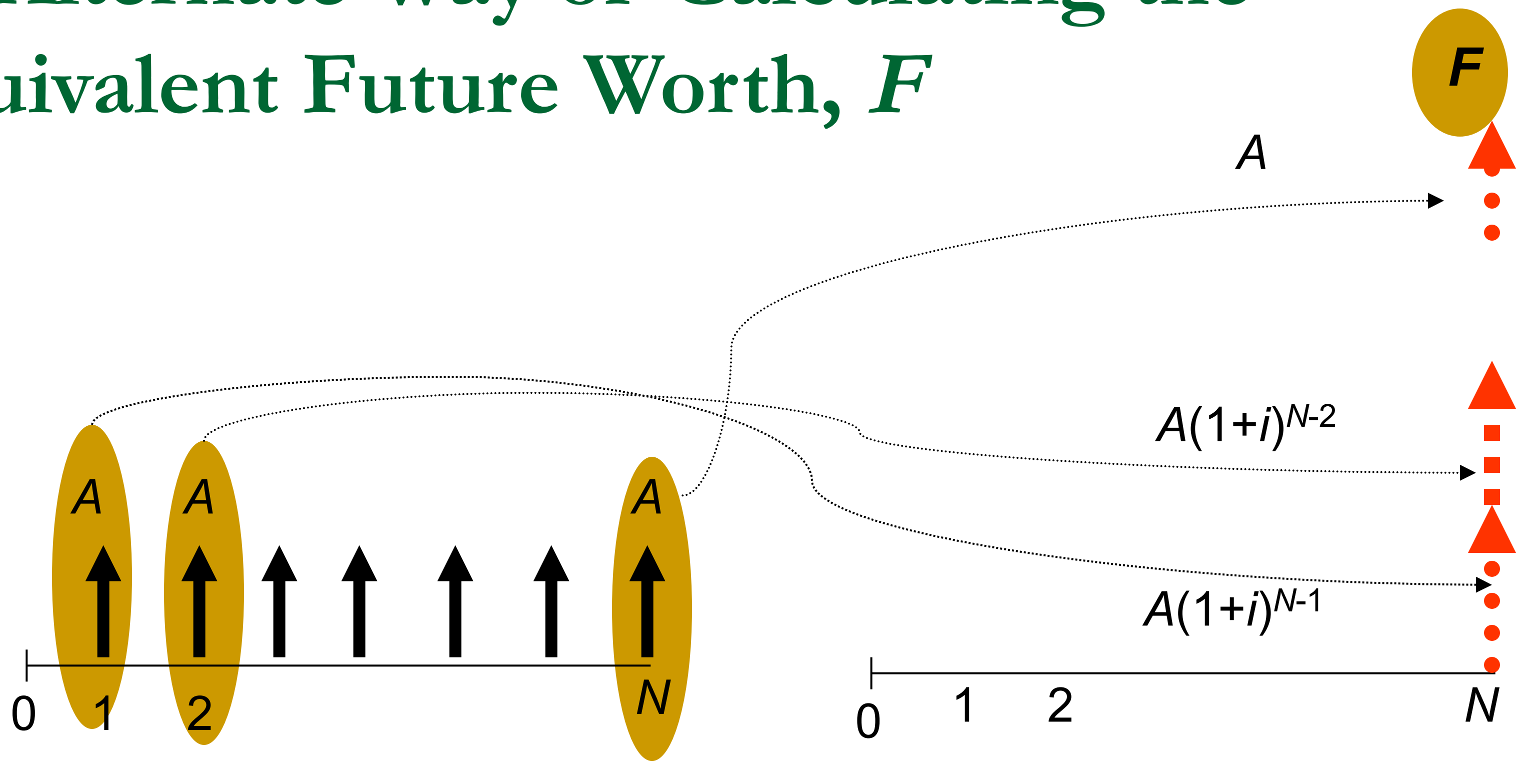


Formula

$$F = A\left[\frac{(1+i)^N - 1}{i}\right] = A(F/A, i, N)$$



An Alternate Way of Calculating the Equivalent Future Worth, *F*

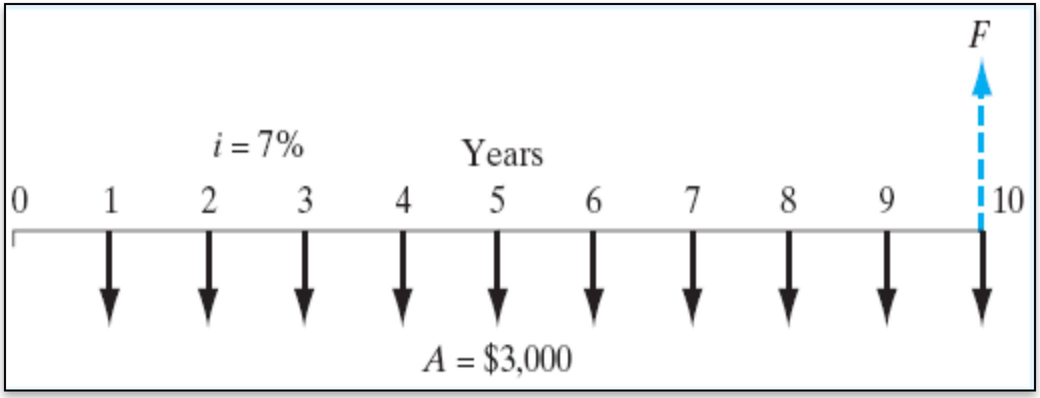


$$F = A(1+i)^{N-1} + A(1+i)^{N-2} + \cdots + A = A\left[\frac{(1+i)^N - 1}{i}\right]$$

Example 3.14 Uniform Series: Find *F*, Given *i*, *A*, and *N*

Given: $A = \$3{,}000$, $N = 10$ years, and $i = 7\%$ per year

Find: F



