INFORMS JOURNAL ON APPLIED ANALYTICS
Vol. 54, No. 4, July–August 2024, pp. 312–328
ISSN 2644-0865 (print), ISSN 2644-0873 (online)
https://pubsonline.informs.org/journal/ijaa

Optimal Transportation Planning for a Do-It-Yourself Retailer with a Zone-Based Tariff

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Received: March 28, 2023
Revised: September 15, 2023; November 17, 2023
Accepted: November 22, 2023
Published Online in Articles in Advance: April 3, 2024

https://doi.org/10.1287/inte.2023.0020

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Abstract. This article describes the development of a decision support tool for a Rich Vehicle Routing Problem (R-VRP) of a major do-it-yourself (DIY) retailer supplying its stores across Europe from multiple depots. The retailer uses external logistic service providers (LSPs) for the delivery to its stores and has two modes to choose from. In the first mode, the retailer proposes delivery tours to LSPs for execution. These tours are billed according to a nonlinear tariff with volume discounts depending on the delivery zones visited and the load carried. The LSPs accept the retailer's tour proposal only if tour duration and distance restrictions are kept. The latter is ensured by a relative detour limit. In the second mode, the retailer assigns single shipments to common carriers that consolidate them with shipments from other customers and bill this based on load and destination. The resulting problem represents an open VRP with two delivery modes, carrier selection and a heterogeneous fleet. Multiple delivery modes are standard in DIY retailing and constitute a general industry problem. The literature on VRPs and current software applications in the industry predominantly considers modeling and solution approaches that rely on linear distance costs, neglecting that nonlinear zone-based tariffs with volume discounts are standard in the freight forwarding business. Our work addresses this issue by developing a decision support tool for the retailer based on an exact algorithm for solving R-VRPs with a nonlinear zone-based tariff scheme and a relative detour limit. The tool is based on an innovative three-component set partitioning algorithm working on a complete set of feasible tours to solve the problem. We show that our approach optimally solves the daily distribution problem of the industry partner with up to 150 stores. Furthermore, implementing the tool enables more comprehensive and structured planning for the retailer and an average of 8% transportation cost savings, translating to total savings of more than €1 million per year for this specific retailer compared with the status quo.

History: This paper was refereed.

Keywords: rich vehicle routing • nonlinear cost • exact approach • set partitioning

Introduction

Do-it-yourself (DIY) retailing accounts for almost $1 trillion in sales globally and continues to grow by 4% annually (Statista 2021). The transportation of goods to the stores is a decisive cost factor and accounts for up to 58% of total distribution costs (Rodrigue 2020). We have developed a tool to address the operational transportation problem at an international DIY retailer (denoted DIY-R). DIY-R is a major European retailer with over 650 retail stores across Europe with approximately 48,000 employees. DIY-R offers a broad product assortment, from gardening supplies to living solutions to technical equipment. The retailer operates various store types, with different sizes and demands, that are supplied from three depots, weekly to triweekly. Up to 150 stores are supplied per day. Like most European DIY companies, DIY-R uses external logistics service providers (LSPs) to supply its stores from central warehouses. The industry standard is the use of multiple delivery modes for the operational problem (Keskin et al. 2014, Dang et al. 2021, Khodabandeh et al. 2021). In the case of DIY-R, these modes are

- *Subcontracted Delivery Tours with LSPs* (SDT) and
- *Subcontracted Single Shipments with common carriers* (SSS).

With regard to SDT, DIY-R proposes delivery tours of one or multiple store orders that an LSP executes. These tours are then exclusively carried out for DIY-R and must adhere to maximum tour duration and distance constraints. The LSPs bill DIY-R according to their nonlinear zone-based tariff. Within this tariff, each store is allocated to a specific delivery zone. The total costs of a tour depend on the furthest zone visited and the total load carried. In addition, the tariff includes an all-unit volume discount, meaning a higher total load leads to lower costs per load unit. Zone-based tariffs are commonly used in the freight transport sector as they substantially simplify the cost calculation of delivery tours. Proposing tours to the LSPs without explicit routing is possible using the tariff. In this case, however, it cannot be guaranteed that the LSPs will accept the tours, owing to potential tour duration and distance violations.

The second mode, SSS, at DIY-R involves shipping a single-store order using common carriers. No delivery tour proposal is required in this case; the common carriers are responsible for the whole delivery process and may deliver the orders of DIY-R together with orders from other customers. The retailer pays a fixed fee that depends on the origin, destination, and load units. The second mode is generally more expensive per load unit than a high-volume delivery tour. It may still be attractive for store orders that cannot be efficiently combined with other orders on delivery tours. Cost-optimal distribution therefore uses a mix of both options.

The current practice at DIY-R is based on a manual, legacy planning process that has evolved over time and mainly relies on spreadsheet calculations. Each store has predetermined delivery days (e.g., every second Monday), and the stores order three days in advance. When all orders of a delivery day are available, the logistics planners allocate the orders either to tours (SDT) or common carriers (SSS). A single store may be served by a combination of SDTs and SSSs, as each store potentially submits multiple orders for different goods. The planners aim to maximize the number of store orders allocated to SDT and predominantly build tours with full truckloads because of the volume discounts granted by LSPs. After the orders are allocated, the planners check the tour feasibility concerning distance and tour duration using a standard map provider. If a tour is nonfeasible, single orders are moved to different tours or to SSS. Once a feasible tour is obtained, the minimum tour costs across all LSPs are determined based on spreadsheet calculations using the zone-based tariff schemes, and an LSP is selected for the execution of the tour. There is no reoptimization once a tour is defined, costs are assessed, and an LSP is chosen. Consequently, manual planning greatly depends on the intuition of a dedicated team of planners and poses some inefficiencies:

- Manual planning consumes a significant share of the workforce and time (up to 32 person-hours) each day, binding valuable resources.
- The entire delivery region is divided into distinct regions to ease manual planning. No comprehensive and consistent planning across regions takes place.
- The sequential planning process without any reoptimization may not be cost-efficient. Because the optimal tour plan is unknown, there is no performance indicator for the cost efficiency of the tours.
- Tour length and duration limitations have sometimes not been adhered to in the past because only a simplified check of tour length has been made with a standard map provider, and the LSPs have consequently rejected tours, resulting in additional replanning efforts.

DIY-R aims to improve its manual process and transportation efficiency. We develop a *Decision-Support System for the Daily Routing* (DSS-DR) for this purpose. Although applying the aforementioned zone-based tariff scheme with volume discounts is standard in freight forwarding, it is not yet widely used in modeling and optimization approaches in the literature and routing software. In both research and applications, the usual method to account for transportation costs is based on the driving distance of the vehicles. However, the costs arising from the zone-based tariffs for the retailers differ because they depend on the delivery zones of the stores supplied and the total load of the tour. The DSS-DR was developed in a joint project over 12 months with DIY-R's supply chain optimization and planning departments to enhance planning efficiency and assess the actual transportation costs at DIY-R. To offer a user-friendly application, the tool is embedded into DIY-R's workflow by interfacing relevant software such as spreadsheets (data input) and a map provider (solution display). Furthermore, we develop an exact three-component set partitioning algorithm that is flexible enough to cope with further constraints and other tariff schemes. Implementing DSS-DR provides an average of 8% distribution cost savings and significantly reduces the manual effort spent on daily transportation planning. Furthermore, using DSS-DR contributes to long-term company success by providing immediate decision support for daily operations and long-term evaluation of the zone-based tariff scheme, and thus for future decisions on the terms contracted with the LSPs.

The remainder of this work is structured as follows. Section *Description of the Business Process* details the underlying planning problem and manual process at DIY-R before Section *Problem Classification and Related Literature* analyzes related literature. Section *Solution Method* presents the methodology, and Section *Benefits and Challenges* details the benefits and challenges of the algorithm developed and its implementation at DIY-R, before Section *Summary and Conclusion* concludes the paper.

Description of the Business Process

The business process coupled with industry standards regarding LSPs and their tariff schemes involves different departments and decision owners at DIY-R. It depends on legacy processes and human factors such as experience and intuition. To understand and emphasize the central role of operational daily tour planning and its interrelation with superordinate strategic and tactical planning at DIY-R, we first describe the entire distribution planning and the underlying zone-based tariff in Section *Tariff and Context of Distribution Planning*. We then delineate the business process at DIY-R in Section *Operational Tour Planning for Each Delivery Day* and highlight shortcomings and improvement opportunities of this process in *Inefficiencies and Improvement Potentials*.

