

### IE 479: Study Set 1

Here are some questions from old Midterms.

**Question 1:** Company AA produces a medical vaccine, which requires special care during transportation. At the plant, the vaccine is packaged so that it will remain safe for 7 days. However, the company is serving over a wide geographical area and not all the customers can be reached in 7 days. Thus the company has rented special centers throughout their service area at which the vaccine can be froze again. There are two different types of centers; at small centers, the vaccine can be frozen almost instantly and will remain safe for 3 more days, at large centers the freezing process takes 1 day but the vaccine becomes good as new and remains safe for an additional 7 days. However, the vaccine can be frozen at most once before it reaches its final destination.

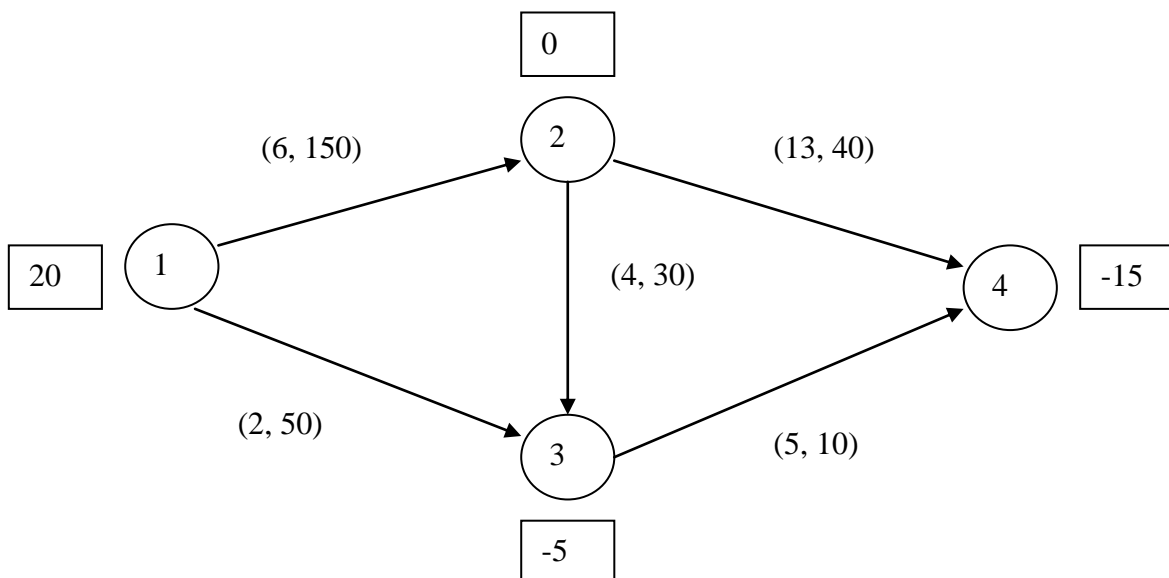
There are 5 main hospitals that the vaccine should be sent to and the demand at hospital  $j$  is  $d_j$ . The vaccine should be ready before certain deadlines, denoted by  $dl_j$ , at each hospital.

Let  $G=(N, A)$  denote the transportation network.  $N=\{0, 1, 2, \dots, 9\}$ . For convenience, let node 0 denote the production plant, nodes 1, 2, ..5 denote the 5 hospitals. Assume node 6 is the large center and nodes 7, 8, and 9 are small centers that the company has rented. Let  $t_{ij}$  denote the travel time along link  $(i, j)$  and  $c_{ij}$  denote the per unit cost of traversing link  $(i, j)$ . Also, let freezing at large centers cost  $l_j$  \$/unit and at small centers cost  $s_j$  \$/unit. Assume that both  $T=[t_{ij}]$  and  $C=[c_{ij}]$  satisfy triangle inequality.

- a) Provide a model for the problem which minimizes the total cost. Explain your variables and write clearly for full credit.
- b) Suppose now that the company AA has not rented the freezing centers yet. There are  $m$  alternative locations for large centers, denoted by set  $M$ , and  $n$  alternative locations for large centers denoted by  $S$ . For convenience let  $M=\{6, \dots, 6+m\}$  and  $S =\{7+m, \dots, 7+m+n\}$ . Let  $f_{lj}$  ( $f_{sj}$ ) denote the fixed cost of locating a large center (small center) at  $j$ . Construct a cost minimizing model which will additionally provide the locations of the centers, if needed.