$$\begin{aligned} F &= \$3{,}000(F/A, 7\%, 10) \\ &= \$3{,}000(13.8164) \\ &= \$41{,}449.20 \end{aligned}$$

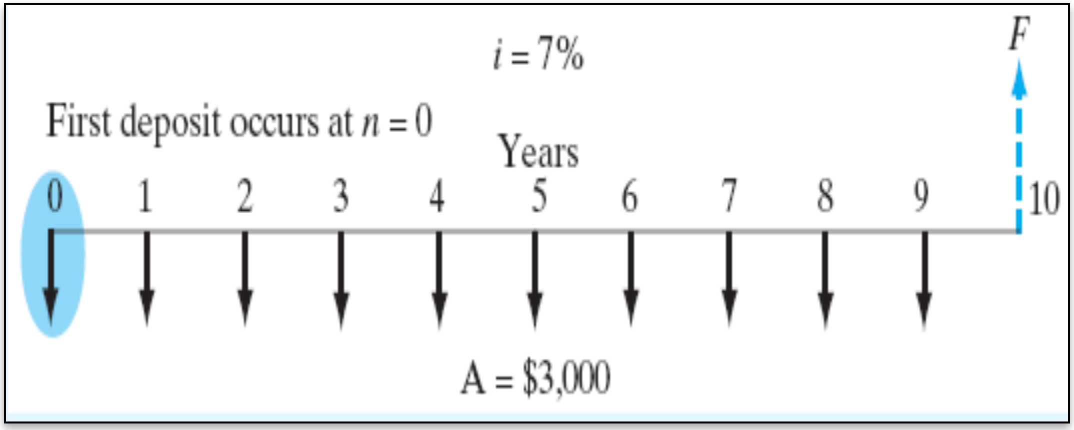
Example 3.15 Handling Time Shifts: Find F, Given i, A, and N

Given: A = \$3,000, N = 10 years, and i = 7% per year

Find: F

$$F = \$3{,}000(F/A, 7\%, 10) = \$3{,}000(13.8164) = \$41{,}449.20$$

$$F_{10} = \$41{,}449.20(1.07) = \$44{,}350.64$$



- Each payment has been shifted to one year earlier, thus each payment would be compounded for one extra year