Tariff and Context of Distribution Planning

The zone-based tariff scheme is standard in freight forwarding and is used by LSPs to bill delivery tours. Based on its postal code, each store of DIY-R belongs to exactly one of several delivery zones defined by the LSPs. Stores that are farther away from a depot are allocated to higher zones. This subdivides the complete distribution area into mutually exclusive delivery zones and reflects the transportation distances between the depot and the stores. The zone-based tariff scheme of each LSP for SDT depends on the depot location, the most remote zone visited, the total load carried, and the number of unloading points. Table 1 is an example of the zone-based tariff scheme with all-units volume discounts for one depot and LSP combination.

The costs per load unit decrease with a higher total load on a tour. This all-units volume discount leads to a nonlinear cost function. Furthermore, the costs per load unit increase nonlinearly related to the most remote zone visited. For the actual costing, the cost factor of the load-zone combination is multiplied by the total load carried on the tour. Finally, each stop is associated with fixed unloading costs. The sum of all unloading costs is added to the total tour costs. For example, if the tour starts at the depot in Zone 1 and visits one store in Zone 1 and two stores in Zone 2 with a total of eight load units, then the highest zone visited (in our example, Zone 2) and the load units carried (eight units) determine the costs per load unit that the retailer needs to pay the LSP. In the example, the transportation costs are $8 \times$ €77.70 and the unloading costs (not indicated in the table) for the three stops are $3 \times$ €63.20, which results in total tour costs of €811.20.

The specific tariffs of LSPs may vary in the number, definition, and assignment of zones; the costs for unloading; and the price per load-zone combination. The structure of the tariff scheme is standard in freight forwarding for retailers, as it simplifies the cost calculation of the tours. It allows the costing of a delivery tour without detailed information on the sequence of the stores visited. This simplification significantly reduces the computational complexity that needs to be handled by the planners, as no explicit VRP needs to be solved. However, this also implies that the retailer does not have exact information on the actual tour costs but relies on the costs resulting from the negotiated tariffs. At DIY-R, a strategic business unit is responsible for negotiating long-term tariff contracts with all LSPs.

Furthermore, the assignment of delivery days to each store poses a mid- to long-term decision problem. It depends on the store's demand and its replenishment processes, as well as on fulfillment capacities in the warehouses and transportation. Like the tariff negotiation, the delivery day assignment is owned by a strategic business unit. Both are input (i.e., predetermined parameters) for the operational transportation planning of the joint project with DIY-R.

Operational Tour Planning for Each Delivery Day

DIY-R operates three depots in close proximity, each with a specific assortment and distinctive inventory (i.e., no duplicate inventories). Consequently, a store may submit up to three different orders for its assigned delivery days. Stores may order from all three depots, but also from just one or two depots. If necessary, the LSP consolidates the orders from the three depots before starting the delivery tour. The orders must be submitted by the stores three days ahead of the delivery day and include all required information on the products and volumes. Once all orders are submitted, the operational tour planning by the operational business unit starts. The orders scheduled for each weekday build the

Table 1. Extraction from a Tariff Scheme Example: Costs per Load Unit

	Most remote zone visited of tour				
Total load of tour (units)	Zone 1 (€)	Zone 2 (€)	Zone 3 (€)	Zone 4 (€)	…
5	114.54	124.32	131.30	138.46	…
6	95.45	103.60	109.42	115.38	…
7	81.82	88.80	93.79	98.90	…
8	71.59	77.70	82.06	86.54	…
…	…	…	…	…	…

planning basis. In detail, four full-time employees designated as planners construct the delivery tours. The planning department further subdivides the entire delivery area into five distinct regions. This division eases transportation planning and limits the number of store orders that need to be handled by each planner. This also means that each store belongs to one of the planning regions. One planner is responsible for a specific region and solves the daily routing problem separately for this region while communicating closely with the corresponding LSPs. The planners collect information about the shipment's content, origin, destination, size, weight, and load units within a database. Based on this information, the supply of all stores is planned for each day. The central aspect of the distribution planning is deciding on the delivery mode: SDT and SSS.

SDT with LSPs. The first and preferred option to supply DIY-R's stores is via delivery tours executed by LSPs, which are generally cheaper than SSS. SDT constitutes a private and exclusive tour of the LSP in which only stores of DIY-R are approached. The manual tour construction logic at DIY-R follows three subsequent steps: (a) allocating store orders to tours, (b) checking duration and distance constraints for each tour, and (c) selecting the LSP.

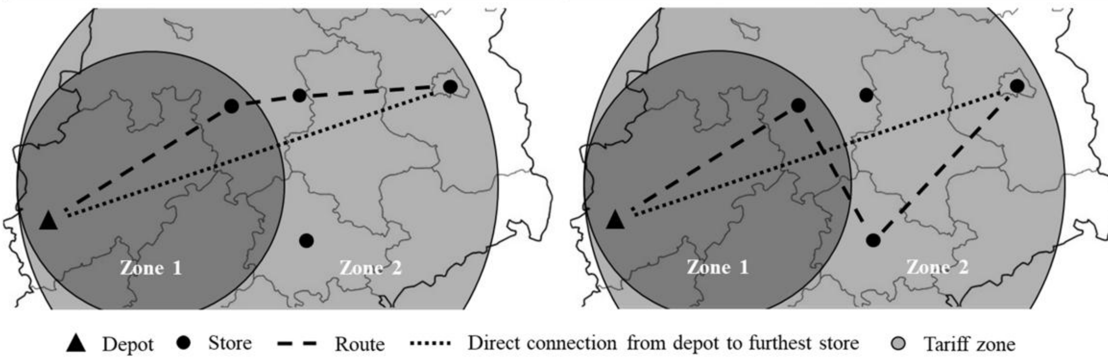
Step (a): Allocating Store Orders to Tours. In the allocation phase, the planners assign store orders gradually to tours based on the proximity of store locations. The main goal of a planner is to achieve high vehicle utilization to exploit the volume discounts of the tariff scheme. The vehicle capacity is checked with the assignment of each additional store order to a tour. There are two types of vehicles with different capacities. The larger truck can hold 34 load units, whereas the smaller one carries only up to 17. Some stores in urban areas with vehicle width, height, and weight restrictions require deliveries by smaller trucks. The planners therefore need to ensure delivery to each store via a suitable vehicle. Furthermore, each store may order from the three different depots. With regard to the route planning for SDT, only the last visited depot before deliveries is decision-relevant, as depots lie in close proximity and thus within the same starting zone across all LSPs. The depot from which the actual delivery tour starts together with the LSP selected determines the tariff scheme. Potential milk runs for order consolidation (i.e., approaching multiple depots) are priced in the tariff schemes. Usually, the orders of one store from different depots are combined on one tour to save transportation costs. Furthermore, time windows are not decision-relevant in our application, mainly because the stores are only delivered on predetermined delivery days, and just-in-time delivery and instant replenishment are usually not necessary.

Step (b): Checking Tour Duration and Distance Constraints. After all the tours are built in Step (a), they are checked in Step (b) concerning tour duration and distance restrictions. To do this, the tours' estimated distance and tour duration are assessed using a general online map service. The tour duration restrictions are based on governmental regulations (e.g., Directive 2003/88/EC of the European Union) that limit the working time of each driver. The tour duration restrictions comprise the driving time and the corresponding service times at all visited locations. Furthermore, because the costs of a tour do not explicitly depend on driven distance, LSPs impose restrictions on the tour length by limiting the out-of-tour distance or detour. This out-of-tour distance measure is widely used in freight forwarding practice and is defined as follows. The assignment of a store to a tour increases the driving distance. This increase in driving distance may not result in a proportional cost increase for the retailer if no new and higher zone (i.e., a more remote and costly zone) is affected, but only the higher volume is billed. For instance, adding a store order of Zone 1 does not increase the distance-related costs if a store order of Zone 2 is already on the tour. Tour costs therefore increase only with respect to the total load, and the additional distance that needs to be covered comes free of charge. This complimentary distance increase that must be covered by the LSP can be considered a "detour" and is limited by the LSPs with a so-called detour factor. In the case of DIY-R, LSPs apply a relative detour factor. It indicates the maximum percentage the distance of a tour may deviate from the direct connection between the depot and the furthest stop—that is, the zone relevant for the pricing (Lindsey et al. 2013, Khodabandeh et al. 2021). The restriction ensures that no extensive tours are constructed that exploit the weaknesses of the zone-based tariff. The relative detour factor fits the zone-based tariff scheme as the possible detour increases with the integration of stores in more remote zones.