**Question 2:** Solve the following MCNFP with network simplex. (Find a bfs and then iterate to reach optimal solution.)

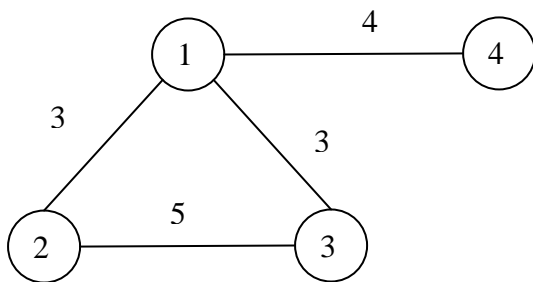
(cost, capacity)



**Question 3:** Solve the covering problem with 6 possible locations with  $f=(5,6,4,4,2,3)$  and 5 customers with  $p_j=3$ . A location covers a customer if it is within 55 km distance.

Distance (km)	Customer				
	1	2	3	4	5
Location 1	65	55	40	80	30
Location 2	70	70	55	80	20
Location 3	80	30	60	75	80
Location 4	30	40	80	40	70
Location 5	60	75	80	10	25
Location 6	20	60	75	30	80

**Question 4:** Consider the following network. Suppose all 4 nodes are customers.



We want to open a DC so as to minimize the longest distance to any customer.

- Find the best location for the DC when the DC will be located at one of the 4 nodes.
- Suppose the DC can be located along the edges also. Find the best location by two different algorithms discussed in class. Show all your work.

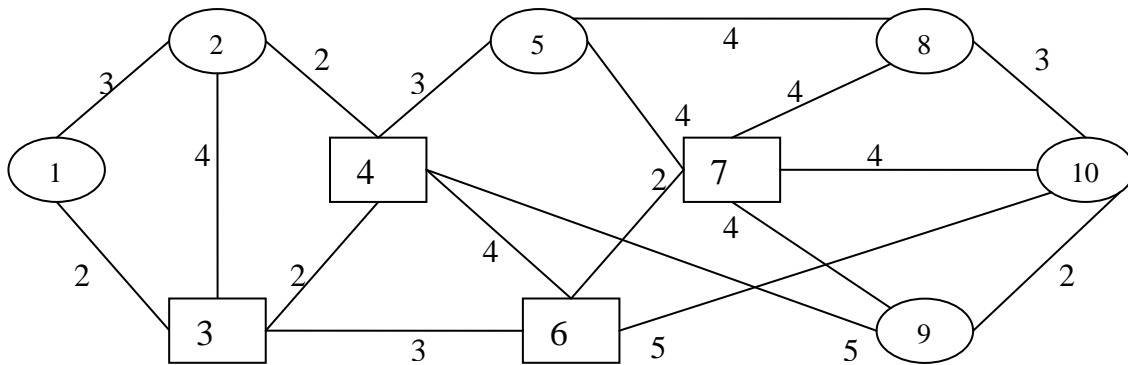
**Question 5:** Mayor of Ankara is planning to establish a 24-hour ambulance service. There are 2 different types of ambulances. Blue-type (which contains basic life support) and Red-type (which contains paramedics). Ambulance base stations are also need to be established to host the ambulances. Let  $M = \{1, \dots, m\}$  denote set of available sites for ambulance base stations and let  $f$  denote the fixed cost of locating a base at any of the sites  $\in M$ . Let  $C^R$  : cost of buying a red-type ambulance and  $C^B$  : cost of buying a blue-type ambulance. Assume all costs are annualized. The main decisions are where to open the bases and what type(s) of ambulances to have at the bases.

The population of Ankara is aggregated into districts  $N = \{1, \dots, n\}$ . Let  $P_i$  : denote the number of people living in district  $i$ . A district is receiving Red-Type ambulance service if it is within 5 minutes drive from a Red-Type ambulance location and similarly Blue-Type ambulance service if it is within 9 minutes drive from a Blue-Type ambulance location. Let  $t_{ij}$  denote the travel time from  $j \in M$  to  $i \in N$ .