Formula – Sinking Fund Factor

Sinking-Fund Factor: Find *A*, Given *i*, *N*, and *F*

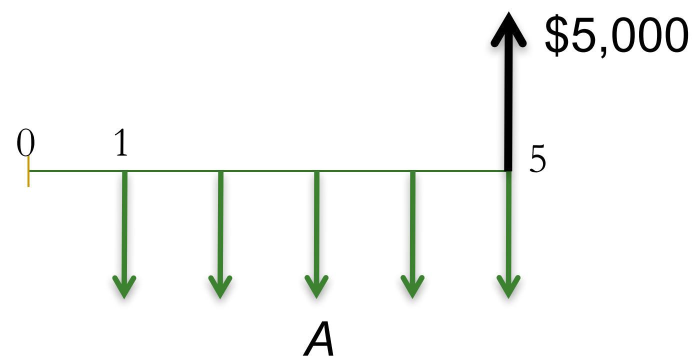
Given: $F = \$5{,}000$, $N = 5$ years, and $i = 7\%$ per year

Find: A

$$A = F\left[\frac{i}{(1+i)^N - 1}\right] = F(A/F, i, N)$$

$$A = \$5{,}000(A/F, 7\%, 5)$$
$$= \$869.50$$

$$A = \$5{,}000(A/F, 7\%, 5)$$
$$= \$869.50$$



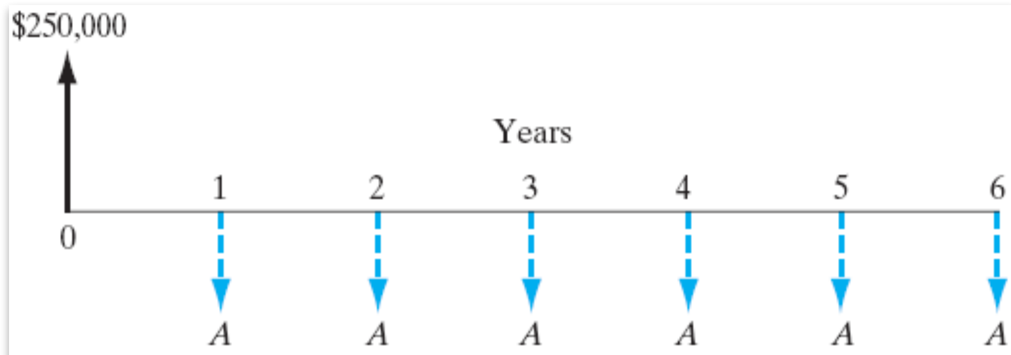
Capital Recovery Factor

Example 3.18 Uniform Series: Find *A*, Given *P*, *i*, and *N*

Given: $P = \$250{,}000$, $N = 6$ years, and $i = 8\%$ per year

Find: A

Formula to use:



$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right] = P(A/P, i, N)$$

$$\begin{aligned} A &= \$250{,}000(A/P, 8\%, 6) \\ &= \$250{,}000(0.2163) \\ &= \$54{,}075 \end{aligned}$$

Excel Solution:

	A	B
1	*P*	$ 250,000
2	*i*	8%
3	*N*	6
4		
5	*A*	$54,078.85

=PMT(8%,6,−250000)

Example 3.19 – Deferred Loan Repayment

Given: P = \$250,000, N = 6 years, and i = 8% per year, but the first payment occurs at the end of year 2