As the detour limitation is essential in our application, we will further illustrate it using a simple example. Figure 1 represents two potential delivery tours. Both tours start from the same depot (triangle) in the West, visit stores (dots) in the Northwest, and end at the identical and most distant store in the East. Tour A visits a further store in the Northeast, whereas Tour B visits another store in the South. According to the tariff scheme, both tours have identical costs according to the zone-based tariff when assuming an equal load. However, the shortest route from the depot visiting all stores differs significantly. The corresponding detours calculated amount to 4% for Tour A and 28% for Tour B. LSPs apply an upper limit for the relative detour. If the

Figure 1. Calculation of the Relative Detour Factor (Example)

Example Tour A: Relative Detour Factor	Example Tour B: Relative Detour Factor
$\frac{Route\ A}{Direct\ Connection\ A} = \frac{498\ km}{477\ km} = 1.04$	$\frac{Route\ B}{Direct\ Connection\ B} = \frac{612\ km}{477\ km} = 1.28$



upper limit is 20%, this will likely result in the LSP's rejection of Tour B. If a tour exceeds the detour limit during the feasibility check by the planner, the tour needs to be adapted, and further iterations are required. The orders affected must be reassigned to different tours or delivered via SSS with common carriers (see *SSS with Common Carriers*).

Step (c): LSP Selection. In the final step of determining the tours with SDT, LSPs are selected based on costs for the tours and the availability of vehicles. This means the planners calculate the costs for each tour obtained from Steps (a) and (b) based on the LSPs' zone-based tariffs. The lowest-cost LSP for a tour is identified based on spreadsheet calculations considering the individual tariff parameters of all available LSPs in a region. Furthermore, the planners must adhere to the LSPs' vehicle availability, meaning that selecting the cheapest LSP is not always possible. After the cost calculation, the final tours are communicated to the selected LSPs, which evaluate the tours in their interest and provide feedback on whether tours are accepted. Each planner follows Steps (a) to (c) sequentially and determines tours only for their delivery area. There is no continuous reoptimization between the different steps.

SSS with Common Carriers. The second mode (SSS) to supply DIY-R's stores is subcontracting single shipments to common carriers. This delivery mode does not require tour building by the retailer and only concerns single orders. The costs of a single shipment depend on the shipments' origin, destination, and load units, and they increase linearly with load and distance. The resulting costs per load unit usually exceed the costs by SDT as long as sufficient capacity utilization is achieved. SSS should only be used in exceptional cases if tours cannot be built efficiently. This applies to small order sizes, for example, that do not justify an additional stop on a tour. Furthermore, the second delivery option eases the process for the planners. They may utilize this option to achieve feasibility if the tour capacity is exceeded or if adding a store to a tour would result in a large detour or violate the tour duration constraint. Common carriers consolidate these deliveries with further deliveries from other customers and perform the delivery to DIY-R stores. This means that the common carriers can usually achieve higher truck utilization for these deliveries. However, the collective shipments still allow competitive costs for the retailer.

Inefficiencies and Improvement Potentials

The business process involving different departments at DIY-R strongly depends on legacy processes, decision owners, and human factors such as experience and intuition. Consequently, the prevailing planning process poses some inherent inefficiencies. We highlight shortcomings and improvement opportunities of this process, which we then address in Section *Solution Method*. The planners communicate closely with the LSPs in their region. Although this allows the gathering of LSP- and region-specific factors, it also implies dependencies and causes problems if a planner is unavailable (e.g., because of illness, holidays, or quitting). It also does not allow optimization across regions, as each planner only attends to their area. This approach results in region-specific planning solutions and inconsistencies across the regions. A further primary driver of the inefficiencies lies in complexity reduction through splitting the entire distribution problem into separate, sequentially solved subproblems (see Steps (a) to (c)) without reoptimization. Additionally, decisions made in the process mostly rely on the experience or intuition of individual planners. There is no control instance or indicator for the cost efficiency of tours planned. Last, manual planning consumes considerable workforce and time and may include human errors that require additional tours or replanning efforts.

Our decision support tool DSS-DR helps to overcome these drawbacks. DSS-DR integrates the different planning steps and achieves a cost-minimal solution for daily distribution planning. This solution also ensures the acceptance of all tours because we directly integrate tour duration and detour restrictions into the tour building. The planning solutions are coherent across all days and regions and no longer depend on individual employees' experience and decision making. DSS-DR covers the complete planning problem across all regions and thus increases overall savings via a concerted planning approach.

Problem Classification and Related Literature

Before we can relate the business problem described to the literature, we need to specify the underlying VRP to define the scope of the related research. The transportation problem and its application at DIY-R have a large number of problem specifics and constraints resulting in a Rich VRP (R-VRP) (see also reviews, taxonomies, and frameworks of, e.g., Crainic et al. 2009, Vidal et al. 2013, Caceres-Cruz et al. 2014, Vidal et al. 2014, Lahyani et al. 2015). Specifically, our daily operational routing problem resembles a VRP with multiple depots, in which each depot has a different assortment. For one tour, multiple depots might have to be visited before deliveries start. This setting can also be seen as a special case of a Pickup and Delivery Problem (Savelsbergh and Sol 1995), in which all pickups occur before the deliveries. Stores in densely populated, urban environments require smaller vehicles for delivery, whereas other stores can be supplied via large trucks. This requirement results in a heterogeneous fleet (HF) of differently sized vehicles. The tariff scheme does not require the vehicles to return to the start depot, turning the problem into an open (O) VRP. External LSPs deliver store orders via two different modes such that the routing problem resembles a VRP-PC, with a private fleet (for the exclusive DIY-R tours) and common carriers. Multiple LSPs or carriers are available in both modes, necessitating a carrier selection. The costs are dependent on a nonlinear zone-based tariff with volume discounts. Tours are restricted by vehicle capacity, tour duration, and distance. The distance restriction is achieved by limiting the relative detour. Following these problem specifics, the underlying planning problem can be denoted HF-O-VRP-PC with multiple depots. In the following, we first highlight related literature on R-VRPs that have a similar setting and scope but apply linear cost functions. Second, we provide an overview of related (R-)VRPs with nonlinear costs. Finally, we summarize the contributions in Table 2 and discuss open areas of research.

Related R-VRPs with Linear Cost Functions

The related R-VRPs are based on similar applications in which tours with multiple depots and tour duration restrictions must be built. However, the problems are based on linear cost functions. Sprenger and Mönch (2012) study an R-VRP arising in the German food industry, including multiple delivery modes in which multiple manufacturers share their fleets. Mancini (2016) solves a related R-VRP of an LSP featuring a heterogeneous fleet. Their VRP variant includes multiple depots, but not every customer can be served by every vehicle or from every depot. Alcaraz et al. (2019) address a similar R-VRP of an LSP with a heterogeneous fleet, additionally considering two delivery modes. They propose heuristics handling outsourcing decisions, and the tours include driving and rest periods according to European regulations. The introduction of the second delivery mode leads to cost savings of 3%–7%. Finally, Kramer et al. (2019) study the case of an LSP delivering pharmaceutical products to healthcare facilities in Italy. Similar to our application, they deal with incompatibilities between vehicles and customers and, hence, a heterogeneous fleet.