- Suppose a district will be considered as receiving service if it receives both type of ambulance services. The budget for the next year is going to be prepared and the Mayor wants to know the least costly way of providing both type of services of every district. Formulate a model for this case.
- Let  $C^*$  be the optimal value found via the model given in part (a). Say the budget can be at most  $C < C^*$ . The mayor may question increasing  $C$  a little but he wants to know how many districts will receive additional service. Provide a method for such an analysis. Be as specific as possible.

- c) Formulate a linear model in which the government has a total budget of C \$ and wants to maximize the number of people receiving service.
- d) Suppose now basic coverage is a must and each district needs to get a blue-type ambulance service. And we want to maximize the number of people receiving red-type ambulance service.
- e) Suppose now a district will receive service as long as it is reached by either a red or a blue-type ambulance. Let  $q$  : denote a level of appreciation / people if served by a blue-type ambulance and  $3q$  denote the appreciation level / people for service with a red – type ambulance. Formulate so that the total appreciation will be maximized. The government also at least Q people receiving service. (Be careful in double counting)
- f) Say there is one hospital located in Ankara, located at k. For each district i in N, Let  $d_{ik}$  denote the travel time from district i to hospital. Suppose now there is an additional requirement on the total journey time of the Red and Blue Type ambulances. The journey is defined from the base till the hospital (base-served district-hospital). Say Red-Type ambulances should finish their journey within 12 minutes and Blue Type ambulances should finish the journey within 14 minutes. Assume that it takes 2 minutes to take the patient at the district stop. Suppose also that each district should receive basic life support. Formulate a model which will additionally maximize the number of people receiving Red-Type ambulance. Say we need to establish p base stations in total

**Question 6:** Consider the following transportation network where the numbers on arcs represent the traversal time of the arc.



- a) Say nodes  $N = \{1, 2, 5, 8, 9, 10\}$  are customers and nodes  $M = \{3, 4, 6, 7\}$  are alternative sites. Construct the coverage matrix  $A = [a_{ij}]$  for  $T = 5$

$$a_{ij} = \begin{cases} 1 & \text{if } d_{ij} \leq T \\ 0 & \text{otherwise.} \end{cases}$$

b) Consider  $\text{Min } \sum_i f_i y_i + \sum_j P_j Z_j$

subject to

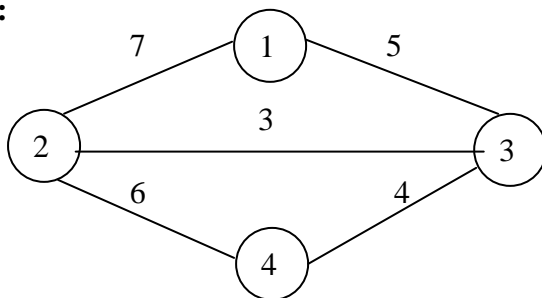
$$\sum_i a_{ij} y_i + Z_j \geq 1 \quad \forall j \in N$$

$$y_i, Z_j \in \{0,1\} \quad \forall j \in N, \forall i \in M$$

Say  $P_j = 3 \quad \forall j$  and  $f = (5,9,6,8)$

Find a solution using the dual greedy approach discussed in class.

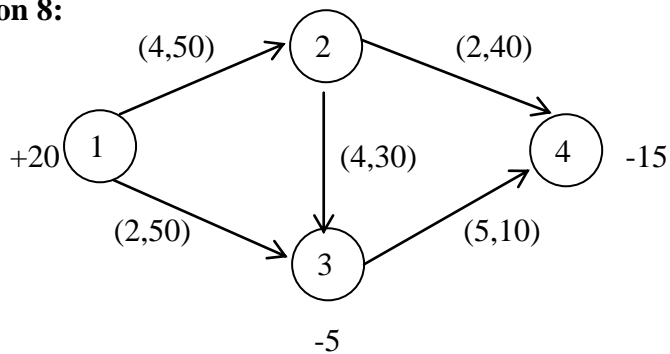
**Question 7:**



Find the optimum 1 center location

- a) Locate on nodes only
- b) Locate anywhere along the arcs

**Question 8:**



Find the optimum shipment structure using network simplex algorithm.  $(c_{ij}, u_{ij})$