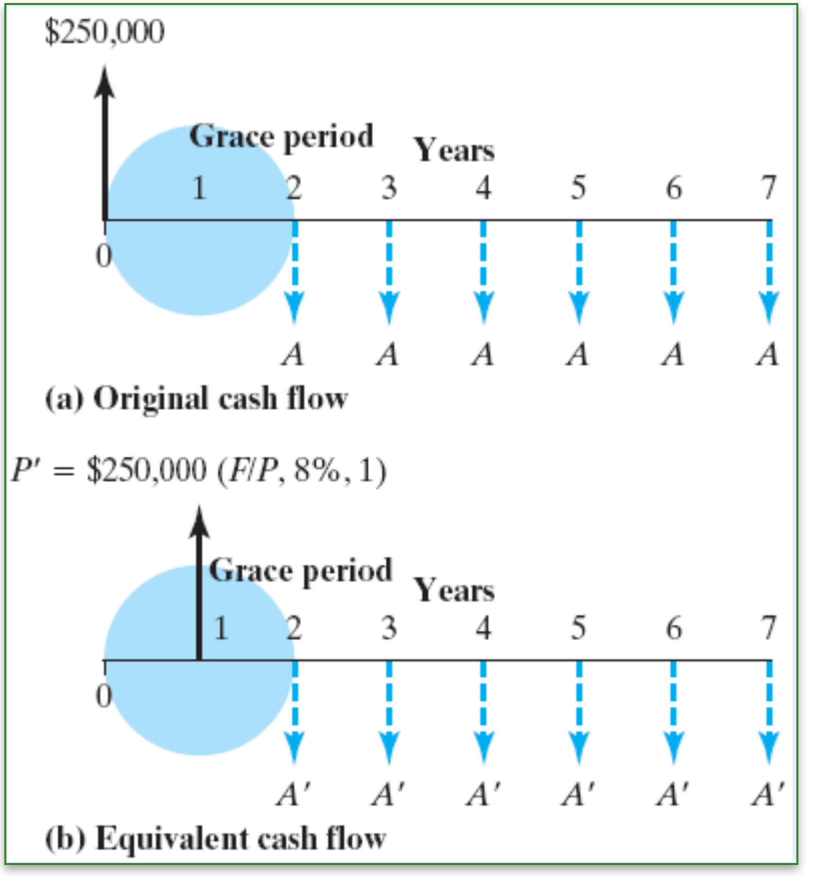
Find: A

- Step 1: Find the equivalent amount of borrowing at the end of year 1:

$$P' = \$250{,}000(F/P, 8\%, 1) = \$270{,}000$$

- Step 2: Use the capital recovery factor to find the size of annual installment:

$$A' = \$270{,}000(A/P, 8\%, 6) = \$58{,}401$$



Present Worth Factor

Example 3.20 Uniform Series: Find *P*, Given *A*, *i*, and *N*

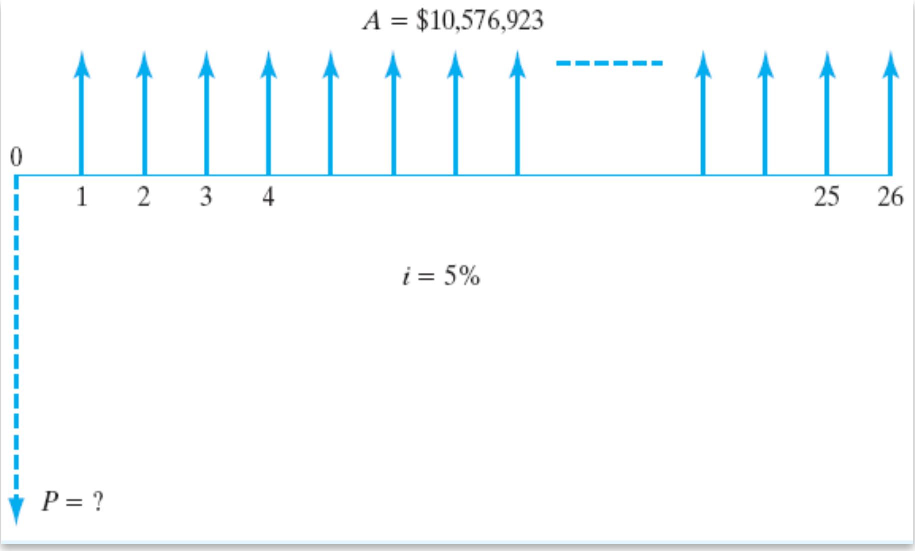
Given: A = \$10,576,923, N = 26 years, and i = 5% per year

Find: P

Formula to use:

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] = A(P/A, i, N)$$

$$\begin{aligned} P &= \$10{,}576{,}923(P/A, 5\%, 26) \\ &= \$152{,}045{,}228 \end{aligned}$$



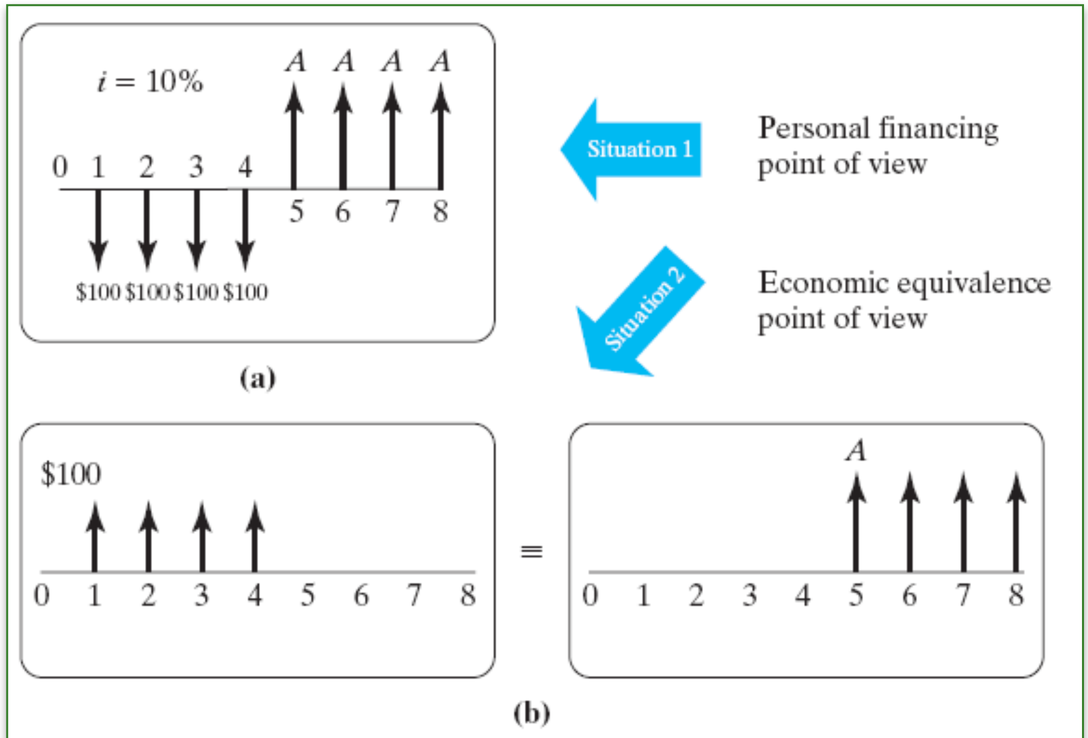
Excel Solution:

```
=PV(5%,26,-10576923)=$152,045,228
```

Composite Cash Flows

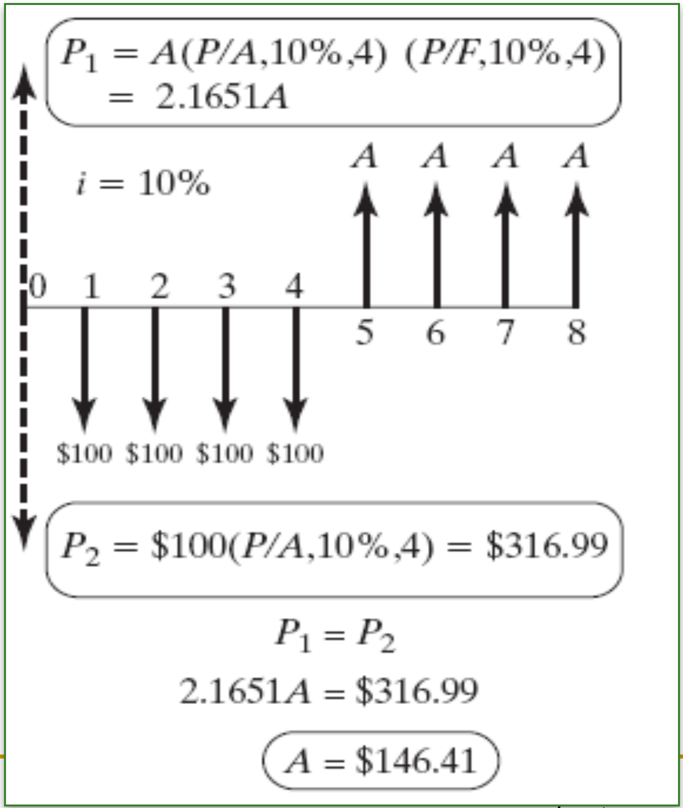
Situation 1: If you make 4 annual deposits of $100 in your savings account which earns a 10% annual interest, what equal annual amount (*A*) can be withdrawn over 4 subsequent years?