(R)-VRPs with Nonlinear Cost Functions

One of the aspects of the application case that is most influential on the problem's complexity and the solution method's design is the nonlinear cost function resulting from the zone-based tariff and the volume discounts. In this paragraph, we therefore focus on the limited literature on (R)-VRPs with nonlinear cost functions, similar to the case of DIY-R. Ceselli et al. (2009) propose an exact algorithm for a software company providing planning tools for R-VRPs with different extensions. The problem includes a heterogeneous fleet, carrier selection, two delivery modes, open tours, time windows at depots and customers, and order splitting. Tours are restricted by maximum length and duration but neglect detour limitations. The routing costs resemble a zone-based tariff and depend on the locations and number of stops, tour distance, and vehicle load. The costs are also subject to volume discounts. The authors propose an algorithm based on column generation and solve the problem with up to 30 customers. Ceschia et al. (2011) provide a solution approach for a class of R-VRPs addressing a heterogeneous fleet, carrier selection, tour duration restrictions, and two delivery modes. The problem stems from an Italian software solutions provider. The authors propose four different cost functions for the routing problem, including nonlinear functions, which are also applicable to model a zone-based tariff with volume discounts. Stenger et al. (2013) propose a VRP with nonlinear costs. This work considers single or multiple depots and two

Table 2. Overview of Related Literature

		Application			Tariff scheme			Solution approach		
Publication	VRP variant[a]	Industry case[b]	Carrier selection	Multiple depots	Zone tariff	Volume discount	Detour limit	Exact	Heuristic	Size[c]
Sprenger and Mönch (2012)	VRP-PC	✓	—	✓	—	—	—	—	Greedy & ACS	200[d]
Mancini (2016)	HF-VRP	—	—	✓	—	—	—	—	ALNS	75
Alcaraz et al. (2019)	HF-VRP-PC	—	—	✓	—	—	—	—	Spec. Heu.	50
Kramer et al. (2019)	HF-VRP	✓	—	✓	—	—	—	—	MS-ILS	300
Ceselli et al. (2009)	HF-O-VRP-PC	✓	✓	✓	✓	✓	—	Col. Gen.	—	30
Ceschia et al. (2011)	HF-VRP-PC	✓	✓	—	✓	✓	—	—	Tabu	56
Stenger et al. (2013)	VRP-PC	—	—	✓	—	✓	—	—	AVNS	483
Dabia et al. (2019)	HF-VRP-PC	—	—	—	—	✓	—	B&C&P	—	100
Khodabandeh et al. (2021)	HF-VRP-PC	✓	✓	✓	—	—	Relative	—	Spec.Heu.	2,369
Linß and Tamke (2022)	VRP	—	✓	—	—	✓	—	B&C	—	20
This paper	HF-O-VRP-PC	✓	✓	✓	✓	✓	Relative	Set Part.	—	150

Note. ✓, considered; —, not considered; ACS, Ant Colony Search; ALNS, Adaptive Large Neighborhood Search; AVNS, Adaptive Variable Neighborhood Search; Col. Gen., Column Generation; Greedy, Greedy Heuristic; MS-ILS, Multistart Iterated Local Search; Set Part., Set Partitioning Algorithm; Spec. Heu., specialized heuristics; Tabu, Tabu Search.
[a]Only extensions that are present in the case at hand.
[b]Application to real-world problem.
[c]Problem sizes of test instances, in number of customers.
[d]The problem size is given only in number of orders.

delivery modes. The costs of one delivery mode are subject to volume discounts. The authors approximate the nonlinear cost of this delivery mode by a piecewise function and apply a heuristic solution algorithm. Dabia et al. (2019) present an R-VRP with a heterogeneous fleet and two delivery modes. Similar to Stenger et al. (2013), they apply a piecewise linear function to account for volume discounts. The authors present the first exact solution algorithm for this family of problems based on two different branch-and-cut-and-price (B&C&P) formulations. Khodabandeh et al. (2021) provide a general framework for R-VRPs arising at an LSP in North America. The framework comprises carrier selection, multiple depots, multiple delivery modes, and different cost functions. One proposed cost function depends on the start and end of a tour and is nonlinear. The authors mention that the framework can address further tour restrictions, including a maximum travel time, distance, and absolute and relative detour. The authors propose a heuristic solution method based on a set partitioning formulation to solve real problem instances.

Linß and Tamke (2022) propose a VRP with Carrier Selection (VRP-CS) from the corrugated packaging industry in Germany. The authors consider three different cost structures for the routing cost, including nonlinear tariff schemes, in which the cost per load unit depends on the total load carried. The problem is modeled using a three-index formulation and solved exactly with a Branch-and-Cut (B&C) algorithm.

Summary

Table 2 summarizes the related literature, provides additional information on the solution methods and problem sizes, and highlights the contribution of our work.

In the first stream of related literature, we highlight R-VRPs motivated by practice that share related aspects with the problem at hand. All of these papers feature linear cost functions and apply heuristic solution approaches. Despite introducing multiple modes and a carrier selection problem, these models do not factor in zone-based tariffs, volume discounts, or detour limits. In the second stream of literature, we summarize (R-)VRPs with similar problem specifics while featuring nonlinear cost functions. The nonlinear costs are primarily based on related volume discounts. The majority of publications apply multiple modes and carrier selection. With the zone-based tariff scheme, it is essential to consider the detour limitation to ensure tour acceptance by LSPs. However, there is no contribution developing an exact solution approach for a VRP with a nonlinear zone-based cost function and a relative detour limitation. We close this gap and contribute to both literature and industry. This study developed an easy-to-use tool to solve an R-VRP that occurs daily at a major European DIY retailer featuring versatile problem aspects. We solve actual instances optimally with up to 150 stores and 450 store orders.

Solution Method

From a solution perspective, the problem at hand has several challenging properties. First, numerous practical extensions (e.g., heterogeneous fleet, two delivery modes) and constraints (such as vehicle capacity, LSP, and truck availability) must be reflected within the problem formulation. Second, there is a zone-based tariff scheme with volume discounts resulting in nonlinear costs. Third, the problem is characterized by a tour duration constraint and a distance-dependent detour limit. To precisely calculate the tour duration (including service time) and distance, an Open Traveling Salesman Problem needs to be solved for each tour. The orders are available three days ahead, and the manual planning takes more than a full working day, binding the workforce of multiple planners. As sufficient computation time and power are available, the goal of DIY-R is to achieve the best possible solution in the given time frame. Despite the problem size of up to 150 stores per day, the problem structure and the scope allow us to address the aforementioned challenges by developing an exact time-efficient algorithm based on decomposition and problem-specific insights. One predominant modeling approach in this context is the set partitioning formulation, introduced by Balinski and Quandt (1964). The formulation uses variables that represent feasible vehicle tours, and the cost coefficients in the objective function reflect the total costs of a tour. The approach is intuitive and easy to follow for all parties involved (i.e., managers and planners). It applies a transparent computation process that fosters the adoption in practice (see, e.g., Guidotti et al. 2018). This was one central requirement at DIY-R for developing the solution approach; we discuss alternative solution approaches in Section *Discussion of Methodology*.

Components of DSS-DR

DSS-DR operates in a three-step approach to increase traceability and consequently splits the planning problem into three mutually independent components:

1. the construction of a set of feasible tours (*set construction component*),
2. the cost calculation of the resulting tours (*costing component*), and
3. the exact solution of the set partitioning model (*solution component*).

Appendix A details the modeling and solution approach for the DSS-DR. In the following, we highlight the major parts only. In the (1) *set construction component*, we generate the complete set of all feasible tours. However, the solution space can be significantly reduced by taking advantage of the hard constraints in our problem. These constraints imply that tours that exceed tour duration and vehicle capacity do not have to be considered. The efficient generation of tours is crucial, as it is directly related to the number of variables and is thus the main driver for the run time. We introduce a tree-based algorithm with efficient pruning strategies for constructing tour candidates. The algorithm starts with a single store order and gradually adds further store orders to a tour. After each addition of an order to a tour, a routine checks the feasibility of the tour concerning tour duration, vehicle capacity, and detour. Violating the tour duration and capacity constraints leads to excluding the tour from further searches (pruning). The violation of the detour factor does not lead to the pruning of a tour, owing to the dynamic nature of the factor—that is, a further insertion of a store order may reduce the detour factor and result in a feasible tour. This is a major driver of the complexity of the problem and the tree-based algorithm. Every feasible tour is then included in the final set to ensure an exact solution. In the (2) *costing component*, we calculate the total costs of each feasible tour obtained. At this stage, the tool interfaces with DIY-R's database to retrieve the individual zone-based tariffs of the LSPs contracted. The tariffs are LSP-specific and depend on the depot a tour starts from (see Section *Description of the Business Process*). Each feasible tour is then priced for each available LSP and depot. This means we add each tour to the final set of candidates multiple times, once for each LSP and depot available. It does not suffice to add the minimal cost tour across all LSPs, as it might be necessary to select another LSP because of fleet size limitations. Last, in the (3) *solution component*, we solve the set partitioning model on the candidate tour set created with Gurobi (version 10.0.1). The structure of the model is as follows: The objective is to minimize delivery costs. The delivery costs consist of costs for the SDT delivery tours obtained in Component 2 for all LSPs and costs for single shipments with common carriers (SSS). The constraints ensure that all store orders are delivered by one of the modes. Furthermore, an LSP can only execute a delivery tour if sufficient vehicles of the correct type are available. After this step, we obtain the cost-minimal delivery tours and assignments to common carriers. To support the next steps of the planning process, the solutions are visualized via Microsoft Excel spreadsheets, and all delivery tours are further plotted on a map. Alongside the actual delivery tours and the SSS assignment, the spreadsheets include information on the selected LSPs, cost, and load units of each tour and shipment. Finally, the SDT tours and SSS assignments are passed on to the LSPs and common carriers.

Discussion of Methodology

Strengths and Limitations of DSS-DR. DSS-DR constitutes a problem-tailored exact approach for the HF-O-VRP-PC with multiple depots at DIY-R. It achieves optimal solutions to the industry problem with nonlinear zone-based tariffs and its numerous restrictions. The methodology is easily explainable to practitioners because it follows the general structure of the previously applied manual approach at DIY-R. The constrained nature of the industry problem permits the evaluation of all feasible tours and substantially limits computation time spent in the search tree. The algorithm can be extended to incorporate further restrictions. On the downside, DSS-DR does not constitute a general tool for distribution problems with significantly different structures, such as less constrained routing and extensively long tours. It may be transferred to other applications with a zone-based tariff and a similar level of "constrainedness."

Alternative Approaches. R-VRPs are complex problems, and the same holds true for our HF-O-VRP-PC with multiple depots. As such, we evaluated alternative approaches for the implementation of DSS-DR in the course of the project. Unlike for our application, the comparatively high run times may not be acceptable for applications for which fast solutions are needed. In these settings, heuristic approaches may be a reasonable alternative when no optimal solution is required (see, e.g., Hu et al. 2022). Metaheuristic solution algorithms are frequently applied in this context (see, e.g., Montoya-Torres et al. 2015). The underlying problem structure at DIY-R may be suitable for methods that decompose the problem in an iterative manner into smaller subproblems that are solved separately and then merge the subproblems to obtain a complete solution. A further option would be ruin-and-recreate methods (such as an Adaptive Large Neighborhood Search) exchange parts of an initial solution and evaluate the exchange of nodes or arcs within a tour. Heuristics usually convince through run-time efficiency—that is, they often require a fraction of time to reach good solutions. Drawbacks of advanced heuristics are potential nonoptimal solutions or a lack of information about the goodness of a solution. Although their general idea is often easy to follow, the actual implementation may be cumbersome to understand, and heuristic elements (e.g., when the search procedure follows certain nature-inspired patterns or uses random elements) are not intuitive for practitioners and tool users, leading to trust issues with regard to the solution quality. DIY-R therefore explicitly expressed the need for an optimal approach in which functioning and computation steps can be easily communicated to all involved parties. We further considered the use of a branch-and-cut approach and modeled a relaxed version of the application case as a MIP, solved by Gurobi. Yet this approach was only suitable for instances of up to 25 stores. Another state-of-the-art approach is branch-and-price (B&P), which assumes a linear cost function and a convex solution space for an efficient solution of the underlying pricing (shortest path) problem. In our case, most efficiency potential is lost in the resource-constrained shortest path problem because the relative detour restriction prevents the pruning of paths. This is because the inclusion of another store may lead to a lower detour. Moreover, the exact approaches mentioned are based on a complex search strategy that requires a deepened understanding of mathematical solution procedures.

Benefits and Challenges

This section discusses the benefits for the retailer resulting from the use of DSS-DR and the challenges that occurred during the implementation. DSS-DR has been developed with DIY-R in regular feedback loops and joint workshops. The direct input of the planners and the DIY-R optimization department during the development of DSS-DR and their feedback on the results contributed significantly to the project's success. This section provides an overview of the results and a comparison with the status quo, applying DSS-DR. Appendix B containes detailed numerical results.

Improvements via Implementation of DSS-DR

The main benefit of DSS-DR is the structured and comprehensive planning process. The tool determines optimal solutions for the real-world routing problem. Embedding DSS-DR into the operational processes helped to identify inconsistencies in the manual process routines and inefficiencies in the tour determination. Furthermore, it reduces the planning effort and working time required by enabling automated calculations. The easy-to-follow methodology encourages practitioners to occupy themselves with the approach and its results. DSS-DR can be further leveraged for price negotiations with LSPs and applied for overarching strategic planning. We detail the benefits of implementing DSS-DR at DIY-R as follows.

Significant Cost Savings. The total cost savings of introducing DSS-DR can be attributed to distribution cost savings, workforce savings, and further positive impacts on other planning tasks. Total savings exceed €1 million per year. The distribution cost reduction amounts to an average of 8% compared with the status quo at DIY-R. In absolute terms, these are savings of around €750,000 per year (see Tables B.1 and B.2). The significant savings can be explained by the integrated planning approach that enables comprehensive planning across all delivery areas. This contrasts with the prevailing planning, in which each planner was only responsible for one distinct area and for optimizing tours within this area. In particular, tour optimization is a major driver for cost savings as more store orders are distributed via delivery tours, and fewer are sent via costly common carriers (see Table B.3). With the application of DSS-DR, the share of store orders shipped with common carriers decreases from an average of 41%–17%. The planners tended to move non-fitting orders to common carriers and thus used the more expensive SSS to ease the planning. Furthermore, manually planned tours do not fully utilize the available tour duration, as vehicle capacity utilization was often the first binding constraint within the manual planning process (see Table B.4). The exploitation of both tour duration and vehicle capacity saves costs in the tariff scheme because zones—not driving times/distances—are billed and volume discounts apply.

DSS-DR guarantees adherence to all contractual requirements while fully exploiting the given boundaries and the cost-minimization potential. Alongside the immediate impact on tour efficiency, DSS-DR enables a more efficient workflow and process, requiring fewer working hours. Considering four planners were needed for manual daily distribution planning, DSS-DR significantly contributes to streamlined and more time-efficient planning. The average daily planning effort could be reduced by 50% using DSS-DR, which reflects personnel cost savings of about €150,000–€200,000 per year. The supply chain business unit profits from the freed-up working capacities of planners because they can be used to address other optimization tasks, such as optimizing delivery frequencies and inventories. Hence, the project laid the foundation for additional process enhancements and further cost savings.

Unification, Planning Support, and User Acceptance. DSS-DR enables coherent solutions across all weekdays and the entire delivery area. Whereas the routing was determined for each delivery region and day independently and was subject to individual expertise in the past, introducing a tool for the entire planning problem eliminates bias and enables unified distribution planning. DSS-DR ensures the same decision logic for each planning day. Major points for improvement are considering the distribution problem as a whole and assessing every feasible tour instead of dividing the distribution problem into subproblems for each planner in the status quo. The primary differentiator of DSS-DR is therefore to evaluate every possible alternative. Furthermore, analyzing the manually planned tours reveals high variations in the detour proposed by the planners (see Table B.5). In many cases, the actual detour violates the defined detour limitation. A high detour usually results in a rejection of tours by the LSPs, requiring intensive replanning efforts and costs. The detour allowed should be fully exploited for cost-efficient planning, but the maximum detour should always be adhered to. DSS-DR ensures both. DSS-DR interfaces DIY-R's planning software and standard tools such as Microsoft Excel. Consequently, planning data (store orders, costs, etc.) are automatically imported, and output data, including single shipments, delivery tours, and selected LSPs, are automatically generated. Additionally, a graphical representation of the results makes them easier to interpret for planners. Interfaces with other software used at the retailer and graphical representation of the results further contribute to the acceptance of the tool by the planning team. Finally, unlike complex black-box approaches such as B&P or advanced heuristics, the solution procedure developed is easy to understand and follows aligned planning steps familiar to planners.