Situation 2: What value of *A* would make the two cash flow transactions equivalent if $i = 10\%$?

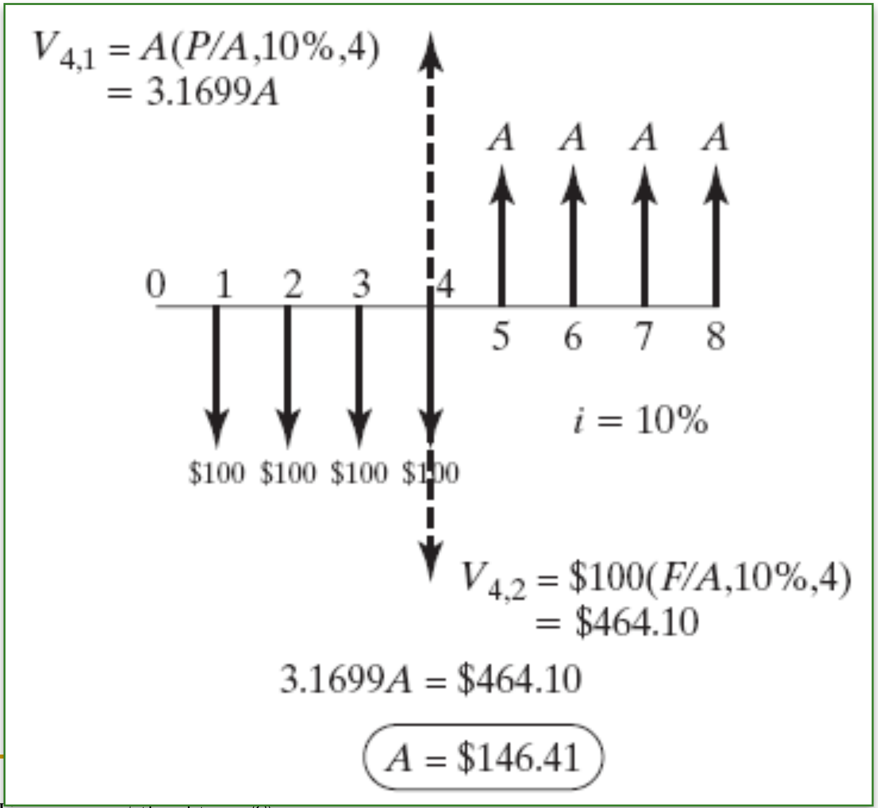


Establishing Economic Equivalence

Method 1: at $n = 0$



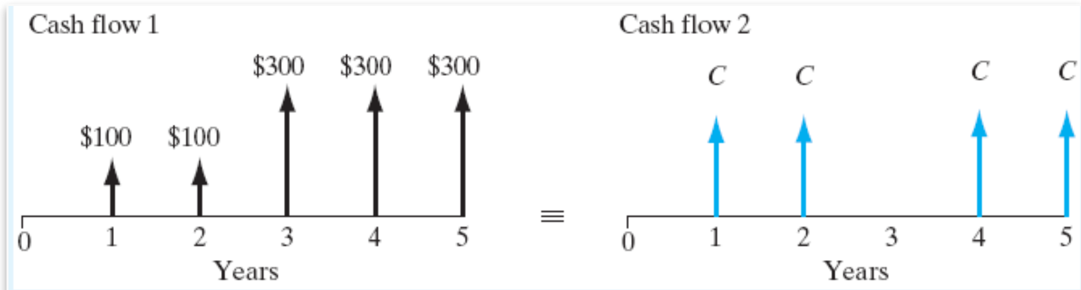
Method 2: At $n = 4$



Example 3.26 Cash Flows with Subpatterns

Given: Two cash flow transactions, and $i = 12\%$

Find: C



Strategy: First select the base period to use in calculating the equivalent value for each cash flow series (say, n = 0). You can choose any period as your base period.

$$P_1 = \$100(P/A, 12\%, 2) + \$300(P/A, 12\%, 3)(P/F, 12\%, 2)$$
$$= \$743.42;$$
$$P_2 = C(P/A, 12\%, 5) - C(P/F, 12\%, 3)$$
$$= 2.8930C$$

$$743.42 = 2.8930C$$

$$C = \$256.97.$$

Example 3.27 Establishing a College Fund

- A couple with a newborn daughter wants to save for their child's college expenses in advance.
- The couple can establish a college fund that pays 7% annual interest.