Adaptability for Future Needs and Applications. DIY-R's manual process strongly depends on hard-to-replace and highly specialized planners who follow legacy patterns. DSS-DR discards many of these dependencies on individual experience and planners' skills by providing a holistic and easy-to-customize framework. This is necessary because of dynamically changing settings. Rapid and frequent changes in markets and supply chains have recently become the norm (e.g., move to omnichannel, lockdown restrictions during the COVID-19 pandemic, inventory rationing due to supply chain issues). Short-term requests to change delivery days and corresponding adaptions of the daily routing problem may cause further disruptions (e.g., due to rush orders and public holidays). Consequently, DIY-R called for an adaptable and flexible approach that is resilient to external changes that impact the entire setting. DSS-DR is based on a solution approach that can be easily customized. Arbitrary objective functions and constraints could be included if needed. DSS-DR only requires an adaption of input data and, for example, updated delivery plans that can be easily integrated into route optimization. This adaptability leads to a tool capable of addressing future organizational changes or industry needs (e.g., different delivery modes and carriers) and builds the basis for future developments (e.g., the introduction of delivery time windows).

Applying DSS-DR for Negotiations and Strategic Planning. The retailer faces challenges due to the zone-based tariff scheme on different hierarchical levels. On a strategic level, specifying and negotiating the cost structure and the specific parameters of the tariff scheme with all LSPs is of interest. However, negotiating tariff costs and structure requires a thorough knowledge of routing costs. DSS-DR provides exact and detailed information of these costs and increases the transparency of possible tariff cost calculations. DIY-R can leverage this information for tariff negotiations. For example, we show that the detour is decreased within the optimal routing solutions (see Table B.5). This results in cost benefits for the LSPs that may be shared with DIY-R. Furthermore, assigning delivery days to each store poses a mid- to long-term decision problem. In this context, DSS-DR may provide decision support to some extent by evaluating a possible changed assignment for single days.

Challenges During the Implementation of DSS-DR

When conducting an optimization project with an industry partner, several challenges must be mastered. In our case, these include identifying the actual planning process and scope, data issues, software development, and the industry partner's specific traceability and flexibility requirements. Although some of these hurdles resulted in deviation from the initially defined timeline, an agile process and regular consultations between all parties significantly helped to overcome all obstacles. The main challenges that we faced are summarized as follows.

Deriving the Explicit Formulation of the Industry Problem and Identifying the Manual Planning Process. A significant challenge that took several months to overcome was the precise specification of the actual decision problem as well as the manual approach of the retailer. The different planners apply skills from their long training to plan delivery tours. A planner may combine certain stores into a tour based on past tours, for example. Experience and intuition lead to decisions that may be hard to verbalize explicitly. DIY-R has not explicitly specified the decision problem and the approach to obtain good solutions. As the manual distribution planning involves different business units and includes numerous limitations and actions, a large amount of implicit knowledge had to be collected and discussed to specify explicit objectives, constraints, steps, and methods. A fundamental difficulty in the clear specification of the problem was the definition of constraints that were previously only intuitively adhered to. This lack of definition led to inconsistent solutions between planners in the manual process. One example of these implicit constraints is the relative detour factor. The factor was not clearly defined in the past, and distance restrictions were incorporated using simple tools for approximation and vague guidelines. The historical tour data highlight that the detours of tours constructed significantly differ between individual planners. A clear definition of the detour allowance is essential to ensure the acceptance of tours by the LSPs and cost-efficient planning. We therefore initiated a process for explicitly defining a binding detour factor by the LSPs. To master the entire tool development process, close collaboration between all teams involved was crucial. All decision-relevant aspects of the problem and the process were analyzed in joint workshops. Once the manual planning process was well defined, the requirements for DSS-DR could be specified. An agile software development process, including regular feedback loops, was established to manage and structure all these tasks. Prototypes were developed, intensively discussed, and tested at certain project statuses with DIY-R. This procedure also revealed shortcomings of the manual process, such as constraint violations and suboptimal rules for assigning store orders to common carriers (SSS).

Data Collection and Interfaces. Some planners relied more or less on their experience—that is, they followed their daily planning routine and did not maintain clean master data. This resulted in coping with poorly accessible, unstructured, and missing data, particularly if the data needed to be retrieved from different systems or if different business units owned them. Some data needed for running DSS-DR were not yet available at DIY-R. For example, no explicit routing in the manual approach at DIY-R meant that the travel distances between stores were not directly available. These data had to be collected via online application interfaces. In general, data for operational tour planning had only been used to a certain extent and were not structured. Consequently, the data could not be used seamlessly as an input to DSS-DR at the beginning of the project. Extensive data cleaning and processing were critical to aligning data inputs. On the other side of the spectrum, the output of DSS-DR needed to be easily understandable and quickly interpretable by the employees of DIY-R. These requirements called for problem-specific interfaces to standard software used at the retailer to integrate DSS-DR into the daily workflow.

Mathematical Complexity of the Decision Problem. Once all procedural hurdles had been mastered and the decision problem had been defined properly, a major challenge was to provide a comprehensive model formulation and solution approach for the problem at hand. Both parts need to be understandable for the users to ensure the acceptance and application of a new tool. DSS-DR was therefore required to map the complex R-VRP of a large retailer, delivering up to 150 stores daily, and needed to be intuitive and easy to grasp at the same time. The set partitioning formulation perfectly suits both requirements because it reduces the problem to the essential decisions, using complete (feasible) tours as input variables for the optimization. From a methodological perspective, the critical requirement was to achieve a preferably optimal solution in the time window between the submission of orders and the delivery phase of three days. Furthermore, the algorithm had to be as traceable as possible to ensure acceptance by the planners. After thoroughly considering all alternatives, we decided on an exact approach that spans three different components, similar to the manual process. We explicitly decided against heuristics and other simplifications, such as restricting the number of stores in one delivery tour, in order to ensure optimality and enable a well-founded cost assessment and comparison. Our approach uses the highly constrained structure of feasible tours to cope with the computational complexity of an exponentially growing number of possible tours as store orders increase. The first component of DSS-DR, a tree-based preprocessing algorithm, uses efficient pruning strategies to avoid exploring nonfeasible tours and significantly reduces total computation times. This enabled us to provide optimal solutions as DIY-R requires and to incorporate all industry-relevant constraints.

Traceability and Acceptance of the Approach. As the team of planners was very confident in their ability to plan efficient delivery tours, one main challenge of the industry project was the acceptance of the software tool. This requires incorporating explicit and tacit knowledge of the planners in the tool so that the planners' decision-making calculus is sufficiently mirrored and planners' decision making is efficiently supported. We considered high transparency of the decision making within the tool and traceability of the algorithm as a primary goal from the beginning of the collaboration. Our project demonstrated that the more the newly developed process is based on the manual planning practice concerning objectives and constraints, the higher the willingness is to replace a manual planning solution with an automated tool. DSS-DR therefore has a very similar logic to the manual planning process. First, tours are built. Second, tour costs are calculated. Third, final tours and LSPs are selected. The solution steps are thus not a black box to the planners. DSS-DR, by name, is meant as a decision support tool and not as a replacement for the manual approach. To master exceptional challenges occurring in the daily process, the planners' multiyear experience is required. This means planners can evaluate different scenarios with DSS-DR.

Summary and Conclusion

We developed and implemented the DSS-DR tool to optimally solve the R-VRP of a European DIY retailer delivering to up to 150 stores per day. The problem includes a zone-based tariff scheme with volume discounts, two delivery modes, detour, and tour duration restrictions. Further extensions include a heterogeneous fleet and carrier selection. DIY-R previously solved this daily planning problem using a manual approach that was strongly reliant on implicit factors such as the experience and intuition of a team of planners. We defined the daily distribution problem and the manual solution process with DIY-R. Working from this definition, we built a decision support tool based on a three-component set partitioning algorithm. It provides optimal solutions to the problem in sufficient time while adhering to the decision logic of the planning team. The underlying algorithm of DSS-DR can be accelerated by using advanced pruning techniques for the tree search. This trades comprehensibility for algorithmic efficiency (Buijs et al. 2016). The first component of DSS-DR generates all feasible tours. The second component calculates the cost of these tours according to the zone-based tariff scheme. The third component uses a commercial solver to select the delivery mode for each store order. DSS-DR interfaces relevant applications for input and output and ensures a user-friendly application and seamless integration into DIY-R's planning.