- Assuming that the child enters college at age 18, the parents estimate that an amount of $40,000 per year (actual dollars) will be required to support the child's college expenses for 4 years.

Example 3.27 Establishing a College Fund

- Determine the equal annual amounts the couple must save until they send their child to college.

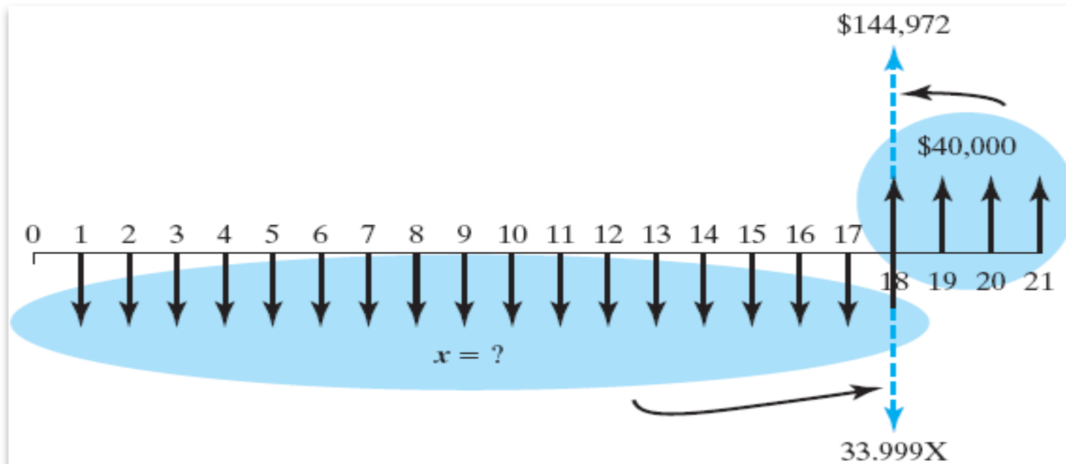
  Assume that: the first deposit will be made on the child's first birthday and the last deposit on the child's 18th birthday. The first withdrawal will be made at the beginning of the freshman year, which also is the child's 18th birthday.

Example 3.27 Establishing a College Fund

Given: Annual college expenses = $40,000 a year for 4 years, i = 7%, and N = 18 years

Find: Required annual contribution (X)

Strategy: It would be computationally efficient if you choose n = 18 (the year she goes to college) as the base period.



Step 1: Find the accumulated deposit balance on the child's 18th birthday.

$$V_{18} = X(F/A, 7\%, 18) = 33.9990X$$

Step 2: Find the equivalent lump-sum withdrawal on the child's 18th birthday:

$$V_{18} = \$40{,}000 + \$40{,}000(P/A, 7\%, 3) = \$144{,}973$$

Step 3: Since the two amounts must be the same, we obtain

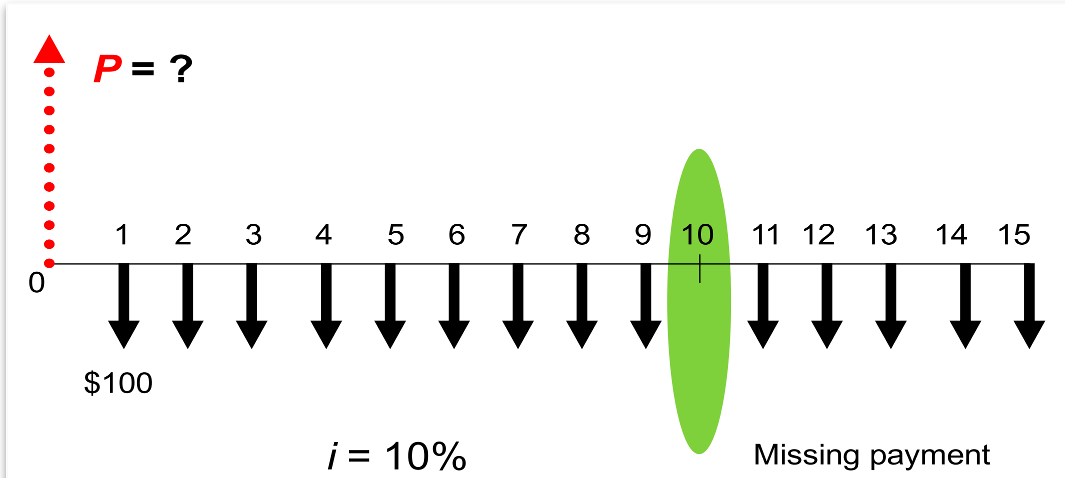
$$33.9990X = \$144{,}973$$
$$X = \$4{,}264$$

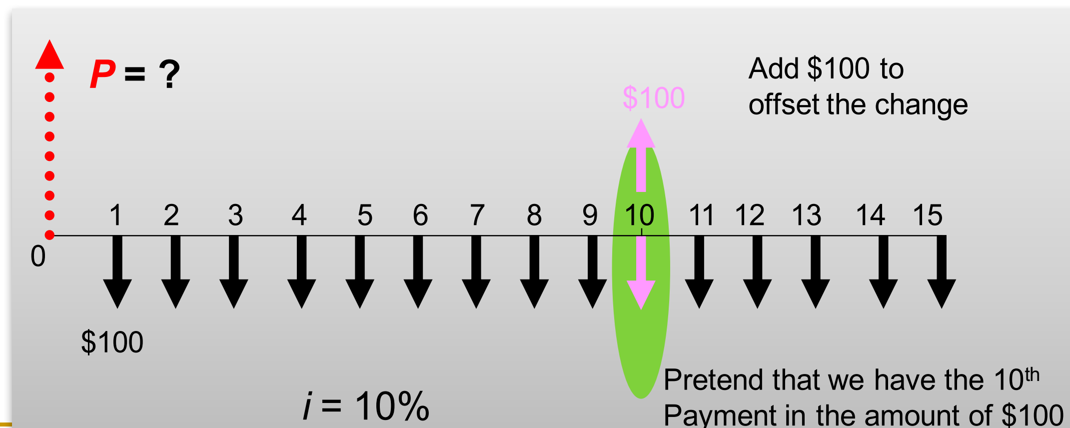
Cash Flows with Missing Payments

Given: Cash flow series with a missing payment, $i = 10\%$

Find: P

Strategy: Pretend that we have the 10th missing payment so that we have a standard uniform series. This allows us to use (P/A,10%,15) to find P. Then we make an adjustment to this P by subtracting the equivalent amount added in the 10th period.





$$P + \$100(P/F,10\%,10) = \$100(P/A,10\%,15)$$

$$P + \$38.55 = \$760.61$$

$$P = \$722.05$$