The industry project led to cost savings of more than €1 million per year, strict adherence to all specified requirements such as detour and driving time limitations, and automation of the whole process. The three-component structure provides easy adaptability to future needs and ensures comprehension and high acceptance by the planning department. The solution to the operational problem enables future projects to improve the decision processes. On the tactical level, a more efficient allocation of stores to weekdays is of interest. On a strategic level, renegotiating the tariff scheme is an open topic. As DSS-DR enables thorough planning and a detailed assessment of routing costs, this knowledge can be leveraged to evaluate costing within existing tariff schemes. In the long run, regular evaluation of the costs of the two delivery modes enables the retailer to compare prices offered by LSPs with the actual delivery costs. The study confirms that the application of operations research methods to real-world problems can lead to significant benefits including large cost savings as shown in other contributions (Fadda et al. 2018, Holguín-Veras et al. 2018, Khodabandeh et al. 2021).

Acknowledgments

The authors thank the industry partner who provided empirical data, practical expertise, and insights and encouraged the publication of this research topic. The authors especially thank the planning team for their support, the codevelopment of DSS-DR, and their exemplary collaboration effort. The project would not have been possible without this close collaboration. The authors thank the supply chain business unit for their valuable feedback on the project and the manuscript. The authors thank the editor Alexandra Newman, the associate editor, and the review team for their valuable suggestions and guidance to improve the paper.

Appendix A. DSS-DR Implementation and Components

Appendix A details the three components of the solution approach, namely (1) *the set construction component*, (2) *the costing component*, and (3) *the solution component* for the optimal allocation of final tours to LSPs and single shipments to common carriers. We describe the integration of the single components in DSS-DR in the pseudocode of *Algorithm* A.1. The notation used is summarized in Table A.1.

1. Set construction component

We use a tree-based structure to construct the set of all feasible vehicle tours (see Figure A.1). The algorithm preprocesses all feasible tours and reduces the solution space by efficiently pruning parts of the tree at the same time. The construction process is initiated for each depot d, and each tour begins at this depot. In the first step, the algorithm appends every store order candidate k of the set of store orders K_d of depot d (with $K_d \subseteq K$). In the following

Table A.1. Notation for the Set Partitioning Model

Index sets	
D	Set of depots
I	Set of feasible tours
I_k	Set of feasible tours that include store order k
I_v	Set of feasible tours that must use a vehicle of type v
J	Set of LSPs for delivery tours
K	Set of store orders
K_d	Set of store orders at depot d
N	Set of common carriers for single order shipments
V	Set of vehicle types
Parameters	
U_{jv}	Total number of vehicles of type v available for LSP j
Cost parameters	
c_{ij}	Costs for subcontracting delivery tour i to LSP j
c_{kn}	Costs for subcontracting single store order k to common carrier n
Decision variables	
x_{ij}	Binary: 1, if a tour i is subcontracted to LSP j; 0 otherwise
y_{kn}	Binary: 1, if a store order k is delivered with common carrier n; 0 otherwise

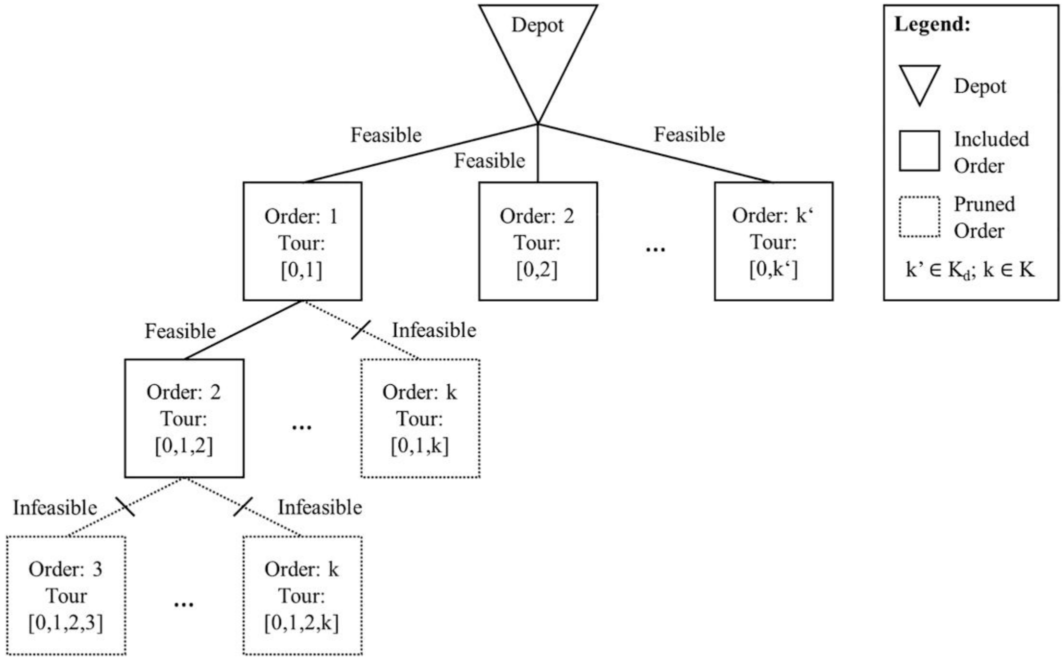
steps, all resulting tours are further appended by the remaining store order candidates from the entire set of store orders K and checked for feasibility. As each store order k is uniquely associated with a certain depot d, traveling from and to this depot is also considered. The implemented algorithm repeats this process until no more feasible tours can be constructed.

There are three criteria for feasibility: The tour must not exceed the capacity limit of the largest applicable truck for the store order, the tour duration, and the relative detour. To prove adherence to the tour duration and detour restrictions, the tour length must be calculated at each node. The shortest distance for a tour is calculated by solving an Open Traveling Salesman problem with a dynamic programming algorithm (Held and Karp 1962). If a tour becomes infeasible because of the vehicle capacity or tour duration constraint, the tour is not further extended with additional store orders, and the branch is pruned. Owing to the possibility of feasible supersets of infeasible tour sets resulting from the relative detour restriction, a violation of the detour constraint only results in excluding the considered tour from the final set. In this case, no pruning can be performed.

2. Costing component

We calculate the tour costs c_{ij} of each feasible tour $i, i \in I$ from (1) according to the specific zone-based tariff schemes for each possible LSP $j, j \in J$, where I denotes the set of feasible

Figure A.1. Representation of the Tree-Based Preprocessing Algorithm



tours and J the set of potential LSPs for delivery tours. In *Algorithm* A.1, the function Costs() invokes this component.

3. Solution component

We formulate the problem as a Set Partitioning Problem to select the optimal set of feasible tours (see similar approaches, e.g., in Balinski and Quandt 1964, Agarwal et al. 1989). In the Set Partitioning Model, the variables represent tours executed by an LSP and assignments of single shipments to common carriers. The corresponding cost parameters reflect the total costs of an LSP tour and a common carrier shipment. This structure ensures a flexible and easy-to-extend model. In the *Algorithm* A.1, the function SP() represents the Set Partitioning Formulation.

$$Min \sum_{i \in I} \sum_{j \in J} c_{ij} \cdot x_{ij} + \sum_{k \in K} \sum_{n \in N} c_{kn} \cdot y_{kn} \tag{A.1}$$

subject to

$$\sum_{i \in I_k} \sum_{j \in J} x_{ij} + \sum_{n \in N} y_{kn} = 1 \qquad \forall k \in K \tag{A.2}$$

$$\sum_{i \in I_v} x_{ij} \leq U_{jv} \qquad \forall j \in J, v \in V \tag{A.3}$$

$$\sum_{i \in I} x_{ij} \leq \sum_{v \in V} U_{jv} \qquad \forall j \in J \tag{A.4}$$

$$x_{ij} \in \{0,1\} \qquad \forall i \in I, j \in J \tag{A.5}$$

$$y_{kn} \in \{0,1\} \qquad \forall k \in K, n \in N \tag{A.6}$$

Objective Function (A.1) minimizes the total transportation costs. These costs include the costs for delivery tours (SDT) and single shipments with common carriers (SSS). Next to the sets I, J, and K introduced above, there are four further sets. Set N denotes all common carriers, and set V includes all vehicle types. Set I_k holds all feasible tours that include store order k, and set I_v includes all feasible tours for vehicle type v. The binary variable x_{ij} is one if tour i served by LSP j is selected and zero otherwise. The costs for directly shipping store order k by common carrier n are represented by the parameter c_{kn}. The amount depends on the order volume and travel distance of the store k and is externally given by the common carriers. The binary variable y_{kn} is one if a store order k is directly shipped by common carrier n and zero otherwise. Constraints (A.2) ensure that each store order is either fulfilled via a delivery tour or shipped with a common carrier. Constraints (A.3) and (A.4) enforce truck availability restrictions. Each LSP has a limited number of differently sized trucks that can be used for fulfilling tours. The maximum number of available trucks of type v of an LSP j is denoted by U_{jv}. Constraints (A.5) and (A.6) determine the domains of the variables.

Pseudocode of DSS-DR

The pseudocode of *Algorithm* A.1 summarizes the functioning of DSS-DR and the three components described above. The code delineates the entire solution procedure. First, the function Generate_tours() generates all feasible tours. Second, the function Costs() prices the feasible tours. Last, the function SP() solves the Set Partitioning Model to determine the cost-optimal delivery plan.

Algorithm A.1. (DSS-DR Pseudocode)

```
Final_tour_set = empty set //feasible delivery tours
Final_tour_set_with_costs = empty set //feasible delivery tours with costs
D = number of Depots
for d = 1 to D do
    Final_tour_set = Final_tour_set.add(Generate_tours(d))
end for
Final_tour_set_with_costs = Costs(Final_tour_set)
Solution = SP(Final_tour_set_with_costs)
return Solution
function Generate_tours(d) //preprocessing algorithm
    Evaluation_tour_set = feasible single-store-order tours from depot d //tours for evaluation
    Final_tour_set_d = empty set //feasible tours starting at depot d
    |K| =total number of store orders
    while Evaluation_tour_set not empty do
        tour = Evaluation_tour_set.get_some_tour()
        Evaluation_tour_set.remove_tour(tour)
        M = highest store order ID of tour
        if M not equal to |K| then
            for O = M + 1 to |K| do
                succ_tour = tour.add_store_order(O)
                if succ_tour.get_shortest_duration() ≤ duration_restriction then
                    if succ_tour.get_capacity() ≤ capacity_restriction then
                        if succ_tour.get_shortest_detour() ≤ max_ detour then
                            succ_tour.add_missing_depots()
                            Final_tour_set_d.add(succ_tour())
                        end if
                        Evaluation_tour_set.add(succ_tour)
                    end if
                end if
            end for
        end if
    end while
return Final_tour_set_d
end function
```

Appendix B. Computational Results

To compare the manual planning approach at DIY-R with DSS-DR, we present the results of both approaches for an exemplary planning week at DIY-R. We use a personal computer with an AMD 5950X processor and 64 GB of memory for all computations. Table B.1 shows that DSS-DR achieves delivery cost savings of up to 10% per planning day and, on average, 8% across the entire week for the defined detour factor of 1.2. In absolute terms, the savings amount to around €750,000 per year for transportation costs.

Table B.2 presents a run-time analysis of our approach across all days of an exemplary week at DIY-R. The table divides the run time into the solution approach's three components. Generally, the first component is the most time-critical. "Wednesday" is a specific delivery day with many small orders and the highest demand, whereas

Table B.1. Comparison of the Manual Process and DSS-DR Regarding the Relative and Absolute Daily Savings in Transportation Costs

Weekdays	Absolute savings[a] (€)	Relative savings[b] (%)
Monday	2,481	6.6
Tuesday	3,583	10.2
Wednesday	3,605	7.5
Thursday	1,547	8.6
Friday	3,086	7.0
Total week	14,303	7.8

[a]Absolute savings using DSS-DR vs. manual approach.
[b]Relative savings using DSS-DR vs. manual approach.

Table B.2. Computation Times for the Three Components of DSS-DR, in Seconds

	Number		Computation time (sec.)				
Weekdays	Stores	Orders	Component 1 (Construction)	Component 2 (Costing)	Component 3 (Solution)	Total time	Number of tours (mil.)[a]
Monday	124	381	15,940	256	249	16,445	1.794
Tuesday	96	297	24,412	452	419	25,283	3.144
Wednesday	143	438	88,354	2,153	2,060	92,567	11.214
Thursday	66	207	348	19	9	376	0.110
Friday	123	378	14,861	324	322	15,507	2.245
Avg. week	110	340	28,783	640	611	30,036	3.701

[a]Tours resulting from the first component of DSS-DR.

Table B.3. Comparison of the Manual Process and DSS-DR Regarding the Number of Tours (SDT), Average Number of Stops per Tour, and Percentage of Store Orders Delivered by Common Carriers (SSS)

	Manual approach			DSS-DR		
Weekdays	Number of tours[a]	Avg. # of stops	Share of SSS (%)	Number of tours[a]	Avg. # of stops	Share of SSS (%)
Monday	27	2.15	39.0	33	2.33	16.0
Tuesday	32	2.31	40.7	41	2.51	13.1
Wednesday	12	2.08	45.0	15	2.33	16.8
Thursday	30	2.27	39.1	38	2.45	21.8
Friday	30	2.33	38.4	35	2.60	17.2
Avg. week	26.2	2.23	40.5	32.4	2.44	17.0

[a]Number of SDT delivery tours.

Table B.4. Comparison of the Manual Process and DSS-DR Regarding Average and Maximum Durations of Delivery Tours (SDT), in Minutes

	Manual approach tour duration		DSS-DR tour duration	
Weekdays	Average	Maximum	Average	Maximum
Monday	144	266	198	479
Tuesday	121	281	154	432
Wednesday	142	265	195	471
Thursday	115	220	150	430
Friday	147	268	217	462
Avg. week	134	261	183	455

"Thursday" is usually a day with lower delivery volume and orders. The run-time performance was obtained using a single core, and no parallelization of the tree search was applied.

Table B.3 shows a comparison of the manual approach and DSS-DR concerning the number of tours (SDT), average number of stops per tour, and percentage of store orders delivered by common carriers (SSS). The manual

Table B.5. Comparison of the Manual Process and DSS-DR Regarding Average and Maximum Detours of Delivery Tours (SDT)

Weekdays	Manual approach detour of tours (%)		DSS-DR detour of tours (%)	
	Average	Maximum	Average	Maximum
Monday	7.9	23.8	8.1	19.3
Tuesday	8.9	24.1	7.8	19.9
Wednesday	10.8	43.3	6.1	19.7
Thursday	7.3	35.0	7.1	18.1
Friday	7.7	23.4	8.6	19.9
Avg. week	8.5	29.9	7.5	19.4

approach relies more on single shipments (SSS), whereas DSS-DR plans more delivery tours (SDT).

Table B.4 shows that while the manual solution underutilizes the maximum tour duration of 480 minutes with a maximum duration of 281 minutes, the optimized approach with DSS-DR plans tours with a duration of up to 479 minutes. This indicates that in the manual approach, planners tend to build smaller tours, which may be ascribed to the high manual planning effort of building larger tours with more stores.

The detours reveal a different story (see Table B.5). On average, the manual approach constructs tours with about 8.5% of detour, whereas DSSDR builds tours with about 7.5% of detour. This lower average detour means that the tours are more cost-efficient for the LSPs. These efficiency gains could potentially become further savings for DIY-R. DSS-DR strictly adheres to the given detour limit of 20%; the manual approach exceeds this limit in 10.7% of the tours created. The highest relative detour is more than 40%.

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Verification Letter

INFORMS has a verification letter on file but it is confidential.

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